ANALELE ŞTIINŢIFICE ALE UNIVERSITĂŢII "AL.I.CUZA" IAŞI
Tomul XL, s.I.a, Matematică, 1994, f1.

# WAVE PROPAGATION IN A PLATE WITH VOIDS

BY

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**0. Introduction.** In what follows we consider the problem of propagation of plane waves in an infinite layer (plate) of constant thickness $2h$, made from a homogeneous material with voids. We approach this problem in two steps. In the first one, following the common procedure, we derive the general form of the solution and frequency equations for symmetric and anti-symmetric vibrations. Then, in the second step, we discuss the obtained equations in terms of circular frequency $\omega$ in the case of anti-symmetric vibrations (bending vibrations). For the analysis we define a characteristic frequency $\omega_0$ proper to the medium and introduce the reduced frequency $\eta = \omega/\omega_0$, which enables us to characterise some specific properties of propagation phenomena. Generally, the plane waves, which propagate in a medium with voids are accomplished by dispersion and attenuation. Although, in the complete form the frequency equations are complicated, however, in some cases they can be simplified and the results become quantitatively. These cases are identified by a certain relation between $\omega$ and $\omega_0$. Thus, if $\omega << \omega_0$, the frequency equation differs, in the fist approximation from that met in the classical elasticity by a term independent of $\omega$ and, we conclude that the waves propagate without dispersion. In the case $\omega >> \omega_0$ the form of dispersion equation becomes more cumbersome in comparison with the above situation but we can derive a correction of order $\varepsilon = \sigma^{-1}$ of the classical Rayleigh solution which indicates explicitely the dispersive and space attenuation character of waves.

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* *This paper represents the written version of the communication presented at the Proceedings of the University "Al.I.Cuza" held in Iaşi, October 21–30, 1994.*

**1. Fundamental Equations.** We start from the equations of a linear theory of materials with voids as was proposed by S.C. C o w i n and J.W. N u n z i a t o in [3].

According to this theory, the field equations governing the deformation of an isotropic homogeneous body consist from the following set:

– *the equation of motion (in the absense of body forces):*

$$\sigma_{ij,j} = \rho\ddot{u}_i \qquad (\ldots)_{,j} = \frac{\partial(\ldots)}{\partial x_j}, \tag{1.1}$$
$$h_{i,i} + g = \rho k_0\ddot{\nu}, \quad \ddot{u}_i = \frac{\partial^2 u_i}{\partial t^2}, \quad i,j = 1,2,3.$$

– *constitutive equations*

$$\sigma_{ij} = (\lambda\theta + \beta\nu)\delta_{ij} + 2\mu\varepsilon_{ij} \tag{1.2}$$
$$h_i = \alpha\nu_{,i}$$
$$g = -\gamma\dot{\nu} - \xi\nu - \beta\theta, \quad \theta = u_{i,i},$$

where $\alpha, \beta, \gamma, \varepsilon, \lambda, \mu, k_0$ are material constants,

– *geometrical relations:*

$$2\varepsilon_{ij} = u_{i,j} + u_{j,i}$$

– *boundary conditions:*

$$\sigma_{ij}n_j = \hat{\sigma}_i, \quad h_j n_j = 0. \tag{1.3}$$

$n_j$ are the components of the outward normala at the body surface and $\sigma_i$ are given surface loads. (The meaning of all field variables and notations are those appearing in [3].)

On substituting (1.2) into (1.1) we obtain the Lamé's form of the equation of motion, namely:

$$\mu u_{i,jj} + (\lambda + \mu)u_{j,ji} + \beta\nu_{,i} = \rho\ddot{u}_i, \tag{1.4}$$
$$\alpha\nu_{,jj} - \gamma\dot{\nu} - \xi\nu - \beta\theta = \rho k_0\ddot{\nu}$$

To solve the problem we assume the displacement vector components and function of volume fraction be of the form:

$$u_1 = u_1(x_1, x_3, t), \quad u_3 = u_3(x_1, x_3, t); \quad u_2 \equiv 0, \quad \nu = \nu(x_1, x_3, t). \tag{1.5}$$



