(iv)(A) ⇒ (D) ⇒ (B). In view of $(2.12)_2$ and $(2.8)_2$, it is evident that $(A) \Rightarrow (D)$. Now, if (D) holds good, then defining

$$\widetilde{D}_x = f(T_x M^{\perp}); \qquad x \in M,$$

in $TM^{\perp}$ and $\overline{D}$ as the complementary subbundle orthogonal to $\widetilde{D}$ in $TM^{\perp}$, we can show that $\varphi(\overline{D}_x) \in T_x M$ and $\varphi(\widetilde{D}_x) = \widetilde{D}_x$ for each $x \in M$, which proves that $(D) \Rightarrow (B)$.

(v)(A) ⇒ (F) ⇒ (B). The proof of $(A) \Rightarrow (F)$ is given in [1]. For $(F) \Rightarrow (B)$, let (F) be true. Defining $\overline{D}$ and $\widetilde{D}$ on $TM^{\perp}$ by projection operators

$$P = I + f^2; \qquad Q = -f^2$$

respectively, it follows that for each $N \in \Gamma(TM^{\perp})$

$$\varphi N + \varphi(f^2 N) = \pi N + fN + (\pi \circ f^2)N + f^3 N = \pi N \in \Gamma(TM),$$

$$\varphi(-f^2 N) = -f^2(fN) \in \Gamma(\widetilde{D}).$$

Thus $(F) \Rightarrow (B)$. This completes the proof of the theorem.

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CURVATURE THEORY OF RECURRENT HAMILTON SPACES WITH GENERALIZED MIRON'S d-CONNECTION

BY

IRENA ČOMIĆ

Introduction.

Let m be an n dimensional and E^* an $n{+}m$ dimensional differentiable manifold and let (E^*, π^*, M) be a dual vector bundle, such that $\pi^*(E^*) = M$. If $u \in E^*$, then $u = (x^1, \dots, x^n, p_1, \dots, p_m) = (x^i, p_a)$. In this paper we shall use two kinds of indices:

$$i, j, h, k, l, m = \overline{1,n} \text{ and } a, b, c, d, e, f = \overline{1,m}.$$

The Einstein summation convention will be used for both kinds of indices. We shall consider the following transformation of coordinate system. If $(x^{i'}, p_{a'})$ are the coordinates of the same point u in the new coordinate system, then

$$(1.1) \qquad (a) \quad x^{i'} = x^{i'}(x^1, \dots, x^n), \ \text{rank } [\partial x^{i'}/\partial x^i] = n$$
$$(b) \quad p_{a'} = M^a_{a'}(x^1, \dots, x^n) p_a, \ \text{rank } [\partial p_{a'}/\partial p_a] = m.$$

If (1.1) is valid, then exists an inverse transformation

$$(1.2) \qquad (a) \quad x^i = x^i(x^{1'}, \dots, x^{n'}) \qquad (b) \quad p_a = M^{a'}_a(x^{1'}, \dots, x^{n'}) p_{a'}$$

The elements $\partial_i = \partial/\partial x^i$ and $\partial^a = \partial/\partial p_a$ of $T(E^*)$ are transforming in the following way

$$(1.3) \qquad \begin{bmatrix} \partial_j \\ \partial^a \end{bmatrix} = \begin{bmatrix} \frac{\partial x^{i'}}{\partial x^i} & (\partial_i M^a_{a'}) p_a \\ 0 & M^a_{a'} \end{bmatrix} \begin{bmatrix} \partial_{i'} \\ \partial^{a'} \end{bmatrix}$$

$$(1.4) \qquad \begin{bmatrix} \partial i' \\ \partial^{a'} \end{bmatrix} = \begin{bmatrix} \frac{\partial x^j}{\partial x^{i'}} & (\partial_{i'} M^{b'}_b) p_{b'} \\ 0 & M^{a'}_b \end{bmatrix} \begin{bmatrix} \partial_j \\ \partial^b \end{bmatrix}$$

Substituting (1.4) into (1.3) we obtain

$$(1.5) \qquad (a) \quad \frac{\partial x^{i'}}{\partial x^i} \frac{\partial x^j}{\partial x^{i'}} = \delta^j_i \qquad (b) \quad M^a_{a'} M^{a'}_a = \delta^a_b$$

$$(c)\quad (\partial_{i'} M_b^{b'}) p_{b'} \frac{\partial x^{i'}}{\partial x^i} + (\partial_i M_{a'}^a) p_a M_b^{a'} = 0.$$

(1.5c) is the consequence of (1.3) and (1.5b).

