In order to construct a constitutive equations theory one may use the way developed in [2] by Eringen. For example, in a linear theory we may take the following constitutive equations:

$$g_i(x, y)=A_{ikl}(x, y)\,[\varepsilon_{kl}(x)+\varepsilon_{kl}(y)],\quad A_{ikl}(x, y)=-A_{ikl}(y, x),$$

$$g_i(x)=a_{ikl}\varepsilon_{kl}+\int_V A_{ikl}(x, y)\,\varepsilon_{kl}(y)\,dY,$$

$$\sigma_{ij}(x)=c_{ijkl}\,\varepsilon_{kl}+\int_V C_{ijkl}(x, y)\,e_{kl}(y)\,dY,$$

where ε_{ij} are the strain components and a_{ikl}, $A_{ikl}(x, y)$, c_{ijkl}, $C_{ijkl}(x, y)$ some coefficients which describe the mechanical properties of the body.

Remark 1 : In the other nonlocal theories in the central point are the so called localization residuals in the balance laws. Here, the localization residuals are removed out by the first postulate. The internal body force g_i is analogous to body force residual in the Eringen theory [2], but, at the last, the body force residual in [2] must vanish and this requirement does not arise in our description.

Remark 2 : If the body opposes internal body forces and internal body couples the stress tensor will be asymmetric, even if the body is acted by surface tractions and external body forces only.

Remark 3 : The proposed scheme is convenient for polar and heat conducting materials. In a next paper we will present some of these extensions.

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ON THE CHANGES IN THE EARTH'S INERTIA TENSOR DUE TO EARTHQUAKE FAULTING

BY

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1. Introduction. In the theory of the Earth's rotation, the calculation of changes in the Earth's inertia tensor accompanying earthquakes is of major interest. The subject was considered in various papers (see e.g. [1]—[6]). In [6], the changes in the Earth's inertia tensor are calculated by a direct application of the elastic reciprocal theorem. The problem is reduced to the finding of the stress distribution which results when the model is steadily rotated about some central axis. Two homogeneous, isotropic spherical Earth models are studied. It is known (see e.g. [6], [7]) that the deformation of a sphere is very greatly affected by heterogeneity of the material. In this paper we consider a piecewise homogeneous Earth model and study changes in the Earth's inertia tensor due to earthquake faulting. The temperature variation is taken into account.

2. Reciprocal relation. Let Ω be a bounded regular (in the sense of Kellog) region with the boundary S_2. Let S_1 be a closed regular surface contained in Ω, satisfying the conditions : $S_1 \cup S_2$ is the boundary of a domain Ω_2, contained in Ω, and S_1 is the boundary of a regular domain Ω_1, contained in Ω, such that $\Omega_1 \cap \Omega_2=\varnothing$, $\Omega_1 \cup \overline{\Omega}_2=\overline{\Omega}$.

Suppose that Ω_1 and Ω_2 are occupied by two elastic materials. Let $C^{(\alpha)}_{ijrs}$, $\beta^{(\alpha)}_{ij}$ be the thermoelastic coefficients of the material which occupies the domain Ω_α $(\alpha=1, 2)$.

Throughout this paper a rectangular Cartesian coordinate system Ox_i $(i=1, 2, 3)$ is used. Let $(\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3)$ be the corresponding unit base vectors. We shall employ the usual summation and differentiation conventions : Latin subscripts-unless otherwise specified-are understood to range over the integers 1, 2, 3, summation over repeated subscripts is implied and subscripts preceded by a comma denote partial differentiation with respect to the corresponding Cartesian coordinate. The position vector to any point in space is $\mathbf{r}=x_i\mathbf{e}_i$.

