$$(m+n)\{\Phi|a_{mn}|\}^{\rho/(m+n)} < (m+n)^{\beta}\text{with} \quad \beta = \sum_{s=1}^{k} \frac{\log A_s^{1/k}}{\log(m+n)} + O(\log(m+n)^{-2}).$$

Since $\lim_{m+n\to\infty}(m+n)^{\beta} = \prod_{s=1}^{k} {A'_s}^{1/k}$, where $A'_s = e\rho_s T_s$ taking lim sup, we get $\limsup_{m+n\to\infty}[(m+n)(\Phi|a_{mn}|)^{\rho/(m+n)}] \le \prod_{s=1}^{k}(e\rho_s T_s)^{1/k}$.
Hence $e\rho_T \le \prod_{s=1}^{k}(e\rho_s T_s)^{1/k} = e\prod_{s=1}^{k}(\rho_s)^{1/k}(\prod_{s=1}^{k} T_s)^{1/k} = \frac{\rho}{p} e \prod_{s=1}^{k}(p_s T_s)^{1/k}$ or $pT \le \prod_{s=1}^{k}(p_s T_s)^{1/k}$, and the theorem is proved.

REFERENCES

1. Dzrbasyan, M. M. — *On the theory of some class of integral functions*, Izv. Akademi Nauk Armyanskoi, S.S.R. VIII, No.4, 1955.
2. Dalal, S. S. — *On the order and type of integral functions of several complex variables*, Jour. of Indian Math. Soc., 33 (1969) 215–220.
3. Goldberg, A. A. — *Elementary remarks on the formulae defining order and type of functions of several variables*, Doklady Akademi Nauk Armyanskoi S.S.R., 29 (1959) 145–151.
4. Mazumdar, N. C. — *On the space of integral functions of several complex variables*, Indian Jour. Mech. and Maths. (Jadavpur University) Vol. XII,No. 1, (1974).
5. Singh Jodh Pal — *On the order and type of entire functions of several complex variables*, Riv. Math. Univ. Parma (2) 10 (1969) 111–121.
6. Srivastava, R. K. and Kumar, V. — *On the order and type of integral functions of several complex variables*, Comp. Math. 17, Fasc 2 (1966) 161–166.

Received 22.VI.1990

Department of Mathematics
University of Kalyani
Kalyani, West Bengal
INDIA.

ANALELE ŞTIINŢIFICE ALE UNIVERSITĂŢII "AL. I. CUZA" IAŞI
Tomul XXXVII, s. I. a. Matematică, 1991, f. 4

ON THE DEFICIENCIES OF DIFFERENTIAL POLYNOMIALS OF MEROMORPHIC FUNCTIONS

BY

S.M. SARANGI and S.J. PATIL

In this paper we use the usual notations of the Nevanlinna theory of meromorphic functions as in [2]. By a meromorphic function we shall always mean a function meromorphic in the complex plane.

If f is a non-constant meromorphic function we shall denote by $S(r,f)$ any quantity satisfying

$$S(r,f) = O(T(r,f)) \quad \text{as} \quad r\to\infty \tag{1}$$

through all values if f is of finite order and as $r\to\infty$ possibly outside a set of finite linear measure if f is of infinite order.

Also we shall always denote by α, α_0, $\alpha_1,\dots$ etc. meromorphic functions satisfying

$$T(r,\alpha), T(r,\alpha_i(z)) = S(r,f) \tag{2}$$

Let P be the differential polynomial in f witch is the sum of a finite number of terms of the form ${f^{(1)}}^{l_1}\,{f^{(2)}}^{l_2}\dots {f^{(m)}}^{l_m}$ where $f^{(1)}$, $f^{(2)},\dots$ are the successive derivatives of f and l_1, $l_2,\dots l_m$ are non-negative integers.

If $l_1+l_2+l_3+\dots+l_m = n$ (a fixed positive integer) in every term of P then P is called a homogeneous differential polynomial in f of degree n.

