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CONFORMAL COMPLEX LANDSBERG SPACES

BY

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Abstract. In this paper, we discuss about the conformal complex Landsberg spaces with some of theirs important subclasses. The study is made with respect to the complex linear connections: Chern-Finsler, Berwald and Rund. The function ρ between conformal Landsberg (G – or strong – Landsberg) spaces (M, L) and $(M, \tilde{L})$ must be constant unless the spaces are purely Hermitian. A special approach is devoted to the projective generalized Berwald manifolds and to the conformal projective complex Berwald spaces, in particular.

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Key words: conformal complex Landsberg, G-Landsberg, strong-Landsberg spaces, conformal projective complex Berwald spaces.

1. Introduction

Many great contributions to the geometry of the conformal real Landsberg and Berwald spaces are due to Berwald [9], Hashiguchi, Ichijyō [12], Matsumoto [14], Ikeda [13], Aikou [3], Vincze [19], see also [7,8,11,15,18]. The conformal complex Berwald spaces are studied by Aikou in [2].

In a previous paper [5] we have introduced and studied two classes of complex Finsler spaces: complex Landsberg spaces and generalized Berwald with some others subclasses: G-Landsberg, strong-Landsberg and, respectively, complex Berwald.

Two complex Finsler manifolds (M, L) and $(M, \tilde{L})$ are called conformal if $\tilde{L} = e^{\rho}L$, where ρ is a real valued function which depends only on the position z. In the present paper our aim is to give an answer how can we check the conformality of a complex Finsler manifold with a complex Landsberg manifold and with a generalized Berwald manifold, respectively.

The study tools are the complex linear connections of Berwald and Rund type. A complex Landsberg space remains complex Landsberg under any conformal change if and only if either the function ρ is a constant or the spaces are purely Hermitian, (Theorem 3.1). We show that the generalized Berwald property is preserved to a conformal change, (Theorem 3.2). A necessary and sufficient condition for that the complex Berwald property to persist under a conformal change of metrics is proven in Theorem 3.3. Also, the conformal flatness is discussed, (Corollary 3.2).

A special conformal change of complex Finsler metrics is given by those conformal projective. In [17] the complex Finsler metrics L and $\tilde{L}$ on a common underlying manifold M, are called projective if these have the same complex geodesics as point sets. Under a projective change the complex Berwald property is preserve in the particular case of two dimensional manifolds, (Theorem 4.2). Moreover, we obtain that a complex Berwald space is transformed in one complex Berwald too, by a conformal projective change, (Theorem 4.3).

The organization of the paper is as follows. In §2, we recall some preliminary properties of the n-dimensional complex Finsler spaces and in particularly we speak about the complex Landsberg and the generalized Berwald spaces. In sections §3 and §4, we prove aforementioned outcomes.

2. Preliminaries

2.1. Complex Finsler spaces

For the beginning we will make a survey of complex Finsler geometry and we will set the basic notions and terminology. For more see [1,4,16,20,21].

Let M be a n-dimensional complex manifold, $z = (z^k)_{k=\overline{1,n}}$ are complex coordinates in a local chart.

The complexified of the real tangent bundle $T_C M$ splits into the sum of holomorphic tangent bundle $T'M$ and its conjugate $T''M$. The bundle $T'M$ is itself a complex manifold, and the local coordinates in a local chart will be denoted by $u = (z^k, \eta^k)_{k=\overline{1,n}}$. They are changed into $(z'^k, \eta'^k)_{k=\overline{1,n}}$ by the rules $z'^k = z'^k(z)$ and $\eta'^k = \frac{\partial z'^k}{\partial z^l}\eta^l$.

A *complex Finsler space* is a pair (M, F), where $F : T'M \to \mathbb{R}^+$ is a continuous function satisfying the conditions:

i) $L := F^2$ is smooth on $\widetilde{T'M} := T'M \backslash \{0\}$;

ii) $F(z,\eta) \geq 0$, the equality holds if and only if $\eta = 0$;

iii) $F(z,\lambda\eta) = |\lambda|F(z,\eta)$ for $\forall\lambda \in \mathbb{C}$;

iv) the Hermitian matrix $\big(g_{i\bar{j}}(z,\eta)\big)$ is positive definite, where $g_{i\bar{j}} := \frac{\partial^2 L}{\partial\eta^i\partial\bar{\eta}^j}$ is the fundamental metric tensor. Equivalently, it means that the indicatrix is strongly pseudo-convex.