In view of (1.5), Eqs. (1.4) become:

$$\mu\Delta u_1 + (\lambda + \mu)\theta_{,1} + \beta\nu_{,1} = \rho\ddot{u}_1, \tag{1.6}$$
$$\mu\Delta u_3 + (\lambda + \mu)\theta_{,3} + \beta\nu_{,3} = \rho\ddot{u}_3,$$
$$\alpha\Delta\nu - \gamma\dot{\nu} - \xi\nu - \beta\theta = \rho k_0\ddot{\nu}$$

In the case of free boundary conditions we must have:

$$\sigma_{31}(\pm h) = \mu(u_{3,1} + u_{1,3}) = 0, \tag{1.7}$$
$$\sigma_{33}(\pm h) = (\lambda + 2\mu)u_{3,3} + \lambda u_{1,1} + \beta\nu = 0,$$
$$h_3(\pm h) = \alpha\nu_{,3} = 0.$$

**2. Solution of the problem.** Let us assume that the direction of propagation of waves is parallel to the $0x_1$-axis. As usually, we represent the displacements $u_1, u_3$ in terms of potentials $\varphi(x_1, x_3, t)$ and $\psi(x_1, x_3, t)$, as follows:

$$u_1 = \varphi_{,1} - \psi_{,3}, \quad u_3 = \varphi_{,3} + \psi_{,1}. \tag{2.1}$$

Inserting (2.1) into (1.6) we obtain the set of the following coupled equations:

$$\Delta\varphi - \frac{1}{c_1^2}\ddot{\varphi} = -\kappa\nu, \tag{2.2}$$
$$\Delta\psi - \frac{1}{c_2^2}\ddot{\psi} = 0$$
$$\Delta\nu - \frac{1}{C_0^2}\nu - \frac{1}{c_0^2}\ddot{\nu} - \gamma_1\dot{\nu}_1 = \beta_1\Delta\phi,$$

where

$$c_1^2 = \frac{\lambda + 2\mu}{\rho}; \quad c_2^2 = \frac{\mu}{\rho}; \quad c_0^2 = \frac{\alpha}{\rho k_0}; \tag{2.3}$$
$$\kappa = \frac{\beta}{\lambda + 2\mu}; \quad \beta_1 = \frac{\beta}{\alpha}, \quad \gamma_1 = \frac{\gamma}{\alpha}, \quad \frac{1}{C_0^2} = \frac{1}{\alpha}\left(\xi - \frac{\beta}{\lambda + 2\mu}\right) > 0, \ [3].$$

The solution of the system (2.2) is sought under the form:

$$\varphi = (A\,\mathrm{sh}\,\delta x_3 + B\,\mathrm{ch}\,\delta x_3)e^{i(\omega t - kx_1)} \tag{2.4}$$
$$\psi = (A_0\,\mathrm{sh}\,\delta_0 x_3 + B_0\,\mathrm{ch}\,\delta_0 x_3)e^{i(\omega t - kx_1)}$$
$$\nu = -\frac{1}{\kappa}\left(\Delta\varphi - \frac{1}{c_1^2}\ddot{\varphi}\right).$$

where $\delta$ and $\delta_0$ are to be determined.

By substituting (2.4) into (2.2), we obtain:

$$\delta_0 = \sqrt{k^2 - \tau^2}, \quad \tau^2 = \frac{\omega^2}{c_2^2} \tag{2.5}$$

$$\delta_{1,2}^2 = \left\{ \frac{1}{2}(a + b - \kappa\beta_1) \pm \frac{1}{2}[(a + b - \kappa\beta_1)^2 - 4(ab - \kappa\beta_1 k^2]^{1/2} \right\}^{1/2} \tag{2.5$'$}$$

where

$$a = k^2 - \sigma^2, \quad b = k^2 + \frac{1}{\ell_0^2} + i\omega\gamma_1 - \varepsilon^2, \quad \sigma^2 = \frac{\omega^2}{c_1^2}; \quad \varepsilon^2 = \frac{\omega^2}{c_0^2}.$$

Note that

$$\delta_1^2 + \delta_2^2 = a + b - \kappa\beta_1, \quad \delta_1^2\delta_2^2 = ab - \beta_1\kappa k^2. \tag{2.5$''$}$$