In this paper we prove the results on homogeneous differential polynomials in f. Some results on differential polynomials in f are already proved by K.L.Hiong [1].

To prove the results in this paper we require the following lemma.

Lemma. *If P is a homogeneous differential polynomial in the derivatives of f of degree $n\ge 1$ then*

(3) (i) $$m(r, P/(f-a)^n) = S(r,f)$$
(4) (ii) $$m(r, P^1/(f-a)^n) = S(r,f)$$
(5) (iii) $$m(r, P^1/(P-b)) = S(r,f)$$

Proof. It is known [2] that

$$m(r, f^{(i)}/(f-a)) = S(r,f) \text{ for } i = 1,2,3,\dots$$

By definition P is the sum of the finite number of terms of the type

$$Q(f) = f^{(1)^{l_1}} f^{(2)^{l_2}} \dots f^{(m)^{l_m}}$$

where $l_1, l_2, \dots l_m$ are non-negative integers satisfying $\sum_{i=1}^{m} l_i = n$. Then

$$\frac{Q(f)}{(f-a)^n} = \alpha \left(\frac{f^{(1)}}{f-a}\right)^{l_1} \cdot \left(\frac{f^{(2)}}{f-a}\right)^{l_2} \cdots \left(\frac{f^{(m)}}{f-a}\right)^{l_m}.$$

So,

$$m\left(r, \frac{Q(f)}{(f-a)^n}\right) \leq m(r,\alpha) + \sum_{i=1}^{m} l_i m\left(r, \frac{f^{(i)}}{f-a}\right) = S(r,f).$$

Thus,

$$m\left(r, \frac{P}{(f-a)^n}\right) = m\left(r, \frac{Q(f)}{(f-a)^n}\right) \leq \sum m\left(r, \frac{Q(f)}{(f-a)^n}\right) + O(1) = S(r,f)$$

(ii) In P^1 every term is also of degree 'n' then clearly it follows that

$$m\left(r, P^1/(f-a)^n\right) = S(r,f),$$

(iii) $$m\left(r, P^1/(P-b)\right) = S(r, P-b) = S(r,P) = S(r,f).$$

Theorem 1. *Let $f(z)$ be a meromorphic function and $\alpha_i(z)$ $(i = 1,2,3\dots)$ be the meromorphic functions satisfying (2). If P is the homogeneous differential polynominal in the derivatives of f with $f(o) \neq 0, a, \infty, P(o) \neq b$ and $P^1(o) \neq 0$ then*

$$[n - o(1)]T(r,f) < N(r,f) + nN\left(n, \frac{1}{f-a}\right) + N\left(r, \frac{1}{P-b}\right) + N_1(r,f) + S(r,f).$$

Proof. To prove the theorem consider the identity.

$$\frac{1}{(f-a)^n} = \frac{1}{b}\left(\frac{P}{(f-a)^n} - \frac{P-b}{P^1}\frac{P^1}{(f-a)^n}\right) \quad b \neq 0.$$

Now,

$$m\left(r, \frac{1}{(f-a)^n}\right) \leq m\left(r, \frac{P}{(f-a)^n}\right) + m\left(r, \frac{P-b}{P^1}\right) + \\ + m\left(r, \frac{P^1}{(f-a)^n}\right) + \log\left|\frac{1}{b}\right| + \log 2.$$

i.e.

$$nm\left(r, \frac{1}{f-a}\right) \leq m\left(r, \frac{P}{(f-a)^n}\right) + m\left(r, \frac{P-b}{P^1}\right) + \\ + m\left(r, \frac{P^1}{(f-a)^n}\right) + \log\left|\frac{1}{b}\right| + \log 2 \tag{6}$$