Any vector $X \in T(E^*)$ is gived by

$$x = X^i \delta_i + X_a \partial^a \tag{1.6}$$

where $\{\delta_j, \partial^a\}$ is the adapted basis in $T(E^*)$ defined by

$$\delta_i = \partial_i - N_{ai}(x,p)\partial^a \tag{1.7}$$

and $N_{ai}(x,p)$ are the coefficients of nonlinear connection. Under coordinate transformation (1.1) they transform in the following way

$$N_{a'j'}(x',p') = M_{a'}^a(x) \frac{\partial x^i}{\partial x^{i'}} N_{aj}(x,p) - p_a \frac{\partial M_{a'}^a(x)}{\partial x^j} \frac{\partial x^i}{\partial x^{i'}}.$$

Now we have

$$(a)\ \delta_j = \delta_{j'} \frac{\partial x^{i'}}{\delta x^i},\quad (b)\ X^i = X^{i'} \frac{\partial x^i}{\partial x^{i'}},\quad (c)\ X^a = X^{a'} M_{a'}^a. \tag{1.8}$$

The linear connection ∇ in $T(E^*)$ with the usual properties is defined by

$$\nabla_{\delta_i}\delta_j = F_j{}^k{}_i \delta_k + F_{jci}\partial^c \qquad \nabla_{\delta_j}\partial^a = F^{ak}{}_j \delta_k + F^a{}_{cj}\partial^c \tag{1.9}$$

$$\nabla_{\partial^a}\delta_j = C_j{}^{ka}\delta_k + C_{jc}{}^a \partial^c \qquad \nabla_{\partial^a}\partial^b = C^{bka}\delta_k + C^b{}_c{}^a \partial^c.$$

If $X, Y \in T(E^*)$, where X is given by (1.6) and $Y = Y^j\delta_j + Y_b\partial^b$, then

$$\nabla_X Y = (Y^k_{|i} X^i + Y^k|^a X_a)\delta_k + (Y_{c|i} X^i + Y_c|^a X_a)\partial^c,$$

where

$$\begin{aligned} &(a)\quad Y^k_{|i} = \delta_i Y^k + F_j{}^k{}_i Y^j + F^{bk}{}_i Y_b,\\ &(b)\quad Y^k|^a = \partial^a Y^k + C_j{}^{ka} Y^j + C^{bka} Y_b,\\ &(c)\quad Y_{c|i} = \delta_i Y_c + F_{jci} Y^j + F^b{}_{ci} Y_b,\\ &(d)\quad Y_c|^a = \partial^a Y_c + C_{jc}{}^a Y^j + C^b{}_c{}^a Y_b. \end{aligned} \tag{1.10}$$

The adapted basis in $T^*(E)$ is $\{dx^i, \delta p_a\}$, where

$$\delta p_a = dp_a + N_{ai}(x,p)dx^i.$$

The bases $\{\delta_i, \partial^a\}$ and $\{dx^i, \delta p_a\}$ are dual to each other.

For the recurrent Hamilton space, i.e. where the field $\lambda(x,p) = \lambda_k dx^k + \lambda^a \delta p_a$ and the metric tensor are connected by the following relations

$$g_{ij|k} = \lambda_k g_{ij}, \qquad g^a_{i|k} = \lambda_k g^a_i, \qquad g^{ab}{}_{|k} = \lambda_k g^{ab},$$

$$g_{ij}|^c = \lambda^c g_{ij}, \qquad g^a_i|^c = \lambda^c g^a_i, \qquad g^{ab}|^c = \lambda^c g^{ab},$$

the connection coefficients and the components of the torsion tensor are obtained in [8].

2.Ricci identities in recurrent Hamilton spaces.

As usual we shall use the notation

$$R = R(X,Y)Z = \nabla_X\nabla_Y Z - \nabla_Y\nabla_X Z - \nabla_{[XY]}Z = R^k\delta_k + R_c\partial^c. \tag{2.1}$$

Since $X, Y, Z \in T(E^*)$ we can write

$$(a)\quad X = X^i\delta_i + X_a\partial^a \qquad (b)\quad Y = Y^j\delta_j + Y_b\partial^b \tag{2.2}$$
$$(c)\quad Z = Z^k\delta_k + Z_c\partial^c.$$

Using the properties of linear connection ∇ we have

$$\begin{aligned} \nabla_Y Z =& \nabla_{(Y^j\delta_j + Y_b\partial^b)}(Z^k\delta_k + Z_c\partial^c) =\\ & Y^j(\delta_j Z^k)\delta_k + Y^j Z^k \nabla_{\delta_j}\delta_k + Y^j(\delta_j Z_c)\partial^c + Y^j Z_c \nabla_{\delta_j}\partial^c +\\ & Y_b(\partial^b Z^k)\delta_k + Y_b Z^k \nabla_{\partial^b}\delta_k + Y_b(\partial^b Z_c)\partial^c + Y_b Z_c \nabla_{\partial^b}\partial^c. \end{aligned} \tag{2.3}$$

Using the above formulae we calculate

$$\nabla_X(\nabla_Y Z) = \nabla_{(X^i\delta_i + X_a\partial^a)}[(\nabla_Y Z)^k\delta_k + (\nabla_Y Z)_c\partial^c]. \tag{2.4}$$