Roughly speaking, the geometry of a complex Finsler space consists of the study of the geometric objects of the complex manifold $T'M$ endowed with the Hermitian metric structure defined by $g_{i\bar{j}}$. Thus, the first step is to study sections of the complexified tangent bundle of $T'M$, which is decomposed in the sum $T_C(T'M) = T'(T'M) \oplus T''(T'M)$. Let $VT'M \subset T'(T'M)$ be the vertical bundle, locally spanned by $\{\frac{\partial}{\partial\eta^k}\}$, and $VT''M$ be its conjugate. At this point, the idea of complex nonlinear connection, briefly (*c.n.c.*), is an instrument in 'linearization' of this geometry. A (*c.n.c.*) is a supplementary complex subbundle to $VT'M$ in $T'(T'M)$, i.e. $T'(T'M) = HT'M \oplus VT'M$. The horizontal distribution $H_uT'M$ is locally spanned by $\{\frac{\delta}{\delta z^k} = \frac{\partial}{\partial z^k} - N_k^j\frac{\partial}{\partial\eta^j}\}$, where $N_k^j(z,\eta)$ are the coefficients of the (*c.n.c.*). The pair $\{\delta_k := \frac{\delta}{\delta z^k}, \dot{\partial}_k := \frac{\partial}{\partial\eta^k}\}$ will be called the adapted frame of the (*c.n.c.*) which obey to the change rules $\delta_k = \frac{\partial z'^j}{\partial z^k}\delta'_j$ and $\dot{\partial}_k = \frac{\partial z'^j}{\partial z^k}\dot{\partial}'_j$. By conjugation everywhere we have obtained an adapted frame $\{\delta_{\bar{k}}, \dot{\partial}_{\bar{k}}\}$ on $T''_u(T'M)$. The dual adapted bases are $\{dz^k, \delta\eta^k\}$ and $\{d\bar{z}^k, \delta\bar{\eta}^k\}$.

Certainly, a main problem in this geometry is to determine a (*c.n.c.*) related only to the fundamental function of the complex Finsler space (M, F).

The next step is the action of a derivative law D on the sections of $T_C(T'M)$. A Hermitian connection D, of $(1,0)$-type, which satisfies in addition $D_{JX}Y = JD_XY$, for all X horizontal vectors and J the natural complex structure of the manifold, is so called Chern-Finsler connection (cf. [1]), in brief $C-F$. The $C-F$ connection is locally given by the following coefficients (cf. [16]):

$$N_j^i = g^{\overline{m}i}\frac{\partial g_{l\overline{m}}}{\partial z^j}\eta^l = L^i_{lj}\eta^l; L^i_{jk} = g^{\bar{l}i}\delta_k g_{j\bar{l}}; C^i_{jk} = g^{\bar{l}i}\dot{\partial}_k g_{j\bar{l}}; L^i_{j\bar{k}} = C^i_{j\bar{k}} = 0, \tag{2.1}$$

where here and further on δ_k is the adapted frame of the $C-F$ (*c.n.c.*) and $D_{\delta_k}\delta_j = L^i_{jk}\delta_i$, $D_{\dot{\partial}_k}\dot{\partial}_j = C^i_{jk}\dot{\partial}_i$, etc. The complex Cartan tensors are $C_{j\bar{h}k} := C^i_{jk}g_{i\bar{h}} = \dot{\partial}_k g_{j\bar{h}}$ with their conjugates.

Let us recall that in [1]'s terminology, the complex Finsler space (M, F) is *strongly Kähler* iff $T^i_{jk} = 0$, *Kähler* iff $T^i_{jk}\eta^j = 0$ and *weakly Kähler* iff $g_{i\bar{l}}T^i_{jk}\eta^j\overline{\eta}^l = 0$, where $T^i_{jk} := L^i_{jk} - L^i_{kj}$. In [10] it is proved that strongly Kähler and Kähler notions actually coincide. We notice that in the particular case of complex Finsler metrics which come from Hermitian metrics on M, so-called *purely Hermitian metrics* in [16], (i.e. $g_{i\bar{j}} = g_{i\bar{j}}(z)$), those kinds of Kähler are same.