Using (2.4) and the notations:

$$\varphi_j(x_3) = A_j \operatorname{sh}\delta_j x_3 + B_j \operatorname{ch}\delta_j x_3, \quad j = \overline{0,3}, \tag{2.6}$$

where $A_j$ and $B_j$ are constants, we can write the general solution of the system (2.2) under the form:

$$\begin{aligned} \varphi &= (\varphi_1 + \varphi_2)e^{i(\omega t - kx_1)} \\ \psi &= \varphi_0 e^{i(\omega t - kx_1)} \\ \nu &= -(\ell_1\varphi_1 + \ell_2\varphi_2)e^{i(\omega t - kx_1)} \end{aligned} \tag{2.7}$$

with

$$\ell_j = \frac{1}{\kappa}(\delta_j^2 - k^2 + \sigma^2), \quad j = 1, 2. \tag{2.7$'$}$$

In view of (2.1), (2.4) and (2.7), the displacement field throughout the plate read as follows:

$$\begin{aligned} u_1(x_1, x_3, t) &= -[ik(\varphi_1 + \varphi_2) + \varphi_0']e^{i(\omega t - kx_1)} \\ u_3(x_1, x_3, t) &= (\varphi_1' + \varphi_2' - ik\varphi_0)e^{i(\omega t - kx_1)} \\ \nu(x_1, x_3, t) &= -(\ell_1\varphi_1 + \ell_2\varphi_2)e^{i(\omega t - kx_1)} \end{aligned}$$



In order to express the stresses, we use the representations (2.1) in the stress-strain relations (1.2). After some transformations we obtain:

$$\begin{aligned} \sigma_{31} &= \mu(2\varphi_{,13} + \psi_{,11} - \psi_{,33}), \\ \sigma_{33} &= -\mu(2\varphi_{,11} + \tau^2\varphi - 2\psi_{,13}) \\ h_3 &= \alpha\nu_{,3}. \end{aligned}$$

When the expressions of functions $\varphi$ and $\psi$ are substituted in these formula with the notations (2.3) and (2.5), one has:

$$\begin{aligned} \sigma_{31}(x_1, x_3, t) &= -\mu[2ik(\varphi_1' + \varphi_2') + (2k^2 - \tau^2)\varphi_0]e^{i(\omega t - kx_1)} \\ \sigma_{33}(x_1, x_3, t) &= -\mu[(2k^2 - \tau^2)(\varphi_1 + \varphi_2) - 2ik\varphi_0']e^{i(\omega t - kx_1)} \\ h_3(x_1, x_3, t) &= -\alpha(\ell_1\varphi_1' + \ell_2\varphi_2')e^{i(\omega t - kx_1)} \end{aligned} \tag{2.8}$$

Solutions (2.7) contain both the symmetric and anti-symmetric modes of vibrations. We can separate these modes, as follows:

(i) *In the case of symmetrical vibrations:*

$$(u_1, u_3, \nu)(-x_1, -x_3, t) = (u_1, -u_3, \nu)(x_1, x_3, t)$$

*which are satisfied when $\varphi$ is even and $\psi$ is odd in variable $x_3$. In view of (2.7) and (2.6) this implies*

$$A_1 = A_2 = 0, \quad B_0 = 0.$$

According to (2.8) the appropriate expression of the stresses are:

$$\begin{aligned} \sigma_{31} &= -[2ik(\delta_1 B_1 \operatorname{sh}\delta_1 x_3 + \delta_2 B_2 \operatorname{sh}\delta_2 x_3) + \\ &\quad + (2k^2 - \tau^2)A_0 \operatorname{sh}\delta_0 x_3]e^{i(\omega t - kx_1)} \\ \sigma_{33} &= -\mu[(2k^2 - \tau^2)(B_1 \operatorname{ch}\delta_1 x_3 + B_2 \operatorname{ch}\delta_2 x_3) - \\ &\quad - 2ik\delta_0 A_0 \operatorname{ch}\delta_0 x_3]e^{i(\omega t - kx_1)} \\ h_3 &= -\alpha[\ell_1\delta_1 B_1 \operatorname{sh}\delta_1 x_3 + \ell_2\delta_2 B_2 \operatorname{sh}\delta_2 x_3]e^{i(\omega t - kx_1)} \end{aligned} \tag{2.9}$$