For $[X,Y]$ we have

$$\begin{aligned} [X,Y] =& [(X^i\delta_i Y^j - Y^i\delta_i X^j) + (X_a\partial^a Y^j - Y_b\partial^b X^j]\delta_j +\\ & [X^i\delta_j Y_b - Y^i\delta_i Y_b) + (X_a\partial^a Y_b - Y_a\partial^a X_b) +\\ & (X^i Y^j - Y^i X^j)(\partial_{[j} N_{|b|i]} + N_{a[i}\partial^a N_{|b|j]}) +\\ & (X^i Y_a - Y^i X_a)\partial^a N_{bi}]\partial^b. \end{aligned} \tag{2.5}$$

After longer calculation we obtain

$$\begin{aligned} R(X,Y)Z =& (X^iY^j - Y^iX^j)2^{-1}[(R_m{}^k{}_{ji}Z^m + R^{dk}{}_{ji}Z_d)\delta_k +\\ & (R_{mcji}Z^m + R^d{}_{cji}Z_d)\partial^c]\\ & + (X_aY^j - Y_aX^j)[(P_m{}^k{}_j{}^a Z^m + P^{dk}{}_j{}^a Z_d)\delta_k +\\ & (P_{mcj}{}^a Z^m + P^d{}_{cj}{}^a Z_d)\partial^c]\\ & + (X_aY_b - Y_aX_b)2^{-1}[(S_m{}^{kba}Z^m + S^{dkba}Z_d)\delta_k +\\ & (S_{mc}{}^{ba}Z^m + S^d{}_c{}^{ba}Z_d)\partial^c], \end{aligned} \tag{2.6}$$

where

$$
\begin{aligned}
(2.7)\ (a)\quad & 2^{-1}R_m{}^k{}_{ji} = \delta_{[i}F_{|m|}{}^k{}_{j]} + F_m{}^h_{[j}F_{|h|}{}^k{}_{i]} + F_{mc[j}F^{ck}{}_{i]} + 2^{-1}C_m{}^{ke}K_{jei}\\
(b)\quad & 2^{-1}R^{dk}{}_{ji} = \delta_{[i}F^{dk}{}_{j]} + F^d{}_{e[j}F^{ek}{}_{i]} + F^{dh}{}_{[j}F_{|h|}{}^k{}_{i]} + 2^{-1}C^{dke}K_{jei}\\
(c)\quad & 2^{-1}R_{mcji} = \delta_{[i}F_{|mc|j]} + F_{me[j}F^e{}_{|c|i]} + F_m{}^h_{[j}F_{|hc|i]} + 2^{-1}C_{mc}{}^eK_{jei}\\
(d)\quad & 2^{-1}R^d{}_{cji} = \delta_{[i}F^d{}_{|c|j]} + F^d{}_{e[j}F^e{}_{|c|i]} + F^{dh}{}_{[j}F_{|hc|i]} + 2^{-1}C^d{}_c{}^eK_{jei}\\
(e)\quad & 2^{-1}K_{iej} = \delta_{[j}N_{|e|i]} + N_{a[i}\partial^aN_{|e|j]},\ K^a{}_{ej} = \partial^aN_{ej} - F^a{}_{ej}
\end{aligned}
$$

$$
\begin{aligned}
(2.8)\qquad (a)\quad & P_m{}^k{}_j{}^a = \partial^aF_m{}^k{}_j + K^a{}_{ej}C_m{}^{ke} - C_m{}^{ka}{}_{|j}\\
(b)\quad & C_m{}^{ka}{}_{|j} = \delta_jC_m{}^{ka} + C_m{}^{ha}F_h{}^k{}_j + C_{me}{}^aF^{ek}{}_j - C_h{}^{ka}F_m{}^h{}_j -\\
& \qquad C^{eka}F_{mej} - C_m{}^{ke}F^a{}_{ej}\\
(c)\quad & P^{dk}{}_j{}^a = \partial^aF^{dk}{}_j + K^a{}_{ej}C^{dke} - C^{dka}{}_{|j}\\
(d)\quad & C^{dka}{}_{|j} = \delta_jC^{dka} + C^d{}_e{}^aF^{ek}{}_j + C^{dha}F_h{}^k{}_j - C_h{}^{ka}F^{dh}{}_j -\\
& \qquad C^{eka}F^d{}_{ej} - C^{dke}F^a{}_{ej}\\
(e)\quad & P_{mcj}{}^a = \partial^aF_{mcj} + K^a{}_{ej}C_{mc}{}^e - C_{mc}{}^a{}_{|j}\\
(f)\quad & C_{mc}{}^a{}_{|j} = \delta_jC_{mc}{}^a + C_m{}^{ha}F_{hcj} + C_{me}{}^aF^e{}_{cj} - C_{hc}{}^aF_m{}^h{}_j -\\
& \qquad C^e{}_c{}^aF_{mej} - C_{mc}{}^eF^a{}_{ej}\\
(g)\quad & P^d{}_{cj}{}^a = \partial^aF^d{}_{cj} + K^a{}_{ej}C^d{}_c{}^e - C^d{}_c{}^a{}_{|j}\\
(h)\quad & C^d{}_c{}^a{}_{|j} = \delta_jC^d{}_c{}^a + C^{dha}F_{hci} + C^d{}_e{}^aF^e{}_c{}^i - C_{mc}{}^aF^{dm}{}_i -\\
& \qquad C^e{}_c{}^aF^d{}_{ei} - C^d{}_c{}^eF^a{}_{ej}
\end{aligned}
$$