2.2. Complex Landsberg spaces

It is well known by [1, 16] that the complex geodesics curves are defined by means of Chern-Finsler (*c.n.c.*). Its local coefficients $N^i_j = g^{\bar{m}i}\frac{\partial g_{l\bar{m}}}{\partial z^j}\eta^l$ always determine a complex spray with coefficients $G^i = \frac{1}{2}N^i_j\eta^j$. Further on, G^i induce a (*c.n.c.*) by $\overset{c}{N}{}^i_j := \dot{\partial}_j G^i$ called *canonical* in [16], where is proved that it coincides with Chern-Finsler (*c.n.c.*) if and only if the complex Finsler metric is Kähler. Using canonical (*c.n.c.*) we associate to it the next complex linear connections:

– one of Berwald type $B\Gamma := (\overset{c}{N}{}^i_j, \overset{B}{L}{}^i_{jk} := \dot{\partial}_k \overset{c}{N}{}^i_j = \overset{B}{L}{}^i_{kj}, \overset{B}{L}{}^i_{j\bar{k}} := \dot{\partial}_{\bar{k}} \overset{c}{N}{}^i_j, 0, 0)$ and

– another of Rund type $R\Gamma := (\overset{c}{N}{}^i_j, \overset{c}{L}{}^i_{jk} := \frac{1}{2}g^{\bar{l}i}(\overset{c}{\delta}_k\, g_{j\bar{l}} + \overset{c}{\delta}_j\, g_{k\bar{l}}), \overset{c}{L}{}^i_{j\bar{k}} := \frac{1}{2}g^{\bar{l}i}(\overset{c}{\delta}_{\bar{k}}\, g_{j\bar{l}} - \overset{c}{\delta}_{\bar{l}}\, g_{j\bar{k}}), 0, 0)$, where $\overset{c}{\delta}_k := \frac{\partial}{\partial z^k} - \overset{c}{N}{}^j_k\, \dot{\partial}_j$.

$R\Gamma$ is only h-metrical and $B\Gamma$ is neither h- nor v- metrical, (for more details see [16]). Note that $2G^i = N^i_j\eta^j = \overset{c}{N}{}^i_j\, \eta^j = \overset{B}{L}{}^i_{jk}\, \eta^j\eta^k$. Additionally, in the Kähler case we have $\overset{c}{\delta}_k = \delta_k$ and so, $L^i_{jk} = \overset{c}{L}{}^i_{jk} = \overset{B}{L}{}^i_{jk}$ and $\overset{c}{L}{}^i_{j\bar{k}} = 0$.

Further on, everywhere in this paper the Berwald and Rund connections will be specified by a super-index, like above (e.g. $\overset{c}{\delta}_k$, $\overset{B}{L}{}^i_{jk}$, $\overset{c}{L}{}^i_{jk}$, $X_{\underset{|\,k}{B}}$, etc.), while for the $C-F$ connection will keep the initial generic notation without super-index (e.g. δ_k, L^i_{jk}, $X_{|\,k}$, etc.).

In complex Finsler geometry we speak about of complex *Landsberg* space iff $\overset{B}{L}{}^i_{jk} = \overset{c}{L}{}^i_{jk}$, equivalently with $C_{\underset{l\bar{r}h}{}\,\underset{|\,0}{B}} = 0$, where $\overset{B}{|}$ is h-covariant derivative with respect to $B\Gamma$ connection. Further, we have defined the class of G-*Landsberg* spaces. It contains the complex Landsberg spaces with $\dot{\partial}_{\bar{k}}G^i = 0$, (see [5]). The necessarily and sufficient conditions for G-Landsberg spaces

