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ON NONLOCAL ELASTIC CONTINUA

BY

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The problem of nonlocal elasticity was largely developed in the last years. Many points of view and many papers arose in the scientific literature, among which we quote [1], [2], [3], [4].

In the present paper we will try to derive the equations of nonlocal elasticity by similarity with the procedure used in classical mechanics of systems of points.

The external influences which a body may receive from the exterior of it are represented by surface forces, external body forces and external body couples, external heat sources, magnetic flux, etc. It is natural to believe that the ways that the body uses to reply to external actions are among internal body forces, internal couples, internal heat sources, internal magnetic flux etc.

Here we will consider the simplest case when the external actions are represented by surface tractions and body forces, and as nonlocal reply the body produces an internal body forces field.

We do not discuss about frame of reference, time and conservation of mass. We will employ the usual notations, sometimes without explanation.

Let V be the region occupied by a continous body and P an arbitrary part of V : $P \subset V$. The influence of the mass contained in the domain $V-P$ to the mass contained in the domain P can be described by the internal body forces acting in the points $x \in P$, and by the contact forces acting on ∂P, the boundary of P.

In order to derive the equations of motion for a nonlocal continuum we will admit the following principles:

I. *The principle of internal forces*

There exists a vectorial function $\mathbf{g}(x, y)$ defined on $V \times V$, called internal body forces density, such that, the contribution $\mathbf{g}_P(x)$ of the mass contained in a region $P \subset V$ to the total internal body force in the point $x \in V$ is given by

$$\mathbf{g}_P(x) = \int_P \rho(y)\, \mathbf{g}(x, y)\, dY, \quad x \in V, \quad y \in P, \tag{1}$$

where $\rho(x)$ is the density of the medium, dX and dY are elements of volume.

We will denote

$$\rho \mathbf{g}(x) = \rho \mathbf{g}_v(x) = \int_V \rho(y) \mathbf{g}(x, y)\, dY, \tag{2}$$

then $\mathbf{g}(x)$ is the total internal body forces density in the point $x \in V$.

The function $\mathbf{g}(x, y)$ has the following properties:

$$\int_P \int_P \rho(y)\, \mathbf{g}(x, y)\, dY dX = \int_P \rho(x)\, \mathbf{g}_p(x)\, dX = 0, \tag{3}$$

$$\int_P \left[\mathbf{r} \times \int_P \rho(y)\, \mathbf{g}(x, y)\, dY\right] dX = \int_P [\mathbf{r} \times \rho(x)\, \mathbf{g}_p(x)]\, dX = 0, \tag{4}$$

where $\mathbf{r}$ is the position of the point x with respect to the coordinate origin.

The formulae (3), (4) tell us that the total internal body forces of P are equivalent to zero,

II. *The principle of linear momentum*

$$\frac{d}{dt}\int_P \rho(x)\, \mathbf{v}(x)\, dX = \int_P \rho(x)\, \mathbf{f}(x)\, dX + \int_P dX \int_{V-P} \rho(y)\, \mathbf{g}(x, y) dY + \int_{\partial P} \mathbf{t}\, d\sigma, \tag{5}$$

where $\mathbf{v}$ is the velocity, $\mathbf{f}(x)$ is the external body force, $\mathbf{t}$ the surface traction on ∂P, the boundary of P, caused by the medium located in $V-P$.

This principle tells that the derivative of the linear momentum with respect to the time is equal to the resultant of external (to P) forces.

III. *The principle of moment of momentum*

$$\frac{d}{dt}\int_P [\mathbf{r} \times \rho(x) \mathbf{v}(x)] dX = \int_P [\mathbf{r} \times \rho(x)\, \mathbf{f}(x)]\, dX + \tag{6}$$

$$+ \int_P \left[\mathbf{r} \times \int_{V-P} \rho(y)\, \mathbf{g}(x, y)\, dY\right] dX + \int_{\partial P} (\mathbf{r} \times \mathbf{t})\, d\sigma$$

and this means that the derivative of the moment of momentum with respect to the time is equal to the resultant moment of external (to P) applied forces.

Using these principles we can derive the equations of motion for our medium.

Taking into account (3) and (4) we have

$$\int_P dX \int_{V-P} \rho(y)\, \mathbf{g}(x, y)\, dY = \int_P \left[\int_V \rho(y)\, \mathbf{g}(x, y)\, dY - \right. \tag{7}$$

$$\left. - \int_P \rho(y)\, \mathbf{g}(x, y)\, dY\right] dX = \int_P \rho(x)\, \mathbf{g}(x)\, dX,$$

$$\int_P \left[\mathbf{r} \times \int_{V-P} \rho(y)\, \mathbf{g}(x, y)\, dY\right] dX = \int_P \left\{\mathbf{r} \times \int_V \left[\rho(y)\, \mathbf{g}(x, y)\, dY - \right.\right. \tag{8}$$

$$\left.\left. - \int_P \rho(y)\, \mathbf{g}(x, y)\, dY\right]\right\} dX = \int_P [\mathbf{r} \times \rho(x)\, \mathbf{g}(x)]\, dX$$

and from (6) we get

$$\int_P (\mathbf{t}_{i,i} + \rho \mathbf{f} + \rho \mathbf{g} - \rho \mathbf{a})\, dX = 0, \tag{9}$$

where $\mathbf{a}$ is the acceleration.