Now by Jensen's formulae,

$$m\left(r, \frac{P-b}{P^1}\right) = m\left(r, \frac{P^1}{P-b}\right) + \\ + \left[N\left(r, \frac{P^1}{P-b}\right) - N\left(r, \frac{P-b}{P^1}\right)\right] + \log\left|\frac{P(0)-b}{P^1(0)}\right|.$$

Also,

$$N\left(r, \frac{P^1}{P-b}\right) - N\left(r, \frac{P-b}{P^1}\right) =$$

$$= N\left(r, P^1\right) + N\left(r, \frac{1}{P-b}\right) - N\left(r, P-b\right) - N\left(r, \frac{1}{P^1}\right).$$

Since $N(r, P-b) = N(r,P)$ and writing $N_1(r,P) = 2N(r,P) - N(r,P^1)$ we have

$$N\left(r, \frac{P^1}{P-b}\right) - N\left(r, \frac{P-b}{P^1}\right) \leq N(r,P) - N_1(r,P) + N\left(r, \frac{1}{P-b}\right).$$

Hence,

$$m\left(r, \frac{P-b}{P^1}\right) \leq m\left(r, \frac{P^1}{P-b}\right) + N(r,P) - N_1(r,P) + \\ + N\left(r, \frac{1}{P-b}\right) + \log\left|\frac{P(0)-b}{P^1(0)}\right|.$$

Now substituting in (6) we have

$$nm\left(r, \frac{1}{f-a}\right) \leq m\left(r, \frac{P}{(f-a)^n}\right) + m\left(r, \frac{P^1}{(f-a)^n}\right) + m\left(r, \frac{P^1}{P-b}\right) +$$

$$+ N(r,P) - N_1(r,P) + N\left(r, \frac{1}{P-b}\right) + \log\left|\frac{P(0)-b}{P^1(0)}\right| + \log\left|\frac{1}{b}\right| + \log 2.$$

Now,

$$nT(r,f) = nT\left(r, \frac{1}{f-a}\right) + \log|f(0)-a| + \log|a| + \log 2,$$

$$nT(r,f) < m\left(r,\frac{1}{f-a}\right) + nN\left(r,\frac{1}{f-a}\right) + \log|f(0)-a| + \log|a| + \log 2.$$

Hence,

$$nT(r,f) < N(r,P) + nN\left(r,\frac{1}{f-a}\right) + N\left(r,\frac{1}{P-b}\right) - N_1(r,P) + Q(r),$$

where

$$Q(r) = m\left(r,\frac{P}{(f-a)^n}\right) + m\left(r,\frac{P^1}{(f-a)^n}\right) + m\left(r,\frac{P^1}{P-b}\right) + \log|f(0)-a| +$$

$$+\log\left|\frac{P(0)-b}{P^1(0)}\right| + \log|a| + \log\left|\frac{1}{b}\right| + \log 2.$$

The poles of P can occur only at the poles of f or at the poles of the coefficients $\alpha(z)$ of P. In view of (2) we ignore the poles of the coefficients $\alpha(z)$, of P. At a pole of f of order k it is easily seen that P has a pole of order at most $nk+mn$. So,

$$(n-o(1))T(r,f) < N(r,f) + n.N\left(r,\frac{1}{f-a}\right) + N\left(r,\frac{1}{P-b}\right) - N_1(r,f) + S(r,f).$$

Theorem 2. *Let P be the homogeneous differential polynominal of degree n in the derivatives of f. Let a, b, c, be distinct finite constants such that $f(0) \neq a, \infty$, $P(0) \neq 0, a, b, c$ and $P^1(0) \neq 0$ or a then for $|z| = r < \rho < R$.*

$$nT(r,f) < nN\left(r,\frac{1}{f-a}\right) + N\left(r,\frac{1}{P-b}\right) + N\left(r,\frac{1}{P-c}\right) - N_1(r,f) + S(r,f).$$