$$
\begin{aligned}
(2.9)\qquad (a)\quad & 2^{-1}S_m{}^{kba} = \partial^{[a}C_m{}^{|k|b]} + C_m{}^{h[b}C_h{}^{|k|a]} + C_{me}{}^{[b}C^{|ek|a]}\\
(b)\quad & 2^{-1}S^{dkba} = \partial^{[a}C^{|dk|b]} + C^{dh[b}C_n{}^{|k|a]} + C^d{}_e{}^{[b}C^{eka]}\\
(c)\quad & 2^{-1}S_{mc}{}^{ba} = \partial^{[a}C_{mc}{}^{b]} + C_m{}^{h[b}C_{hc}{}^{a]} + C_{me}{}^{[b}C^{|e|}{}_c{}^{a]}\\
(d)\quad & 2^{-1}S^d{}_c{}^{ba} = \partial^{[a}C^{|d|}{}_c{}^{b]} + C^{dh[b}C_{hc}{}^{a]} + C^d{}_e{}^{[b}C^{|e|}{}_c{}^{a]}.
\end{aligned}
$$

On the other side starting from (1.10) for $W = \nabla_Y Z$ we get

$$W = (Y^jZ^k{}_{|j} + Y_bZ^k|^b)\delta_k + (Y^jZ_{c|j} + Y_bZ_c|^b)\partial^c$$

and

$$(2.10)\qquad \nabla_X\nabla_YZ = \nabla_XW = (X^iW^k_{|i} + X_aW^k|^a)\delta_k + (X^iW_{c|i} + X_aW_c|^a)\partial^c.$$

We may obtain $\nabla_{[X,Y]}Z$ from

$$(2.11)\qquad \nabla_{[X,Y]}Z = ([XY]^jZ^k{}_{|j} + [XY]_bZ^k|^b)\delta_k + ([XY]^jZ_{c|j} + [XY]_bZ_c|^b)\partial^c,$$

where $[XY]$ is given by (2.5). From (2.1), (2.10), (2.11) and (2.5) we obtain

$$
\begin{aligned}
(2.12)R = {} & \nabla_X\nabla_YZ - \nabla_Y\nabla_XZ - \nabla_{[X,Y]}Z =\\
& (X^iY^j - Y^iX^j)[(Z^k_{[|j|i]} + F_{[j}{}^h{}_{i]}Z^k{}_{|h} + F_{[j|b|i]}Z^k|^b + 2^{-1}K_{jbi}Z^k|^b)\delta_k +\\
& \quad (Z_{c[|j|i]} + F_{[j}{}^h{}_{i]}Z_{c|h} + F_{[j|b|i]}Z_c|^b + 2^{-1}K_{jbi}Z_c|^b)\partial^c] +\\
& (X_aY^j - Y_aX^j)[(Z^k_{|j}|^a - Z^k|^a{}_{|j} - F^{ai}{}_jZ^k_{|i} + C_{jb}{}^aZ^k|^b + C_j{}^{ha}Z^k_{|h} +\\
& \quad K^a{}_{bj}Z^k|^b)\delta_k + (Z_{c|j}|^a - Z^c|^a{}_{|j} - F^{ah}{}_jZ_{c|h} + C_{jb}{}^aZ_c|^b + C_j{}^{ha}Z_{c|h} +\\
& \quad K^a{}_{bj}Z_c|^b)\partial^c] +\\
& (X_aY_b - Y_aX_b)[(Z^k|^{[b}|^{a]} + C^{[b}{}_d{}^{a]}Z^k|^d + C^{[b|j|a]}Z^k{}_{|j})\delta_k +\\
& \quad (Z_c|^{[b}|^{a]} + C^{[b|j|a]}Z_{c|j} + C^{[b}{}_d{}^{a]}Z_c|^d)\partial^c].
\end{aligned}
$$