To obtain this equation we have used the divergence theorem and the equation of continuity

$$\frac{d\rho}{dt} + \rho\, \mathrm{div}\, \mathbf{v} = 0. \tag{10}$$

Since the equation (9) must hold for an arbitrary P and the integrand is supposed to be continuous, it results that

$$\mathbf{t}_{i,i} + \rho \mathbf{f} + \rho \mathbf{g} = \rho \mathbf{a} \tag{11}$$

and in a rectangular Cartesian system of coordinates x_1, x_2, x_3 we have the well-known equations of motion

$$\sigma_{ji,j} + \rho f_i + \rho g_i = \rho a_i. \tag{12}$$

From (6), using the continuity equation (10) and (8) we obtain

$$\int_P [\mathbf{r} \times (\rho \mathbf{a} - \rho \mathbf{f} - \rho \mathbf{g})]\, dX = \int_{\partial P} (\mathbf{r} \times \mathbf{t})\, d\sigma$$

and, at last, with the divergence theorem

$$\int_P [\mathbf{r} \times (\rho \mathbf{a} - \rho \mathbf{f} - \rho \mathbf{g} - \mathbf{t}_{i,i})]\, dX +$$

$$+ \int_P [(\sigma_{32} - \sigma_{23})\, \mathbf{e}_1 + (\sigma_{13} - \sigma_{31})\, \mathbf{e}_2 + (\sigma_{21} - \sigma_{12})\, \mathbf{e}_3]\, dX = 0.$$

The first term in the above equation is zero by virtue of (11) and consequently

$$\sigma_{ij} = \sigma_{ji},$$

because this equation must hold for arbitrary P.

Now, it remains to prescribe appropriate constitutive equations for g_i and σ_{ij}.

In order to construct a constitutive equations theory one may use the way developed in [2] by E r i n g e n. For example, in a linear theory we may take the following constitutive equations:

$$g_i(x, y) = A_{ikl}(x, y)\,[\varepsilon_{kl}(x) + \varepsilon_{kl}(y)], \quad A_{ikl}(x, y) = -A_{ikl}(y, x),$$

$$g_i(x) = a_{ikl}\varepsilon_{kl} + \int_V A_{ikl}(x, y)\,\varepsilon_{kl}(y)\,dY,$$

$$\sigma_{ij}(x) = c_{ijkl}\,\varepsilon_{kl} + \int_V C_{ijkl}(x, y)\,e_{kl}(y)\,dY,$$

where ε_{ij} are the strain components and a_{ikl}, $A_{ikl}(x, y)$, c_{ijkl}, $C_{ijkl}(x, y)$ some coefficients which describe the mechanical properties of the body.

Remark 1 : In the other nonlocal theories in the central point are the so called localization residuals in the balance laws. Here, the localization residuals are removed out by the first postulate. The internal body force g_i is analogous to body force residual in the Eringen theory [2], but, at the last, the body force residual in [2] must vanish and this requirement does not arise in our description.

Remark 2 : If the body opposes internal body forces and internal body couples the stress tensor will be asymmetric, even if the body is acted by surface tractions and external body forces only.

Remark 3 : The proposed scheme is convenient for polar and heat conducting materials. In a next paper we will present some of these extensions.

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ON THE CHANGES IN THE EARTH'S INERTIA TENSOR DUE TO EARTHQUAKE FAULTING

BY

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1. Introduction. In the theory of the Earth's rotation, the calculation of changes in the Earth's inertia tensor accompanying earthquakes is of major interest. The subject was considered in various papers (see e.g. [1]—[6]). In [6], the changes in the Earth's inertia tensor are calculated by a direct application of the elastic reciprocal theorem. The problem is reduced to the finding of the stress distribution which results when the model is steadily rotated about some central axis. Two homogeneous, isotropic spherical Earth models are studied. It is known (see e.g. [6], [7]) that the deformation of a sphere is very greatly affected by heterogeneity of the material. In this paper we consider a piecewise homogeneous Earth model and study changes in the Earth's inertia tensor due to earthquake faulting. The temperature variation is taken into account.

2. Reciprocal relation. Let Ω be a bounded regular (in the sense of Kellog) region with the boundary S_2. Let S_1 be a closed regular surface contained in Ω, satisfying the conditions : $S_1 \cup S_2$ is the boundary of a domain Ω_2, contained in Ω, and S_1 is the boundary of a regular domain Ω_1, contained in Ω, such that $\Omega_1 \cap \Omega_2 = \emptyset$, $\Omega_1 \cup \bar{\Omega}_2 = \bar{\Omega}$.

Suppose that Ω_1 and Ω_2 are occupied by two elastic materials. Let $C^{(\alpha)}_{ijrs}$, $\beta^{(\alpha)}_{ij}$ be the thermoelastic coefficients of the material which occupies the domain Ω_α $(\alpha = 1, 2)$.

Throughout this paper a rectangular Cartesian coordinate system Ox_i $(i = 1, 2, 3)$ is used. Let $(\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3)$ be the corresponding unit base vectors. We shall employ the usual summation and differentiation conventions : Latin subscripts-unless otherwise specified-are understood to range over the integers 1, 2, 3, summation over repeated subscripts is implied and subscripts preceded by a comma denote partial differentiation with respect to the corresponding Cartesian coordinate. The position vector to any point in space is $\mathbf{r} = x_i\mathbf{e}_i$.

In this paper we consider the linear theory of classical thermoelasticity. Let u_i denote the components of the displacement vector. The components of the strain tensor are given by