Proof. Consider the identity

$$(f-a)^n = P\frac{(f-a)^n}{P}$$

$$m(r,(f-a)^n) < m(r,P) + m\left(r,\frac{(f-a)^n}{P}\right).$$

Now by Jensen–Nevanlinna formulae

$$nm(r,f-a) < m(r,P) + m\left(r,\frac{P}{(f-a)^n}\right) + \left[N\left(r,\frac{P}{(f-a)^n}\right) -\right.$$

$$\left.-N\left(r,\frac{(f-a)^n}{P}\right)\right] + \log\left|\frac{f(0)-a}{P(0)}\right|.$$

Now

$$N\left(r,\frac{P}{(f-a)^n}\right) - N\left(r,\frac{(f-a)^n}{P}\right) =$$

$$= N(r,P) + N\left(r,\frac{1}{(f-a)^n}\right) - nN(r,f-a) - N\left(r,\frac{1}{P}\right).$$

Hence,

$$nT(r,f) < T(r,P) + nN\left(r,\frac{1}{f-a}\right) - N\left(r,\frac{1}{P}\right) +$$

$$+m\left(r,\frac{P}{(f-a)^n}\right) + \log\left|\frac{(f(0)-a)^n}{P(0)}\right|.$$

Since

$$T(r,f-a) > T(r,f) - \log|b| - \log 2$$

We have

$$(7) \quad n.T(r,f) < T(r,P) + nN\left(r,\frac{1}{f-a}\right) - N\left(r,\frac{1}{P}\right) + m\left(r,\frac{P}{(f-a)^n}\right) +$$

$$+\log\left|\frac{(f(0)-a)^n}{P(0)}\right| + n.\log|a| + \log 2,$$

Now applying the second Nevanlinna theorem to P with constants b, c and o, we have

$$T(r,P) < N\left(r,\frac{1}{P}\right) + N\left(r,\frac{1}{P-b}\right) + N\left(r,\frac{1}{P-c}\right) -$$

$$-\left[N_1(r,P) + N\left(r,\frac{1}{P^1}\right)\right] + S(r,f).$$

Hence

$$n.T(r,f) < n.N\left(r,\frac{1}{f-a}\right) + N\left(r,\frac{1}{P-b}\right) + N\left(r,\frac{1}{P-c}\right) -$$

$$-\left[N_1(r,P) + N\left(r,\frac{1}{P^1}\right)\right] + Q(r),$$

where

$$Q(r) = S(r,P) + m\left(r,\frac{P}{(f-a)^n}\right) + \log\left|\frac{(f(0)-a)^n}{P(0)}\right| + n\log|a| + n.\log 2.$$

Clearly it follows that $S(r,P) = S(r,f)$ and also by the Lemma

$$m(r,P/(f-a)^n) = S(r,f).$$

Hence, $Q(r,f) = S(r,f)$ and $N_1(r,P) \geq N_1(r,f)$. Hence we have

$$n.T(r,f) < n.N\left(r,\frac{1}{f-a}\right) + N\left(r,\frac{1}{P-b}\right) + N\left(r,\frac{1}{P-c}\right) - \\ - \left[N_1(r,f) + N\left(r,\frac{1}{P^1}\right)\right] + S(r,f)$$

Theorem 3. *If P is the homogeneous differential polynomial of degree n and a, b, c are the constants then*

$$\delta(a) + (m+1)[\delta(b,P) + \delta(c,P)] \leq 2(m+1).$$

Proof. Now by definition of deficiency we have

$$N\left(r,\frac{1}{f-a}\right) < (1-\delta(a)+\epsilon_1)T(r,f),$$
$$N\left(r,\frac{1}{P-b}\right) < (1-\delta(b,P)+\epsilon_2)T(r,P),$$
$$N\left(r,\frac{1}{P-c}\right) < (1-\delta(c,P)+\epsilon_3)T(r,P).$$

Now,

$$m(r,P) < m(r,f) + m(r,P/f) = m(r,f) + S(r,f),$$
$$N(r,P) \leq n(m+1)N(r,f) + S(r,f).$$