are $C\overset{B}{}_{l\bar{r}h\,|\,0} = 0$ and $C\overset{B}{}_{r\bar{0}h\,|\,\bar{0}} = 0$. A reinforcement of this tensorial characterization gives rise to a subclass of G-Landsberg, namely *strong Landsberg* iff $C\overset{B}{}_{l\bar{r}h\,|\,0} = 0$ and $C\overset{B}{}_{j\bar{r}h\,|\,\bar{0}} = 0$, equivalently with either $C\overset{B}{}_{j\bar{r}h\,|\,\bar{k}} = 0$ or $C\overset{B}{}_{l\bar{r}h\,|\,k} = 0$ and $\dot{\partial}_{\bar{h}} G^i = 0$. Because any Kähler space is a complex Landsberg space, the substitution of the Landsberg condition with the Kähler condition in the definition of the G-Landsberg spaces leads to another subclass of this, called us *G-Kähler*. These spaces coincide with the category of complex Berwald spaces defined by Aikou in [2], i.e. the complex Finsler spaces which are Kähler and $L^i_{jk}(z)$. An extension of the complex Berwald spaces, directly related to the $B\Gamma$ connection, is called by us *generalized Berwald* in [5]. It is with the coefficients $\overset{B}{L}{}^i_{jk}$ depending only on the position z, equivalently with either $\dot{\partial}_{\bar{h}} G^i = 0$ or $B\Gamma$ is of $(1,0)$-type. The intersection between complex Landsberg spaces and generalized Berwald spaces give the G-Landsberg spaces. Among above classes of complex Finsler spaces we have the inclusions: complex Berwald space $\subset$ strong Landsberg $\subset$ G-Landsberg $\subset$ complex Landsberg.

3. Conformal complex Landsberg metrics

Let (M, L) be a complex Finsler space and $\tilde{L}$ be another complex Finsler metric on the manifold M. The complex Finsler metrics L and $\tilde{L}$ on a common underlying manifold M, are called conformal iff $\tilde{L} = e^{\rho(z)} L$, where ρ is a real valued function which depends only on the position z. The following Lemma gives the link between the geometrical objects of the conformal complex Finsler spaces $(M, \tilde{L})$ and (M, L).

Lemma 3.1. *Let L and $\tilde{L}$ be two conformal complex Finsler metrics. Then,*

i) $\tilde{G}^i = G^i + \frac{1}{2}\frac{\partial \rho}{\partial z^j}\eta^j\eta^i$; $\dot{\partial}_{\bar{h}}\,\tilde{G}^i = \dot{\partial}_{\bar{h}}\,G^i$;

ii) $\overset{c}{\tilde{N}}{}^i_j = \overset{c}{N}{}^i_j + \frac{1}{2}(\frac{\partial \rho}{\partial z^j}\eta^i + \frac{\partial \rho}{\partial z^l}\eta^l\delta^i_j)$; $\overset{c}{\tilde{\delta}}_j = \overset{c}{\delta}_j - \frac{1}{2}(\frac{\partial \rho}{\partial z^j}\eta^i + \frac{\partial \rho}{\partial z^l}\eta^l\delta^i_j)\,\dot{\partial}_i$;

iii) $\overset{B}{\tilde{L}}{}^i_{jk} = \overset{B}{L}{}^i_{jk} + \frac{1}{2}(\frac{\partial \rho}{\partial z^j}\delta^i_k + \frac{\partial \rho}{\partial z^k}\delta^i_j)$; $\overset{B}{\tilde{L}}{}^i_{j\bar{k}} = \overset{B}{L}{}^i_{j\bar{k}}$;

iv) $\tilde{g}_{i\bar{j}} = e^{\rho(z,\eta)} g_{i\bar{j}}$; $\overset{c}{\tilde{L}}{}^i_{jk} = \overset{c}{L}{}^i_{jk} + \frac{1}{2}(\frac{\partial \rho}{\partial z^j}\delta^i_k + \frac{\partial \rho}{\partial z^k}\delta^i_j - \frac{\partial \rho}{\partial z^l}\eta^l C^i_{jk})$;

v) $\tilde{N}^i_j = N^i_j + \frac{\partial \rho}{\partial z^j}\eta^i$; $\tilde{\delta}_j = \delta_j - \frac{\partial \rho}{\partial z^j}\ \eta^i\ \dot{\partial}_i$; $\tilde{L}^i_{jk} = L^i_{jk} + \delta^i_j \frac{\partial \rho}{\partial z^k}$; $\tilde{C}^i_{jk} = C^i_{jk}$.

The proof results by direct computations.