In this paper we consider the linear theory of classical thermoelasticity. Let u_i denote the components of the displacement vector. The components of the strain tensor are given by

(2.1) $$e_{ij}=\frac{1}{2}(u_{i,j}+u_{j,i}).$$

The constitutive equations are (see e.g. [8])

(2.2) $$t_{ij}=C^{(\alpha)}_{ijrs}e_{rs}-\beta^{(\alpha)}_{ij}\theta \text{ in } \Omega_\alpha.$$

where t_{ij} are the components of the stress tensor and θ is the temperature measured from the absolute constant temperature T_0. The constitutive coefficients have the symmetry properties

(2.3) $$C^{(\alpha)}_{ijrs}=C^{(\alpha)}_{jirs}=C^{(\alpha)}_{rsij},\quad \beta^{(\alpha)}_{ij}=\beta^{(\alpha)}_{ji}.$$

The equations of equilibrium are

(2.4) $$t_{ji,j}+\rho^{(\alpha)}f_i=0 \text{ in } \Omega_\alpha,$$

where $\rho^{(\alpha)}$ is the density of the material which occupies the domain Ω_α, and f_i are the components of body force vector.

We assume the two materials to be welded together along the interface S_1. Let $[\varphi]_\alpha$ be the restriction of the function φ to Ω_α. The displacement vector and the stress vector must be continuous in passing from one medium to another, so that we have the conditions

(2.5) $$[u_i]_1=[u_i]_2,\quad [t_{ji}]_1\eta_j=[t_{ji}]_2\eta_j \text{ on } S_1,$$

where η_i are the direction cosines of the vector normal to S_1, outward to Ω_1.

We assume that the domain Ω_2 contains an internal regular surface Σ, so that $\Sigma\cap S_\alpha=\varnothing(\alpha=1,2)$, on which the slip displacements are to be prescribed. The sides of Σ are denoted by Σ^+ and Σ^-. Let ν_i the components of the unit normal vector $\underset{\sim}{\nu}$ of Σ, directed from the $(-)$ to $(+)$ side. We consider the conditions

(2.6) $$u_i^+-u_i^-=b_i,\quad t^+_{ji}\nu_j=t^-_{ji}\nu_j \text{ on } \Sigma,$$

where φ^+ and φ^- are the limits of $\varphi(x)$ as x approaches a point on Σ^+ and Σ^-, respectively, and b_i are prescribed functions.

Let us consider the boundary conditions

(2.7) $$t_{ji}n_j=\tilde{t}_i \text{ on } S_2,$$

where n_i are the components of the outward unit normal of S_2, and $\tilde{t}_i$ are prescribed functions.

In what follows, we assume that the temperature variation θ is given. Let us consider two elastic states $\{u'_i, e'_{ij}, t'_{ij}\}$ and $\{u''_i, e''_{ij}, t''_{ij}\}$ corresponding to the systems of loadings $\{f'_i, b'_i, \tilde{t}'_i, \theta'\}$ and $\{f''_i, b''_i, \tilde{t}''_i, \theta''\}$, respectively. If we denote

(2.8) $$W^{(\alpha)}(\mathbf{u}',\mathbf{u}'')=\frac{1}{2}C^{(\alpha)}_{ijrs}[e'_{ij}e''_{rs}]_\alpha \text{ in } \Omega_\alpha,$$

then, on the basis of (2.3), we have

(2.9) $$W^{(\alpha)}(\mathbf{u}',\mathbf{u}'')=W^{(\alpha)}(\mathbf{u}'',\mathbf{u}').$$

Using the relations (2.1), (2.4)–(2.7) and the divergence theorem we obtain

(2.10) $$\begin{aligned}2\sum_{\alpha=1}^{2}\int_{\Omega_\alpha}W^{(\alpha)}(\mathbf{u}',\mathbf{u}'')\,dv&=\sum_{\alpha=1}^{2}\int_{\Omega_\alpha}\rho^{(\alpha)}f'_iu''_i\,dv+\int_{S_2}\tilde{t}'_iu''_i\,da-\\&-\int_{\Sigma}T'_ib''_i\,da+\sum_{\alpha=1}^{2}\int_{\Omega_\alpha}\beta^{(\alpha)}_{ij}\theta'e''_{ij}\,dv,\end{aligned}$$