From (2.1), (2.6) and (2.12) equating the coordinates beside the basis vectors δ_k and ∂^c we get

$$
\begin{aligned}
(2.13)\ & (X^iY^j - Y^iX^j)2^{-1}(R_m{}^k{}_{ji}Z^m + R^{dk}{}_{ji}Z_d) +\\
& (X_aY^j - Y_aX^j)(P_m{}^k{}_j{}^aZ^m + P^{dk}{}_j{}^aZ_d) +\\
& (X_aY_b - Y_aX_b)2^{-1}(S_m{}^{kba}Z^m + S^{dkba}Z_d) =\\
& (X^iY^j - Y^iX^j)(Z^k_{[|j|i]} + F_{[j}{}^h{}_{i]}Z^k{}_{|h} + F_{[j|b|i]}Z^k|^b + 2^{-1}K_{jbi}Z^k|^b) +\\
& (X_aY^j - Y_aX^j)(Z^k{}_{|j}|^a - Z^k|^a{}_{|j} - F^{ai}{}_jZ^k{}_{|i} + C_{jb}{}^aZ^k|^b + C_j{}^{ha}Z^k{}_{|h} +\\
& \qquad K^a{}_{bj}Z^k|^b) +\\
& (X_aY_b - Y_aX_b)(Z^k|^{[b}|^{a]} + C^{[b}{}_d{}^{a]}Z^k|^d + C^{[b|j|a]}Z^k{}_{|j}).
\end{aligned}
$$

$$
\begin{aligned}
(2.14)\ & (X^iY^j - Y^iX^j)2^{-1}(R_{mcji}Z^m + R^d{}_{cji}Z_d) +\\
& (X_aY^j - Y_aX^j)(P_{mcj}{}^aZ^m + P^d{}_{cj}{}^aZ_d) +\\
& (X_aY_b - Y_aX_b)2^{-1}(S_{mc}{}^{ba}Z^m + S^d{}_c{}^{ba}Z_d) =\\
& (X^iY^j - Y^iX^j)(Z_{c[|j|i]} + F_{[j}{}^k{}_{i]}Z_{c|k} + F_{[j|b|i]}Z_c|^b + 2^{-1}K_{jbi}Z_c|^b) +\\
& (X_aY^j - Y_aX^j)(Z_{c|j}|^a - Z_c|^a{}_{|j} - F^{ah}{}_jZ_{c|h} + C_{jb}{}^aZ_c|^b + C_j{}^{ha}Z_{c|h} +\\
& \qquad K^a{}_{bj}Z_c|^b) +\\
& (X_aY_b - Y_aX_b)(Z_c|^{[b}|^{a]} + C^{[b|j|a]}Z_{c|j} + C^{[b}{}_d{}^{a]}Z_c|^d).
\end{aligned}
$$

Theorem 2.1. *In the recurrent Hamilton spaces supplied with generalized connection the Ricci identities have the form*

$$
\begin{aligned}
(2.15)(a)\quad & 2^{-1}(R_m{}^k{}_{ji}Z^m + R^{dk}{}_{ji}Z_d) = \\
& Z^k_{[|j|i]} + F_{[j}{}^k{}_{i}Z^k{}_{|h} + F_{[j|b|i]}Z^k|^b + 2^{-1}K_{jbi}Z^k|^b \\
(b)\quad & P_m{}^k{}_j{}^aZ^m + P^{dk}{}_j{}^aZ_d = \\
& Z^k{}_j|^a - Z^k|^a{}_{|j} - F^{ai}{}_jZ^k{}_{|i} + C_{jb}{}^aZ^k|^b + C_j{}^{ha}Z^k|h + K^a{}_{bj}Z^k|^b \\
(c)\quad & 2^{-1}S_m{}^{kba}Z^m + S^{dkba}Z_d = Z^k|^{[b|a]} + C^{[b}{}_d{}^{a]}Z^k|^d + C^{[b|j|a]}Z^k{}_{|j} \\
(d)\quad & 2^{-1}(R_{mcji}Z^m + R^d{}_{cji}Z_d) = \\
& Z_{c|[j|i]} + F_{[j}{}^h{}_{i|}Z_{c|h} + F_{[j|b|i]}Z_c|^b + 2^{-1}K_{jbi}Z_c|^b \\
(e)\quad & P_{mcj}{}^aZ^mP^d{}_{cj}{}^aZ_d = \\
& Z_{c|j}|^a - Z_c|^a{}_{|j} - F^{ah}{}_jZ_{c|h} + C_{jb}{}^aZ_c|^b + C_j{}^{ha}Z_{c|h} + K^a{}_{bj}Z_c|^b \\
(f)\quad & 2^{-1}(S_{mc}{}^{ba}Z^m + S^d{}_c{}^{ba}Z_d) = Z_c|^{[b}|^{a]} + C^{[b|j|a]}Z_{c|j} + C^{[b}{}_d{}^{a]}Z_c|^d.
\end{aligned}
$$

Proof. Equating the coefficients beside bivectors $(X^iY^j - Y^iX^j)$, $(X_aY^j - Y_aX^j)$, $(X_aY_b - Y_aX_b)$ in (2.13) and (2.14) we obtain (2.15).