Theorem 3.1. *Let (M, L) and $(M, \tilde{L})$ be two conformal complex Finsler spaces. The conformal L and $\tilde{L}$ metrics are both complex Landsberg if and only if either the function ρ is a constant or the spaces are purely Hermitian.*

Proof. We suppose that (M, L) is a complex Landsberg space. Thus, using Lemma 3.1 iii) and iv) we obtain $\tilde{L}^{i\ c}_{jk} = \tilde{L}^{i\ B}_{jk} - \frac{1}{2}\frac{\partial \rho}{\partial z^l}\eta^l C^i_{jk}$. From here we have $(M, \tilde{L})$ is a complex Landsberg space if and only if $\frac{\partial \rho}{\partial z^l}\eta^l C^i_{jk} = 0$, which is equivalently with either ρ is a constant or $C^i_{jk} = 0$. Lemma 3.1 v) gives $\tilde{C}^i_{jk} = C^i_{jk} = 0$, i.e. the spaces are purely Hermitian. □

Theorem 3.2. *Let (M, L) and $(M, \tilde{L})$ be two conformal complex Finsler spaces. (M, L) is generalized Berwald if and only if $(M, \tilde{L})$ is also generalized Berwald.*

The proof results immediately by Lemma 3.1 i).
As consequences of the above two theorems we have.

Corollary 3.1. *The conformal function ρ between the G-Landsberg (strong-Landsberg) spaces (M, L) and $(M, \tilde{L})$ compulsory needs be constant unless the spaces are purely Hermitian.*

Proposition 3.1. *Let (M, L) and $(M, \tilde{L})$ be two conformal complex Finsler spaces. If one of the metrics L and $\tilde{L}$ is Kähler then the other is also Kähler if and only if the function ρ is a constant.*

Proof. We suppose that (M, L) is a Kähler space. So, by Lemma 3.1 v) it results $\tilde{L}^i_{jk} - \tilde{L}^i_{kj} = \delta^i_j \frac{\partial \rho}{\partial z^k} - \delta^i_k \frac{\partial \rho}{\partial z^j}$ and so forth $\tilde{L}$ is Kähler iff ρ is a constant. □

Theorem 3.3. *Let (M, L) and $(M, \tilde{L})$ be two conformal complex Finsler spaces. If one of the metrics L and $\tilde{L}$ is complex Berwald then the other is also complex Berwald if and only if the function ρ is a constant.*

As in the real case, the *conformal flatness* means that the complex Finsler space $(M, \tilde{L})$ is conformal to a locally Minkowski space (M, L), i.e. $\tilde{L}(z, \eta) = e^{\rho(z)} L(\eta)$.

Corollary 3.2. *Let (M, L) and $(M, \tilde{L})$ be two conformal complex Finsler spaces. If $(M, \tilde{L})$ is conformal flatness then it is generalized Berwald.*

An example of conformal flatness metric is given by the complex version of *Antonelli - Shimada* metric

$$\tilde{L}_{AS}(z, w; \eta, \theta) := e^{2\rho} \left(|\eta|^4 + |\theta|^4\right)^{\frac{1}{2}}, \text{ with } \eta, \theta \neq 0, \tag{3.1}$$

on a domain D from $\widetilde{T'M}$, $\dim_C M = 2$, such that its metric tensor is nondegenerated. We relabeled the local coordinates z^1, z^2, η^1, η^2 as z, w, η, θ, respectively. $\rho(z, w)$ is a real valued function and $|\eta^i|^2 := \eta^i \overline{\eta}^i$, $\eta^i \in \{\eta, \theta\}$, ([16]). It is conformal with the locally Minkowski metric $L(z, w; \eta, \theta) := \left(|\eta|^4 + |\theta|^4\right)^{\frac{1}{2}}$. A direct computation leads to

$$\overset{B}{\tilde{L}}{}^z_{zz} = 2\frac{\partial \rho}{\partial z};\ \overset{B}{\tilde{L}}{}^z_{zw} = \overset{B}{\tilde{L}}{}^z_{wz} = \frac{\partial \rho}{\partial w};\ \overset{B}{\tilde{L}}{}^w_{zz} = 0;\ \overset{B}{\tilde{L}}{}^w_{zw} = \overset{B}{\tilde{L}}{}^w_{wz} = \frac{\partial \rho}{\partial z};\ \overset{B}{\tilde{L}}{}^w_{ww} = 2\frac{\partial \rho}{\partial w}$$

which depend only on z and w. Thus $\tilde{L}_{AS}$ is generalized Berwald.