where

(2.11) $$T'_i=t'^+_{ji}\nu_j=t'^-_{ji}\nu_j.$$

From (2.9) and (2.10) we get the following reciprocal relation

(2.12) $$\begin{aligned}&\sum_{\alpha=1}^{2}\int_{\Omega_\alpha}\rho^{(\alpha)}f'_iu''_i\,dv+\int_{S_2}\tilde{t}'_iu''_i\,da-\int_{\Sigma}T'_ib''_i\,da+\sum_{\alpha=1}^{2}\int_{\Omega_\alpha}\beta^{(\alpha)}_{ij}\theta'e''_{ij}\,dv=\\&=\sum_{\alpha=1}^{2}\int_{\Omega_\alpha}\rho^{(\alpha)}f''_iu'_i\,dv+\int_{S_2}\tilde{t}''_iu'_i\,da-\int_{\Sigma}T''_ib'_i\,da+\sum_{\alpha=1}^{2}\int_{\Omega_\alpha}\beta^{(\alpha)}_{ij}\theta''e'_{ij}\,dv.\end{aligned}$$

Now let us consider the elastic state $\{u_i, e_{ij}, t_{ij}\}$ corresponding to the system of loading $\{f'_i=F_i, b'_i=0, \tilde{t}'_i=0, \theta'=0\}$ and the elastic state $\{v_i, \varepsilon_{ij}, \sigma_{ij}\}$ corresponding to the system of loading $\{f''_i=0, b''_i=b_i, \tilde{t}''_i=0, \theta''=\theta\}$. Then, from (2.12) we obtain

(2.13) $$\sum_{\alpha=1}^{2}\int_{\Omega_\alpha}\rho^{(\alpha)}F_iv_i\,dv=\int_{\Sigma}T_ib_i\,da+\sum_{\alpha=1}^{2}\int_{\Omega_\alpha}\beta^{(\alpha)}_{ij}\theta e_{ij}\,dv,$$

where $T_i=[t_{ji}]_2\nu_j$.

Let $\mathbf{s}$ be the unit vector in the slip direction at every point of Σ. We have $b_i=s_i b$, where b is the slip magnitude. From (2.13) we get

(2.14) $$\sum_{\alpha=1}^{2}\int_{\Omega_\alpha}\rho^{(\alpha)}F_iv_i\,dv=\int_{\Sigma}Tb\,da+\sum_{\alpha=1}^{2}\int_{\Omega_\alpha}\beta^{(\alpha)}_{ij}\theta e_{ij}\,dv,$$

where

(2.15) $$T=T_is_i=[t_{ji}]_2\nu_js_i,$$

is the shear stress induced on Σ by the self-equilibrating field of body force F_i.

3. Changes in the inertia tensor of a piecewise homogeneous Earth model. The moment of inertia about the axis (d) passing through the origin and having the orientation of a unit vector $\mathbf{N}$ is

$$I_{(N)}=\sum_{\alpha=1}^{2}\int_{\Omega_\alpha}\rho^{(\alpha)}\zeta^2 dv, \tag{3.1}$$

where $\boldsymbol{\zeta}=\mathbf{r}-(\mathbf{N}\mathbf{r})\,\mathbf{N}$. We can write $I_{(N)}=I_{ij}N_iN_j$ where

$$I_{ij}=\sum_{\alpha=1}^{2}\int_{\Omega_\alpha}\rho^{(\alpha)}(r^2\delta_{ij}-x_ix_j)\,dv.$$

is the inertia tensor.

When the body Ω is deformed, we have a change in moment of inertia

$$\delta I_{(N)}=N_iN_j\delta I_{ij}=2\sum_{\alpha=1}^{2}\int_{\Omega_\alpha}\rho^{(\alpha)}\zeta_i u_i dv, \tag{3.3}$$

to first order.