(ii) *In the case of anti-symmetrical vibrations, the displacement field is characterized by:*

$$(u_1, u_3, \nu)(-x_1, -x_3, t) = (-u_1, u_3, -\nu)(x_1, x_3, t)$$

*which correspond to $\varphi$ odd and $\psi$ even in the variable $x_3$.*

In virtue of the same formula (2.7) and (2.6), we must have:

$$B_1 = B_2 = 0, \quad A_0 = 0.$$

Consequently, from (2.8), we obtain:

$$\begin{aligned}
\sigma_{31} &= -[2ik(\delta_1 A_1 \operatorname{ch}\delta_1 x_3 + \delta_2 A_2 \operatorname{ch}\delta_2 x_3) + \\
&\quad + (2k^2 - \tau^2) B_0 \operatorname{ch}\delta_0 x_3] e^{i(\omega t - kx_1)} \\
\sigma_{33} &= -\mu[(2k^2 - \tau^2)(\delta_1 A_1 \operatorname{sh}\delta_1 x_3 + \delta_2 A_2 \operatorname{sh}\delta_2 x_3) - \\
&\quad - 2ik\delta_0 B_0 \operatorname{sh}\delta_0 x_3] e^{i(\omega t - kx_1)} \\
h_3 &= -\alpha[\ell_1 \delta_1 A_1 \operatorname{ch}\delta_1 x_3 + \ell_2 \delta_2 A_2 \operatorname{ch}\delta_2 x_3] e^{i(\omega t - kx_1)}
\end{aligned} \tag{2.10}$$

By sustituting (2.9) and (2.10) into (1.7) we arrive, in both cases to some homogeneous system of three equations for unknown constants $B_1, B_2, A_0$ and, respectively, $A_1, A_2, B_0$. The condition of consistency for these constants lieds to two transcendental equations for the wave number $k$. These equations have the form:

– *For symmetrical vibrations:*

$$\frac{2k^2 - \tau^2}{4k^2\delta_0} = \frac{\delta_1\delta_2(\ell_1 - \ell_2)\operatorname{cth}\delta_0 h}{\ell_1\delta_1 \operatorname{cth}\delta_2 h - \ell_2\delta_2 \operatorname{cth}\delta_1 h}, \tag{2.11}$$

– *For anti-symmetrical vibrations:*

$$\frac{2k^2 - \tau^2}{4k^2\delta_0} = \frac{\delta_1\delta_2(\ell_1 - \ell_2)\operatorname{th}\delta_0 h}{\ell_1\delta_1 \operatorname{th}\delta_2 h - \ell_2\delta_2 \operatorname{th}\delta_1 h} \tag{2.12}$$

Solving these equations we will obtain, generally, complex values for the wave number $k(\omega) = k_R(\omega) + k_i(\omega)$. Therefore we can write:

$$e^{i(\omega t - kx_1)} = e^{ik_R(\omega t/k_R - x_1)} e^{-k_i x_1}.$$

The phase velocity of the waves are given by $c = \dfrac{\omega}{k_R}$, and the exponential factor containing $k_i$, indicates the fact that the propagation phenomena is accomplished with attenuation along $0x_1$.

Moreover, since the phase velocity depends on frequency $\omega$, $c = c(\omega)$, the waves are subjected to dispersion. The analysis of equations (2.11) and (2.12) under the general form presents some difficulties. However, we can obtain some concret results in two extremal cases; namely $k_R h >> 1$ or $k_R h << 1$. These cases will be investigated below. In this order we introduce the following dimensionless quatities:



$$\varsigma = \frac{\omega^2}{k^2 c_2^2}, \quad \delta_s = \frac{\delta_s}{k}, \quad s = \overline{0,3}, \quad m = \frac{c_2^2}{c_1^2}, \quad n = \frac{c_2^2}{c_0^2}. \tag{2.13}$$

With these notations, from $(2.5)'$ and $(2.5)''$, after some calculus, we obtain:

$$\begin{aligned}
\delta_0^2 &= 1 - \varsigma, \\
\delta_1^2 + \delta_2^2 &= 2 - m\varsigma + M(\omega)\varsigma, \\
\delta_1^2\delta_2^2 &= (1 - m\varsigma)[1 + M(\omega)\varsigma] - \frac{\kappa\beta_1 c_2^2}{\omega^2}\varsigma,
\end{aligned} \tag{2.14}$$

where

$$M(\omega) = -n + \frac{c_1^2}{c_0^2\omega^2} + \frac{i\gamma_1 c_2^2}{\omega} - \frac{\kappa\beta_1 c_2^2}{\omega^2}.$$

**3. The analysis of the frequency equations.** In what follows we focus our attention to the frequency equation (2.12), which gives the wave number for anti symmetrical vibrations in terms of $\omega$, considered as independent variable with real values.

