functions (on the circle), which we regard as a subspace of L^∞. Sarason showed that $H^\infty + C$ is a closed subalgera of L^∞. It is contained in every closed subalgebra of L^∞ that contains H^∞ as a proper subset. We obtain the following result, answering a question posed by Sarason.

Theorem 2. *To each $f \in L^\infty$ there corresponds a $g_0 \in H^\infty + C$ such that*

$$\|f - g_0\| = \inf \{\|f - g\| : g \in H^\infty + C\}.$$

The connection between this result and the previous one is made by means of the theory of Hankel matrices. If $f \in L^\infty$ let

$$H_f = \begin{vmatrix} \hat{f}(-1) & \hat{f}(-2) & \hat{f}(-3) \cdots \\ \hat{f}(-2) & \hat{f}(-3) & \hat{f}(-4) \cdots \\ \hat{f}(-3) & \hat{f}(-4) & \hat{f}(-5) \cdots \\ \cdots & \cdots & \cdots \end{vmatrix}$$

(viewed as an operator on l^2).

Nehari showed that $\|H_f\| = \operatorname{dist}\ (f, H^\infty)$. Hartman showed that H_f is compact if and only if $f \in H^\infty + C$, and Adamian, Arov, and Krein showed that $\|H_f\|_e = \operatorname{dist}\ (f, H^\infty + C)$. It can be shown that Theorem 2 is equivalent to the assertion that each Hankel operator has a closest compact operator which is itself Hankel.

Analele ştiinţifice ale Universităţii „Al. I. Cuza" Iaşi
Supliment la tomul XXV, s. I a 1979
Cu ocazia **centenarului naşterii Acad. Prof. Al. Myller**

NON-BOUNDED INTERTWINING OPERATORS OF BUNDLE SHIFTS *)

BY

ION SUCIU

1. Bundle shifts. In what follows Ω will be a bounded connected open set in the complex plane whose boundary $\partial\Omega$ consists of finite number of analytic Jordan curves. Denote by D the unit disk and by μ the normalized Lebesque measure on ∂D.

From the construction of the universal covering space for Ω we obtain the following:

— A group G of linear fractional transformations A that map D onto D.

— An open G-invariant subset ω of ∂D such that $\mu(\partial D \setminus \omega) = 0$.

— A simply connected G-invariant subset D' containing $D \setminus \omega$.

— An open set Ω' containing the closure of Ω.

— A holomorphic covering map Π from D' onto Ω' with the group of deck transformations G and such that $\Pi(D) = \Omega$ and $\Pi(\omega) = \partial\Omega$.

Let m be the harmonic measure, supported in $\partial\Omega$, of the point $\Pi(0) = z_0$ in Ω. The measure μ lifts to the universal cover the measure m, in the sense that, for any $f \in L^1(m)$, we have

$$\int_{\partial\Omega} f(z)\, dm(z) = \int_{\partial D} f(\Pi(\lambda))\, d\mu(\lambda). \tag{1.1}$$

Let $\mathcal{E}$ be a separable complex Hilbert space. Denote by $H^2_{\mathcal{E}}(\Omega)$ the space of all $\mathcal{E}$-valued analytic functions on Ω which are dominated in norm by positive harmonic functions. Then, putting

$$\|h\| = \inf \{[u(z_0)]^{1/2},\ u \text{ positive harmonic in } \Omega,\ u(z) \geq \|h(z)\|,\ z \in \Omega\} \tag{1.2}$$

we obtain a norm on $H^2_{\mathcal{E}}(\Omega)$ with respect to which, $H^2_{\mathcal{E}}(\Omega)$ becomes a Hilbert space. Any function h in $H^2_{\mathcal{E}}(\Omega)$ has m a.e. nontangentially boundary limit $\tilde{h}$ and the map $h \to \tilde{h}$ is an isometric isomorphism of $H^2_{\mathcal{E}}(\Omega)$ with a subspace $H^2_{\mathcal{E}}(\partial\Omega)$ of $L^2_{\mathcal{E}}(m)$. Hence

*) Lecture presented at the Romanian-American Seminar on Operator Theory and Applications, 20—24 March 1978.

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