Let us suppose that the body is rotated with angular velocity $\boldsymbol{\omega}$ about the axis (d). This creates the centrifugal body force

$$\mathbf{f}^*=\omega^2\mathbf{r}-(\boldsymbol{\omega}\mathbf{r})\,\boldsymbol{\omega}. \tag{3.4}$$

where $\boldsymbol{\omega}=\omega\mathbf{N}$. If $\omega=1$, then $\mathbf{f}^*=\boldsymbol{\zeta}$. Let us denote by $(\mathcal{A})$ the problem of determining a solution corresponding to the system of loading $\Lambda^{(N)}=\{f_i=\zeta_i,\ b_i'=0,\ \tilde{t}_i=0,\ \theta'=0\}$. We denote by $\mathcal{S}^{(N)}=\{u_i^{(N)},\ e_{ij}^{(N)},\ t_{ij}^{(N)}\}$ the elastic state corresponding to the system $\Lambda^{(N)}$. From (2.13) — (2.15) we obtain

$$\sum_{\alpha=1}^{2}\int_{\Omega_\alpha}\rho^{(\alpha)}\zeta_i v_i dv=\int_{\Sigma}T^{(N)}b\,da+\sum_{\alpha=1}^{2}\int_{\Omega_\alpha}\beta_{ij}^{(\alpha)}\,\theta e_{ij}^{(N)}dv, \tag{3.5}$$

where

$$T^{(N)}=[t_{ji}^{(N)}]_2\,\nu_j s_i. \tag{3.6}$$

The relations (3.3) and (3.5) lead to

$$\delta I_{(N)}=N_iN_j\delta I_{ij}=2\int_{\Sigma}T^{(N)}\,b\,da+2\sum_{\alpha=1}^{2}\int_{\Omega_\alpha}\beta_{ij}^{(\alpha)}\theta e_{ij}^{(N)}\,dv. \tag{3.7}$$

In the case of isotropic solids we have $\beta_{ij}^{(\alpha)}=\beta^{(\alpha)}\delta_{ij}$, where δ_{ij} is Kronecker's delta. In this case the relation (3.7) becomes

$$\delta I_{(N)}=N_iN_j\delta I_{ij}=2\int_{\Sigma}T^{(N)}\,b\,da+2\sum_{\alpha=1}^{2}\int_{\Omega_\alpha}\beta^{(\alpha)}\theta e_{rr}^{(N)}\,dv. \tag{3.8}$$

We therefore conclude that the change in moment of inertia is determined by the temperature variation and the slip magnitude which occur in faulting, and the steady rotation solution $\mathcal{S}^{(N)}$.

It is easy to see that the functions $u_i^{(N)}$, $e_{ij}^{(N)}$, $t_{ij}^{(N)}$ have the form

$$u_i^{(N)}=U_i^{(rs)}N_rN_s,\quad e_{ij}^{(N)}=E_{ij}^{(rs)}N_rN_s,\quad t_{ij}^{(N)}=P_{ij}^{(rs)}N_rN_s,$$

where $U_i^{(rs)}$, $E_{ij}^{(rs)}$, $P_{ij}^{(rs)}$ are symmetrical in r and s.

From (3.6) we get

$$T^{(N)}=S^{(rs)}\,N_rN_s,$$

where

$$S^{(rs)}=[P_{ji}^{(rs)}]_2\,\nu_j s_i. \tag{3.9}$$

Since (3.8) holds for any choice of $\mathbf{N}$, we can write

$$\delta I_{ij}=2\int_{\Sigma}S^{(ij)}\,b\,da+2\sum_{\alpha=1}^{2}\int_{\Omega_\alpha}\beta^{(\alpha)}\theta E_{rr}^{(ij)}\,dv. \tag{3.10}$$

It is easy to extend the result to the case when Ω is composed of n elastic bodies with different constitutive coefficients.