Hence,

$$T(r,P) \leq n(m+1)T(r,f) + S(r,f),$$

$$nT(r,f) < n(1-\delta(a)+\epsilon_1).T(r,f) + \\ (1-\delta(b,P)+\epsilon_2+1-\delta(c,P)+\epsilon_3).n(m+1)T(r,f) + S(r,f).$$

i.e.

$$T(r,f) < \{1+2(m+1)-\delta(a)-m+1)\delta(b,P)-(m+1)\delta(c,P)+\epsilon\}T(r,f)+O(T(r,f)).$$

Hence we have

$$\delta(a) + (m+1)[\delta(b,P) + \delta(c,P)] < 2(m+1).$$

REFERENCES

1. K. L. Hiong — *Sur la limitation de $T(r,f)$ sans intervention des poles*, Bull. Sec. Sciences Math. Paris 80 (1956).
2. W. K. Hayman — *Meromorphic Functions*, Oxford Press, 1964.

Received 23.XI.1990

Department of Mathematics
Karnatak University,
Dharwad–580 003, INDIA

ANALELE ŞTIINŢIFICE ALE UNIVERSITĂŢII "AL. I. CUZA" IAŞI
Tomul XXXVII, s. I. a. Matematică, 1991, f. 4

QUELQUES THEOREMES D'INVERSION POUR LES MULTIFONCTIONS

PAR

C. C. CHOU

1. Introduction. Ces derniers années, divers auteurs ont généralisé le théorème classique d'homomorphisme de Lyusternik-Grave [12] pour les multifonctions. En dehors du cas vectoriel, des résultats sont également obtenus pour des multifonctions d'un espace métrique ou quasi-métrique dans un autre ([5],[11]). Dans ces articles, la métrique intervient de façon essentielle. Or le problème d'ouverture d'une multifonction est un problème topologique et il nous semble naturel de chercher des critères d'overture liées plus intrinsèquement à la topologie qu'à l'emploi d'une métrique.C'est dans ce but que nous définissons dans la section suivante les suites convergentes d'ensembles et les suites sommables d'ensembles. Grâce à ces notions, nous obtenons nos pricipaux résultats d'ouverture qui nous permettent en particulier de retrouver le main-théorème de Borwein–Zhuang [5], le théorème de Robinson–Ursescu [15][16].

Dans la section 2, nous considérons d'abord une multifonction d'un espace vectoriel topologique dans un autre et nous démontrons une relation fondamentale énoncée dans le théorème 2.2. Cette relation nous permet de déduire un théorème d'inversion locale quand l'espace d'arrivée est à base dénombrable. Nous montrerons ensuite comment on peut transposer nos résultats dans le cas non vectoriel.

Dans la section 3, nous considérons des multifonctions transformant les ensembles convexes en des ensembles convexes, et un résultat similaire au théorème 2.2 est donné.

Enfin dans la section 4, nous utilisons des multifonctions approximantes, plus particulièrement des approximantes coniques, pour étudier le problème d'ouverture.

2. Les principaux resultats d'ouverture pour une multifonction d'une espace topologique dans un autre. Dans cette section, nous décrirons un cadre général où la méthode itérative de Newton peut être appliquée pour obtenir quelques résultats d'ouverture.

Nous désignons par E, G,.. des espaces topologiques; par $\mathbf{E}$, $\mathbf{G}$,.. des espaces vectoriels topologiques; par $\mathbf{F}$, $\mathbf{F_1}$,.. des espaces de Fréchet; par $\mathcal{N}_{\mathbf{E}}$, $\mathcal{N}_{\mathbf{G}}\ldots$ le filtre de voisinages ouverts de zéro de $\mathbf{E}$, $\mathbf{G}$ etc. Nous disons qu'on espace uniforme (pas forcément séparé) est complet si tout filtre de Cauchy a un point adhérent.