As we observed, the waves throughout the plate are subjected to dispersion and attenuation. Setting

$$v = \frac{\omega}{k} = c_2\sqrt{\varsigma},$$

for the phase velocity $c$, we obtain

$$c = \frac{\omega}{k_R} = \frac{1}{Re(v^{-1})} = \frac{1}{Re\{c_2\sqrt{\varsigma}\}^{-1}}. \tag{3.1}$$

The measure of attentuation is given by the coefficient of attenuation:

$$d = \frac{1}{\omega\, Im\{c_2\sqrt{\varsigma}\}^{-1}}, \tag{3.2}$$

where $Re\{\ldots\}$ and $Im\{\ldots\}$ denote, respectively, the real and imaginary parts of the expressions inside the brackets.

In order to derive some quantitative results concerning the propagation phenomena, we define the characteristic frequence $\omega_0$, by the relation:

$$\omega_0^2 = \frac{c_1^2}{\ell_0^2},$$

where $c_1$ is the velocity of longitudinal waves and $\ell_0$ is the characteristic length of the medium [3].

In addition, considering $\omega_0$ as a reference frequency we introduce the reduced dimensionless frequency

$$\eta = \frac{\omega}{\omega_0}. \tag{3.3}$$

With this notation, the formula (2.14) can be rewritten as follows:

$$\begin{aligned} &\delta_0^2 = 1 - \varsigma, \\ &\delta_1^2 + \delta_2^2 = 2 - (m+n)\varsigma + m\varsigma\left(\frac{A}{\eta^2} + \frac{iC}{\eta}\right) \\ &\delta_1^2\delta_2^2 = (1 - m\varsigma)(1 + n\varsigma) + (1 - m\varsigma)\left(\frac{A}{\eta^2} + \frac{iC}{\eta}\right) - \frac{\kappa\beta_1\ell_0^2}{\eta^2} m\varsigma. \end{aligned} \tag{3.4}$$

where

$$A = 1 - \kappa\beta_1\ell_0^2, \quad C = \gamma_1 C_1 \ell_0.$$

In view of (3.4), the frequency equation (2.12) one reduces to an equation giving $\varsigma$ as a function of $\omega$ and parameter $\eta$: $\varsigma = \varsigma(\omega, \eta)$.

The following considerations refers to the cases $k_R h >> 1$ and respectively $k_R h << 1$.

**1. The case** $k_R h >> 1$. (the short wave lengths approximation) In this case the function th(...)$h$ appearing into (2.12) will be substituted approximately by unity. By using (2.13) and (2.7)$'$, after some calculus (2.12) can be written under the form:

$$(2-\varsigma)^2(\delta_1^2 + \delta_2^2 + \delta_1^2\delta_2^2 - 1 + m\varsigma) = 4\delta_0\delta_1\delta_2(\delta_1 + \delta_2). \tag{3.5}$$

At this point we will separate the problem into two parts as for as $\omega << \omega_0$ (or $\eta << 1$), respectively $\omega >> \omega_0$ (or $\eta >> 1$).

**1a. The case $\omega << \omega_0$.** If $\omega << \omega_0$, then we have

$$\frac{\omega^2}{\omega_0^2} = \eta^2 << 1$$



and the terms containing $\eta^{-2}$ greater than that of order $\eta^{-1}$. Expanding the terms appearing in (3.5) in power serie of reduced frequency $\eta$, we obtain:

$$\begin{aligned} &\delta_1^2 + \delta_2^2 + \delta_1\delta_2 - 1 + m\varsigma = \frac{m\varsigma}{\eta^2}(1 + 0(\eta)), \\ &\delta_1\delta_2(\delta_1 + \delta_2) = \frac{m\varsigma}{\eta^2}\sqrt{1 + \kappa\beta_1\ell_0^2 - m\varsigma}(1 + 0(\eta)) \end{aligned}$$