3. Bianchi identities for the recurrent Hamilton spaces with generalized Miron's d-connection.

Let us denote as usually:

$$
(3.1)\qquad (\nabla_XT)(YZ) = \nabla_X(T(Y,Z)) - T(\nabla_XY,Z) - T(Y,\nabla_XZ),
$$

$$
\begin{aligned}
(3.2)\quad (\nabla_XR)(Y,Z,U) =& \nabla_X(R(Y,Z,U)) - R(\nabla_XY,Z,U) - R(Y,\nabla_XZ,U) - \\
& R(Y,Z,\nabla_XU),
\end{aligned}
$$

where X, Y, Z, U are vector fields from $T(E^*)$, then from (2.1) and

$$
(3.3)\qquad T(XY) = \nabla_XY - \nabla_YX - [XY]
$$

we obtain

Theorem 3.1. *The Bianchi identities for the recurrent Hamilton spaces with generalized Miron's d-connection have the form:*

$$
(3.4)\qquad \sigma_{XYZ}\{(\nabla_XT)(Y,Z) - R(X,Y)Z - T(T(X,Y)Z)\} = 0
$$

$$
(3.5)\qquad \sigma_{XYZ}\{(\nabla_XR)(Y,Z,U) + R(T(X,Y),Z)U\} = 0.
$$

Proof. The proof is obtained from (3.1) - (3.3).

Taking ∇_UR (R given by (2.6) and (2.12)) and the cyclic permutation of UXY we get the Bianchi identities in the explicite form:

$$
\begin{aligned}
(3.6)\quad & 2^{-1}(R_h{}^k{}_{ji|m}Z^h + R_h{}^k{}_{ji}Z^h_{|m} + R^{dk}{}_{ji|m}Z_d + R^{dk}{}_{ji}Z_{d|m}) = \\
& Z^k_{|[j|i]|m} + F_{[j}{}^h{}_{i]|m}Z^k_{|h} + F_{[j}{}^h{}_{i]}Z^k_{|h|m} + F_{[j|b|i]|m}Z^k|^b + F_{[j|b|i]}Z^k|^b{}_{|m} + \\
& F_{[j|b|i]}Z^k|^b{}_{|m} - 2^{-1}K_{ibj|m}Z^k|^b - 2^{-1}K_{ibj}Z^k|^b{}_{|m}
\end{aligned}
$$

$$
\begin{aligned}
(3.7)\quad & 2^{-1}(R_{hcji|m}Z^h + R_{hcji}Z^h_{|m}R^d{}_{cji|m}Z_d + R^d{}_{cji}Z_{d|m}) = \\
& Z_{c|[j|i]|m} + F_{[j}{}^h{}_{i]|m}Z_{c|h} + F_{[j}{}^h{}_{i]}Z_{c|h|m} + F_{[j|b|i]|m}Z_c|^b + \\
& F_{[j|b|i]}Z_c|^b{}_{|m} + F_{[j|b|i]}Z_c|^b_{|m} - 2^{-1}K_{ibj|m}Z_c|^b - 2^{-1}K_{ibj}Z_c|^b_{|m}
\end{aligned}
$$

$$
\begin{aligned}
(3.8)\quad & 2^{-1}(R_h{}^k{}_{ij}|^aZ^h + R_h{}^k{}_{ij}Z^h|^a + R^{dk}{}_{ij}|^aZ_d + R^{dk}{}_{ij}Z_d|^a + \\
& 2^{-1}[(P_h{}^k{}_{j}{}^a_{|i}Z^h + P_h{}^k{}_j{}^aZ^h_{|i}P^{dk}{}_{j}{}^a_{|i}Z_d + P^{dk}{}_j{}^aZ_{d|i}) - (i,j)] = \\
& = 2^{-1}[(Z^k_{|j}|^a_{|i} + Z^k{}^a_{|i|j} + Z^k_{|i|j}|^a - F^{ah}{}_{j|i}Z^k_{|h} - F^{ah}{}_jZ^k_{|h|i} + \\
& C_{jb}{}^aZ^k|^b + C_{jb}{}^aZ^k|^b_{|i} + C_j{}^{ha}_{|i}Z^k_{|h} + C_j{}^{ha}Z^k_{|h|i} + K^a{}_{bj}Z^k|^b + \\
& K^a{}_{bj}Z^k|^b{}_{|i} - (i,j)] + F_{[i}{}^h{}_{j]}|^aZ^k_{|h} + F_{[i}{}^h{}_{j]}Z^k_{|h}|^a + F_{[i|b|j]}|^aZ^k|^b + \\
& F_{[i|b|j]}Z^k|^b|^a + 2^{-1}K_{ibj}|^aZ^k|^b + 2^{-1}K_{ibj}Z^k|^b|^a
\end{aligned}
$$