In what follows we study an inhomogeneous Earth model. We assume that the domain Ω occupied by the Earth is a sphere of radius a_2 and center at 0. We suppose that $\overline{\Omega}_2$ is the spherical shell $\{x: a_1\leq x_ix_i\leq a_2\}$, $(a_1<a_2)$, and Ω_1 is the sphere of radius a_1 and center at O. We consider the case when the domains $\Omega_\alpha(\alpha=1,2)$ are occupied by two different homogeneous and isotropic elastic materials. In this case we have

$$C_{ijrs}^{(\alpha)}=\lambda^{(\alpha)}\delta_{ij}\delta_{rs}+\mu^{(\alpha)}(\delta_{ir}\delta_{js}+\delta_{is}\delta_{jr}),\qquad \beta_{ij}^{(\alpha)}=\beta^{(\alpha)}\delta_{ij},$$

where $\lambda^{(\alpha)}$, $\mu^{(\alpha)}$, $\beta^{(\alpha)}$ are constants.

In order to calculate the changes in the inertia tensor we must find the solution of the problem $(\mathcal{A})$ (in the absence of any slip on Σ).

4. The problem (A). In this section we give the solution of the problem $(\mathcal{A})$ for the piecewise homogeneous Earth model considered in Section 3. We shall establish the solution when the body force vector is given by (3.4). Obviously, if $\omega=1$ then this solution reduces to the solution of the problem $(\mathcal{A})$.

The constitutive equations become

$$t_{ij}=\lambda^{(\alpha)}e_{rr}\delta_{ij}+2\mu^{(\alpha)}e_{ij}-\beta^{(\alpha)}\theta\delta_{ij}\quad\text{in }\Omega_\alpha. \tag{4.1}$$

In this case, the equilibrium equations reduce to

$$\mu^{(\alpha)}u_{i,jj}+(\lambda^{(\alpha)}+\mu^{(\alpha)})u_{j,ji}+\rho^{(\alpha)}f_i^*=0\quad\text{in }\Omega_\alpha, \tag{4.2}$$

where

$$f_i^*=\omega^2x_i-(\omega_jx_j)\,\omega_i,\quad \omega=\text{const.},$$

We can write

$$f_i^*=\left(\frac{1}{3}\omega^2r^2+V\right)_{,i}$$

where $r^2=x_ix_i$, and

$$V=-\frac{1}{2}\left(\omega_i\omega_j-\frac{1}{3}\omega^2\delta_{ij}\right)x_ix_j. \tag{4.3}$$

The conditions (2.5) become

$$[u_i]_1=[u_i]_2,\quad [t_{ji}]_1 n_j=[t_{ji}]_2 n_j\quad\text{on } r=a_1. \tag{4.4}$$

The boundary conditions lead to

$$t_{ji}n_j = 0 \text{ on } r = a_2. \tag{4.5}$$

The problem consists in the determination of a solution of the equations (4.2) which satisfies the conditions (4.4) and (4.5).

Let us introduce the notations

$$\tau^{(\alpha)} = -\frac{2(2\lambda^{(\alpha)}+7\mu^{(\alpha)})}{5\lambda^{(\alpha)}+7\mu^{(\alpha)}}, \quad \gamma^{(\alpha)} = \frac{3\lambda^{(\alpha)}+8\mu^{(\alpha)}}{\mu^{(\alpha)}}, \tag{4.6}$$

$$k^{(\alpha)} = \frac{\rho^{(\alpha)}}{\lambda^{(\alpha)}+2\mu^{(\alpha)}}, \qquad (\alpha = 1, 2).$$