Therefore, we can write (3.5) as follows:

$$(2-\varsigma)^2\frac{m\varsigma}{\eta^2}(1 + 0(\eta)) - \frac{4m\varsigma}{\eta^2}\sqrt{1-\varsigma}\sqrt{1 + \kappa\beta_1\ell_0^2 - m\varsigma}(1 + 0(\eta)) = 0.$$

From this relation, by neglecting the terms of order $\eta$, one has:

$$(2-\varsigma)^2 - 4\sqrt{1-\varsigma}\sqrt{1 + \kappa\beta_1\ell_0^2 - m\varsigma} = 0. \tag{3.6}$$

Since this equation is independent of $\omega$, we conclude that the plane waves propagate in the approximation $0(\eta)$ with no dispersion.

Further, in view of (2.13), (3.6) can be written under equivalent form as:

$$\left(2 - \frac{c^2}{c_2^2}\right)^2 = 4\left[\left(1 - \frac{c^2}{c_2^2}\right)\left(1 - \kappa\beta_1\ell_0^2 - \frac{c^2}{c_1^2}\right)\right]^{1/2} \tag{3.7}$$

which represents the Rayleigh equation describing the surface wave propagation in a semi infinite medium with voids, the influence of voids structure being pointed out by the term $\kappa\beta_1\ell_0^2$.

Therefore, we conclude tht in this case the waves are not subjected to dispersion and attenuation.

**1b. The case $\omega >> \omega_0$.** When $\omega >> \omega_0$, we have

$$\frac{\omega^2}{\omega_0^2} = \eta^2 >> 1, \quad \text{or} \quad \frac{1}{\eta^2} << 1.$$

Setting $\eta^2 = z_1$ and neglecting the terms of order $z_1^2$, from (3.4), we find:

$$\delta_0^2 = 1 - \varsigma, \; \delta_1^2 = (1 - m\varsigma), \; \delta_2^2 = (1 - m\varsigma + m\varsigma z_1). \tag{3.8}$$

By introducing these expressions into (3.5), after some operations, we obtain the equation:

$$\frac{(2-\varsigma)^2}{4\sqrt{1-\varsigma}\sqrt{1-m\varsigma}} = 1 \tag{3.9}$$

which characterises just the propagation of a surface wave in a classical elastic medium. Therefore, in this case, the presence of voids does not affect the propagation phenomena.

**2. The case** $k_R h << 1$. (the long flexural wave approximation) In the investigation of this case, we will approximate the function $\mathrm{th}(\ldots)h$, as follows:

$$\mathrm{th}\,\delta_i h = \delta_i h(1 - \delta_i^2 h^2).$$

Using this approximation, the dispersion equation (2.12) can be written under the form:

$$\frac{(2-\varsigma)^2}{4\delta_0^2} = \frac{(\ell_1 - \ell_2)\delta_0(1 - \frac{1}{3}\delta_0^2 h^2)}{\ell_1 - \ell_2 - \frac{1}{3}(\ell_1\delta_2^2 - \ell_2\delta_1^2)h^2}.$$

When the expression of $\ell_i$ given by $(2.7)'$ are taken into account the above relation can be transformed to obtain

$$(3.11)\qquad \frac{(2-\varsigma)^2}{4(1-\varsigma)} = \frac{3-(1-\varsigma)(hk)^2}{3-(1-m\varsigma)(hk)^2}.$$

From the equation one gets $\varsigma$ as function of $(hk)^2$, which indicates that the anti-symmetrical waves are subjected to dispersion. In addition, since the influence of voids structure do not appear, we conclude that the propagation phenomena is similar to that in the classical elastic medium.

When in (3.11) we effect some calculus in which the terms $\varsigma\lambda^2$ are dropped, we arrive to the formulae:

$$\varsigma \simeq \frac{4}{3}(hk)^2(1-m).$$

From this relation, we obtain for the phase velocity the expression:

$$c = \frac{4}{3}(hk)^2\,\frac{\lambda+\mu}{\lambda+2\mu}\,\frac{\mu}{\rho}.$$

This relation shows that the phase velocity decreases to zero with increasing wave length. The same property is true for the group velocity.

Finally, we observe that asimilar study can be made in the case of symmetric vibrations of the plate (see [4]).



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