$$
\begin{aligned}
(3.9)\quad & 2^{-1}(R_{hcij}|^aZ^h + R_{hcij}Z^h|^a + R^d{}_{cij}|^aZ_d + R^d{}_{cij}Z_d|^a) + \\
& 2^{-1}[(P_{hcj}{}^a_{|i}Z^h + P_{hcj}{}^aZ^h_{|i} + P^d{}_{cj}{}^a_{|i}Z_d + P^d{}_{cj}{}^aZ_{d|i}) - (i,j)] = \\
& = 2^{-1}[(Z_{c|j}|^a_{|i} + Z_c|^a_{|i|j} + Z_{c|i|j}|^a - F^{ah}{}_{j|i}Z_{c|h} - F^{ah}{}_jZ_{c|h|i} + \\
& C_{jb}{}^a_{|i}Z_c|^b + C_{jb}{}^aZ_c|^b_{|i} + C_j{}^{ha}_{|i}Z_{c|h} + C_j{}^{ha}Z_{c|h|i} + \\
& K_{abj|i}Z_c|^b + K^a{}_{bj}Z_c|^b_{|i}) - (i,j)] + F_{[i}{}^h{}_{j]}|^aZ_{c|h} + F_{[i}{}^h{}_{j]}Z_{c|h}|^a + \\
& F_{[i|b|j]}|^aZ_c|^bF_{[i|b|j]}Z_c|^b|^a - 2^{-1}K_{ibj}Z_c|^b - 2^{-1}K_{ibj}Z_c|^b|^a
\end{aligned}
$$

$$
\begin{aligned}
(3.10)\quad & 2^{-1}(S_h{}^{kba}_{|i}Z^h + S_h{}^{kba}Z^h_{|i} + S^{dkba}_{|i}Z_d + S^{dkba}Z_{d|i}) - \\
& - 2[(P_h{}^k{}_i{}^a|^bZ^h + P_h{}^k{}_i{}^aZ^h|^b + P^{dk}{}_i{}^a|^hZ_d + P^{dk}{}_i{}^aZ_d|^b) - (a,b)] = \\
& = 2^{-1}[(Z^k|^b|^a_{|i} + Z^k|^a_{|i}|^b + Z^k_{|i}|^b|^a) - (a,b)] + C^{[b}{}_d{}^{a]}{}_{|i}Z^k|^d + \\
& C^{[b}{}_d{}^{a]}Z^k|^d{}_{|i} + C^{[b|h|a]}{}_{|i}Z^k_{|h} + C^{[b|h|a]}Z^k_{|h|i} + 2^{-1}[(F^{ah}{}_i|^bZ^k_{|h} + \\
& F^{ah}{}_iZ^k_{|h}|^b - C_{ie}{}^a|^bZ^k|^e - C_{ie}{}^aZ^k|^e|^b - C_i{}^{ha}|^bZ^k_{|h} - \\
& C_i{}^{ha}Z^k_{|h}|^b - K^a{}_{ei}Z^k|^e - K^a{}_{ei}Z^k|^e|^b) - (a,b)]
\end{aligned}
$$

$$
\begin{aligned}
(3.11)\quad & 2^{-1}(S_{hc}{}^{ba}{}_{|i}Z^h + S_{hc}{}^{ba}Z^h_{|i} + S^d{}_c{}^{ba}{}_{|i}Z_d + S^d{}_c{}^{ba}Z_{d|i})- \\
& -2^{-1}[(P_{hci}{}^a|^bZ^h + P_{hci}{}^aZ^h|^b + P^d{}_{ci}{}^a|^bZ_d + P^d{}_{ci}{}^aZ_d|^b) - (a,b)] = \\
& = 2^{-1}[(Z_c|^b|^a_{|i} + Z_c|^a_{|i}|^b + Z_{c|i}|^b|^a) - (a,b)] + \\
& C^{[b}{}_d{}^{a]}{}_{|i}Z_c|^d + C^{[b}{}_d{}^{a]}Z_c|^d_{|i} + C^{[b|h|a]}{}_{|i}Z_{c|h} + C^{[b|h|a]}Z_{c|h|i} + \\
& 2^{-1}[F^{ah}{}_i|^bZ_{c|h} + F^{ah}{}_iZ_{c|h}|^b - C_{ie}{}^a|^bZ_c|^e - C_{ie}{}^aZ_c|^e|^b - \\
& C_i{}^{ha}|^bZ_{c|h} - C_i{}^{ha}Z_{c|h}|^b - K^a{}_{ei}|^bZ_c|^e - K^a{}_{ei}Z_c|^e|^b) - (a,b)]
\end{aligned}
$$

$$
\begin{aligned}
(3.12)\quad & 2^{-1}(S_h{}^{kba}|^eZ^h + S_h{}^{kba}Z_h|^e + S^{dkba}|^eZ_d + S^{dkba}Z_d|^e) = \\
& Z^k|^{[b}|^{a]}|^e + C^{[b}{}_d{}^{a]}|^eZ^k|^d + C^{[b}{}_d{}^{a]}Z^k|^d|^e + \\
& C^{[b}{}_j{}^{a]}|^eZ^k_{|j} + C^{[b}{}_j{}^{a]}Z^k_{|j}|^e.
\end{aligned}
$$

$$
\begin{aligned}
(3.13)\quad & 2^{-1}(S_{hc}{}^{ba}|^eZ^h + S_{hc}{}^{ba}Z^h|^e + S^d{}_c{}^{ba}|^eZ_d + S^d{}_c{}^{ba}Z_d|^e = \\
& Z_c|^{[b}|^{a]}|^e + C^{[b}{}_d{}^{a]}Z_c|^d + C^{[b}{}_d{}^{a]}Z_c|^d|^e + C^{[b|j|a]}|^eZ_{c|j} + \\
& C^{[b|j|a]}Z_{c|j}|^e,
\end{aligned}
$$

where $(Q_{ab}) - (a,b) = Q_{ab} - Q_{ba}$: (Q_{ab}) is any expression in witch indices a, b are present.