It may be shown that the functions u_i defined by

$$u_i = \left(\tau^{(1)}A_1 - \frac{1}{7}k^{(1)}\right)Vx_i + \left(A_1r^2 + A_2 - \frac{1}{14}k^{(1)}r^2\right)V_{,i} + \left(B_1 - \frac{1}{15}\omega^2 k^{(1)}r^2\right)x_i \text{ in } \Omega_1,$$

$$u_i = \left[\tau^{(2)}A_3 + (\gamma^{(2)}-5)A_4r^{-4} - 5A_5r^{-7} - \frac{1}{7}k^{(2)}\right]Vx_i + \left(A_3r^2 + A_4r^{-3} + A_5r^{-5} + A_6 - \frac{1}{14}r^2k^{(2)}\right)V_{,i} + \left(B_2 + B_3r^{-2} - \frac{1}{15}\omega^2k^{(2)}r^2\right) \text{ in } \Omega_2, \tag{4.7}$$

where A_i $(i = 1, 6)$, B_j are arbitrary constants, satisfy the equations (4.2). We note that

$$e_{ij} = \left(A_1r^2 + A_2 - \frac{1}{14}k^{(1)}r^2\right)V_{,ij} + \frac{1}{2}\left[(2+\tau^{(1)})A_1 - \frac{2}{7}k^{(2)}\right](x_iV_{,j} + x_jV_{,i}) + \left(\tau^{(1)}A_1 - \frac{1}{7}k^{(1)}\right)V\delta_{ij} + B_1\delta_{ij} - \frac{1}{15}k^{(1)}\omega^2(\delta_{ij}r^2 + 2x_ix_j) \text{ in } \Omega_1,$$

$$e_{ii} = [(4+5\tau^{(1)})A_1 - k^{(1)}]V + 3B_1 - \frac{1}{3}k^{(1)}\omega^2r^2 \text{ in } \Omega_1,$$

(4.8)

$$e_{ij} = \left(A_3r^2 + A_4r^{-3} + A_5r^{-5} + A_6 - \frac{1}{14}k^{(2)}r^2\right)V_{,ij} + \frac{1}{2}\left[(2+\tau^{(2)})A_3 + (\gamma^{(2)}-8)A_4r^{-5} - 10A_5r^{-7} - \frac{2}{7}k^{(2)}\right](x_iV_{,j} + x_jV_{,i}) + \left[\tau^{(2)}A_3\delta_{ij} + (5-\gamma^{(2)})A_4(5x_ix_jr^{-2} - \delta_{ij})r^{-5} + 35A_5r^{-9}x_ix_j - 5A_5r^{-7}\delta_{ij} - \frac{1}{7}k^{(2)}\delta_{ij}\right]V + B_2\delta_{ij} + B_3(\delta_{ij}r^{-3} - 3r^{-5}x_ix_j) -$$

$$- \frac{1}{15}k^{(2)}\omega^2(\delta_{ij}r^2 + 2x_ix_j) \text{ in } \Omega_2,$$

$$e_{ii} = [(4+5\tau^{(2)})A_3 - 6A_4r^{-5} - k^{(2)}]V + 3B_2 - \frac{1}{3}k^{(2)}\omega^2r^2 \text{ in } \Omega_2.$$

In view of these relations we obtain

$$a_1[t_{ji}]_1 n_j = (A_1L^{(1)} + K^{(1)})Vx_i + (A_1N^{(1)}a_1^2 + 2\mu^{(1)}A_2 + Q^{(1)}a_1^2)V_{,i} + (P^{(1)}B_1 + S^{(1)}\omega^2a_1^2)x_i,$$

$$a_1[t_{ji}]_1 n_j = (A_3L^{(2)} + A_4M^{(2)}a_1^{-5} + 40\mu^{(2)}A_5a_1^{-7} + K^{(2)})Vx_i + (A_3N^{(2)}a_1^2 + A_4P^{(2)}a_1^{-3} - 8\mu^{(2)}A_5a_1^{-5} + 2\mu^{(2)}A_6 + Q^{(2)}a_1^2)V_{,i} + (P^{(2)}B_2 - 4\mu^{(2)}B_3a_1^{-3} + S^{(2)}\omega^2a_1^2)x_i \text{ on } r = a_1, \tag{4.9}$$