4. Projective complex Berwald metrics

In this section we will discuss about of projective complex Finsler spaces. For beginning we suppose that (M, L) is a weakly Kähler space. Thus, the complex geodesic curve are solutions of the equations

$$\frac{d^2 z^k}{dt^2} + 2G^k(z^k(t), \eta^k(t)) = 0. \tag{4.1}$$

Under mentioned assumptions we have $\overset{c}{\delta}_k\ L = 0$. We consider $\tilde{L}$ an arbitrary complex Finsler metric on M. In [17] the complex Finsler metrics L and $\tilde{L}$ on a common underlying manifold M, are called *projective* iff they have the same complex geodesics as point sets, which is equivalently with $\tilde{G}^k = G^k$ and $\overset{c}{\tilde{\delta}}_k\ \tilde{L} = \overset{c}{\delta}_k\ L = 0$. From here we have deduced that $\overset{c}{\tilde{\delta}}_k = \overset{c}{\delta}_k$ and thus $\overset{c}{\delta}_k\ (\tilde{L} - L) = 0$. This means that $\tilde{L} = L + P$, with $P(z, \lambda\eta) = |\lambda|^2 P(z, \eta)$, for $\forall \lambda \in \mathbb{C}$, and $\overset{c}{\delta}_k\ P = 0$. On the other hand, the condition $\overset{c}{\delta}_k\ (\tilde{L} - L) = 0$ implies that $(M, \tilde{L})$ is weakly Kähler, too. Moreover, between the geometrical objects of the projective complex Finsler spaces $(M, \tilde{L})$ and (M, L) we have: $\overset{c}{\tilde{\delta}}_j = \overset{c}{\delta}_j$; $\overset{c}{\tilde{N}}{}^i_j = \overset{c}{N}{}^i_j$; $\overset{B}{\tilde{L}}{}^i_{jk} = \overset{B}{L}{}^i_{jk}$ and $\overset{B}{\tilde{L}}{}^i_{j\bar{k}} = \overset{B}{L}{}^i_{j\bar{k}}$.

Theorem 4.1. *Let (M, L) and $(M, \tilde{L})$ be two projective complex Finsler spaces. The space (M, L) is generalized Berwald if and only if $(M, \tilde{L})$ is generalized Berwald too.*

For complex Finsler manifolds of dimension two we have proven in [6] that if we request the weakly Kähler and generalized Berwald conditions, then the manifolds are Berwald. Thus, we obtain.

Theorem 4.2. *Let M be a 2-dimensional complex manifold and, L and $\tilde{L}$ be two projective complex Finsler metrics. (M, L) is Berwald if and only if $(M, \tilde{L})$ is also Berwald.*

Proof. The weakly Kähler property of L and $\tilde{L}$ together with Theorem 4.1 get the proof. □

The fundamental metric tensors of the projective complex Finsler spaces (M, L) and $(M, \tilde{L})$ are related by $\tilde{g}_{i\bar{j}} = g_{i\bar{j}} + p_{i\bar{j}}$, where $p_{i\bar{j}} := \frac{\partial^2 P}{\partial \eta^i \partial \bar{\eta}^j}$. Choosing $p_{i\bar{j}}$ a conformal metric with $g_{i\bar{j}}$, i.e. $p_{i\bar{j}} = e^{\sigma(z)} g_{i\bar{j}}$, where σ is a real valued function which depends only on the position z, it results $\tilde{g}_{i\bar{j}} = (1 + e^{\sigma(z)}) g_{i\bar{j}}$. In [17] is proven that such of spaces are projective if and only if σ is a constant and they are called *conformal projective*. A characterization for conformal projective complex Landsberg spaces is subsequently.

Theorem 4.3. *Let (M, L) and $(M, \tilde{L})$ be two conformal projective complex Finsler spaces. (M, L) is Landsberg if and only if $(M, \tilde{L})$ is also Landsberg.*

Proof. Because L and $\tilde{L}$ are conformal projective it results $\tilde{\overset{c}{L}}{}^i_{jk} = \overset{c}{L}{}^i_{jk}$ and $\tilde{\overset{B}{L}}{}^i_{jk} = \overset{B}{L}{}^i_{jk}$. Thus, $\overset{c}{L}{}^i_{jk} = \overset{B}{L}{}^i_{jk}$ if and only if $\tilde{\overset{c}{L}}{}^i_{jk} = \tilde{\overset{B}{L}}{}^i_{jk}$. □

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