4. Special cases. We obtain the d-connection in Hamilton spaces if we put

$$(4.1)\qquad F_{jci} = 0, \qquad F^{ak}{}_j = 0, \qquad C_{jc}{}^a = 0, \quad C^{bka} = 0$$

in (1.9).

Theorem 4.1. *Under conditions (4.1) the following relations are valid.*

$$(4.2)\quad R^{dk}{}_{ji} = 0, \quad R_{hcji} = 0, \quad P^{dk}{}_j{}^a = 0, \quad P_{mcj}{}^a = 0, \quad S^{dkba} = 0, \quad S_{mc}{}^{ba} = 0.$$

Proof. Substituting (4.1) into (2.7), (2.8) and (2.9) we obtain (4.2).

Theorem 4.2. *Under conditions (4.1) and* $Z = Z^k\delta_k(Z_c = 0)$ *the Ricci identities have the form*

$$
\begin{aligned}
(4.3)\qquad & (a)\quad Z^k{}_{|[j|i]} + F_{[j}{}^h{}_{i]}Z^k_{|h} = 2^{-1}R_h{}^k{}_{ji}Z^h + 2^{-1}K_{ibj}Z^k|^b \\
& (b)\quad Z^k{}_{|j}|^a - Z^k|^a_{|j} + C_j{}^{ha}Z^k_{|h} + K^a{}_{bj}Z^k|^b = P_h{}^k{}_j{}^aZ^h \\
& (c)\quad Z^{k[}|^b|^{a]} + C^{[b}{}_d{}^{a]}Z^k|^d = 2^{-1}S_h{}^{kba}Z^h.
\end{aligned}
$$

Proof. Substituting (4.1) and $Z_c = 0$ into (2.15) we obtain (4.3).

Theorem 4.3. *Under conditions (4.1) and* $Z^k = 0$ *the Ricci identities have the form*

$$
\begin{aligned}
(4.4)\qquad & Z_{c|[j|i]} + F_{[j}{}^h{}_{i]}Z_{c|h} = 2^{-1}R^d{}_{cji}Z_d + 2^{-1}K_{idj}z_c|^d \\
& Z_{c|j}|^a - Z_c|^a_{|j} + C_j{}^{ha}Z_{c|h} + K^a{}_{bj}Z_c|^b = P^d{}_{cj}{}^aZ_d \\
& Z_c|^{[b|a]} + C^{[b}{}_d{}^{a]}Z_c|^d = 2^{-1}S^d{}_c{}^{ba}Z_d.
\end{aligned}
$$

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DIRECTION DEPENDENT f–STRUCTURES SATISFYING $f^3 + f = 0$

BY

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Introduction. In [8],[9] K. Yano has defined the notion of f–structure on a manifold satisfying $f^3 + f = 0$ and such that the rank of f is equal to a constant k everywhere. It is the purpose of the present paper to start a study of f–structures which depend on direction.

1. Preliminaries. Let N be a real differentiable manifold of dimension n. Denote by TN the tangent bundle over N and by π the canonical projection of TN to N. Also, denote by $d\pi : TTN \to TN$ the differential of π and define the vertical subbundle VTN of TTN as the kernel of $d\pi$. A complementary distribution HTN to VTN in TTN is called a nonlinear connection on TN (see M. Matsumoto [6]). Of course, the fibres of vector bundles VTN and HTN are of the same dimension n. It is well known, VTN is an integrable distribution on TN.

Let (x^i, y^i) be a coordinate system on TN and $\{\delta/\delta x^i, \partial/\partial y^i\}$ be a local field of frames on TN adapted to the decomposition $TTN = HTN \oplus VTN$, where

$$\frac{\delta}{\delta x^i} = \frac{\partial}{\partial x^i} - N_i^j(x,y)\frac{\partial}{\partial y^j},$$

and $N_i^j(x,y)$ are n^2 differentiable functions locally defined on TN. The automorphism

$$P: \Gamma(TTN) \to \Gamma(TTN),$$

which is defined by

$$PX = X_V^i(x,y)\frac{\delta}{\delta x^i} + X_H^i(x,y)\frac{\partial}{\partial y^i},$$

for

$$X = X_H^i(x,y)\frac{\delta}{\delta x^i} + X_V^i(x,y)\frac{\partial}{\partial y^i},$$

is an almost product structure on TN, i.e, $P^2 = I$, where $\Gamma(TTN)$ is the $\mathcal{F}(TN)$–module of all differentiable cross sections of TTN. We keep the same notation for any other bundle. The almost product structure P will be called natural almost