where

$$L^{(\alpha)} = -\frac{2\mu^{(\alpha)}(19\lambda^{(\alpha)} + 14\mu^{(\alpha)})}{5\lambda^{(\alpha)} + 7\mu^{(\alpha)}}, \quad M^{(\alpha)} = -24(\lambda^{(\alpha)} + \mu^{(\alpha)}),$$

$$N^{(\alpha)} = (4+\tau^{(\alpha)})\mu^{(\alpha)}, \quad P^{(\alpha)} = 3\lambda^{(\alpha)} + 2\mu^{(\alpha)},$$

$$Q^{(\alpha)} = -\frac{3}{7}\mu^{(\alpha)}k^{(\alpha)}, \quad K^{(\alpha)} = -\frac{1}{7}(7\lambda^{(\alpha)} + 6\mu^{(\alpha)})k^{(\alpha)}, \tag{4.10}$$

$$S^{(\alpha)} = -\frac{1}{15}(5\lambda^{(\alpha)} + 6\mu^{(\alpha)})k^{(\alpha)}, \qquad (\alpha = 1, 2).$$

The conditions (4.4), (4.5) reduce to the following systems for the unknown constants

$$\sum_{q=1}^{6} a_{pq}A_q = b_p, \ (p = \overline{1,6}), \ c_{ij}B_j = \beta_i, \tag{4.11}$$

where we have used the notations

$$a_{11} = \tau^{(1)}, \ a_{12} = a_{16} = 0, \ a_{13} = -\tau^{(2)}, \ a_{14} = (5-\gamma^{(2)})a_1^{-5}, \ a_{15} = 5a_1^{-7},$$

$$a_{21} = -a_{23} = a_1^2, \ a_{22} = -a_{26} = 1, \ a_{24} = -a_1^{-3}, \ a_{25} = -a_1^{-5},$$

$$a_{31} = L^{(1)}, \ a_{32} = a_{36} = 0, \ a_{33} = -L^{(2)}, \ a_{34} = -M^{(2)}a_1^{-5}, \ a_{35} = -40\mu^{(2)}a_1^{-7},$$

$$a_{41} = N^{(1)}a_1^2, \ a_{42} = 2\mu^{(1)}, \ a_{43} = -N^{(2)}a_1^2, \ a_{44} = -P^{(2)}a_1^{-3}, \ a_{45} = 8\mu^{(2)}a_1^{-5}, a_{46} = -2\mu^{(2)},$$

$$a_{5\alpha} = a_{56} = 0, \ a_{53} = L^{(2)}, \ a_{54} = M^{(2)}a_2^{-5}, \ a_{55} = 40\mu^{(2)}a_2^{-7},$$

$$a_{6\alpha} = 0, \ a_{63} = N^{(2)}a_2^2, \ a_{64} = P^{(2)}a_2^{-3}, \ a_{65} = -8\mu^{(2)}a_2^{-5}, \ a_{66} = 2\mu^{(2)},$$

$$b_1 = 2a_1^{-2}, \ b_2 = \frac{1}{7}(k^{(1)} - k^{(2)}), \ b_3 = K^{(2)} - K^{(1)}, \ b_4 = (Q^{(2)} - Q^{(1)})a_1^2,$$

$$b_5 = -K^{(2)}, \ b_6 = -Q^{(2)}a_2^2,$$

$$c_{11} = -c_{12} = 1, \ c_{13} = -a_1^{-3}, \ c_{21} = P^{(1)}, \ c_{22} = -P^{(2)},$$

$$c_{23}=4\,\mu^{(2)}a_1^{-3},\quad c_{31}=0,\quad c_{32}=P^{(2)},\quad c_{33}=-4\,\mu^{(2)}a_2^{-3},$$

$$\beta_1=\frac{1}{15}\,(k^{(1)}-k^{(2)})\omega^2a_1,\quad \beta_2=(S^{(2)}-S^{(1)})\,\omega^2a_1^2,\quad \beta_3=-S^{(2)}\omega^2a_2^2.$$

The systems (4.11) determine the constants $A_s\ (s=\overline{1,6})$, B_i. If

$$\lambda^{(1)}=\lambda^{(2)}=\lambda,\quad \mu^{(1)}=\mu^{(2)}=\mu,\quad \rho^{(1)}=\rho^{(2)}=\rho,$$

then we obtain

$$A_1=A_3=-\frac{(7\lambda+6\mu)\,(5\lambda+7\mu)\,\rho}{14\mu\,(19\lambda+14\mu)\,(\lambda+2\mu)},\qquad A_2=A_6=\frac{\rho(4\lambda+3\mu)\,a_2^2}{\mu(19\lambda+14\mu)},$$

$$B_1=B_2=\frac{(15\lambda+6\mu)\,\rho\omega^2a_2^2}{15\,(3\lambda+2\mu)\,(\lambda+2\mu)},\qquad A_4=A_5=B_3=0.$$

In this case the solution reduces to the solution for a homogeneous sphere (see e.g. [6], [7]).

In order to obtain the solution of the problem (A) we take $\omega=1$. To obtain $U_i^{(rs)}$, $E_{ij}^{(rs)}$, $P_{ij}^{(rs)}$ we simple write

$$V=-\frac{1}{2}\left(N_iN_j-\frac{1}{3}N_kN_k\delta_{ij}\right)x_ix_j,\quad 1=N_kN_k,$$

and identify the symmetricized coefficients of N_rN_s. From (3.9) and (4.8) we obtain

$$E_{mm}^{(ij)}=-\frac{1}{2}\left[(4+5\gamma^{(1)})\,A_1-k^{(1)}\right]\left(x_ix_j-\frac{1}{3}\,\delta_{ij}r^2\right)+3B_1\delta_{ij}-$$
$$-\frac{1}{3}\,k^{(1)}r^2\delta_{ij}\ \text{ in } \Omega_1,$$

$$E_{mm}^{(ij)}=-\frac{1}{2}\left[(4+5\gamma^{(2)})\,A_3-6A_4r^{-5}-k^{(2)}\right]\left(x_ix_j-\frac{1}{3}\,\delta_{ij}r^2\right)+$$
$$+3B_2\delta_{ij}-\frac{1}{3}\,k^{(2)}r^2\delta_{ij}\ \text{ in } \Omega_2,$$

$$S^{(ij)}=-\mu^{(2)}\left(A_3r^2+A_4r^{-3}+A_5r^{-5}+A_6-\frac{1}{14}\,k^{(2)}r^2\right)(\nu_is_j+\nu_js_i)- \tag{4.12}$$
$$-\frac{1}{2}\,\mu^{(2)}r\Big[(2+\gamma^{(2)})\,A_3+(\gamma^{(2)}-8)\,A_4r^{-5}-10\,A_5r^{-7}-$$
$$-\frac{2}{7}\,k^{(2)}\Big]\left[s^*\,(\nu_ix_j+\nu_jx_i)+\nu^*\,(s_ix_j+s_jx_i)-\frac{4}{3}\,r^2\nu^*s^*\delta_{ij}\right]-$$
$$-2\mu^{(2)}\left\{\frac{5}{2}\left(x_ix_j-\frac{1}{3}\,r^2\delta_{ij}\right)\left[A_4(5-\gamma^{(2)})\,r^{-5}+7A_5r^{-7}\right]-\right.$$
$$\left.-\left(3B_3r^{-3}+\frac{2}{15}\,k^{(2)}\right)\delta_{ij}\right\}\nu^*s^*,$$

where $rs^*=\mathbf{r}\cdot\mathbf{s}$, $r\nu^*=\boldsymbol{\nu}\cdot\mathbf{r}$.

In view of the relations (3.10) and (4.12) we can obtain the changes in the inertia tensor. Numerical evaluations could be performed.

The result is particulary simple for a strike-slip fault for then $\nu^*=s^*=0$.

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