2.12 (a) and Lemma 2.14, $T_>(C)$ is open in $R(X, Y)$. It turns out that $T_>(C)$ and $R(X, Y_C) \cup R(X_C, Y) - T_>(C)$ are the two components of $R(X, Y_C) \cup R(X_C, Y)$. A similar argument applies for $C \in \mathcal{C}_2^-(Z)$.

Theorem 2.18. *Let (X, Y) be an ordered couple of nonvoid, connected spaces, with $Z = X \cap Y$ nonvoid and closed in both X and Y. Suppose that $\mathcal{C}(X-Z)$, $\mathcal{C}(Y-Z)$, and $\mathcal{C}(Z)$ are nonvoid and finite. The following hold:*

(a) *if $\mathcal{C}_2^+(Z) \cup \mathcal{C}_2^-(Z) \neq \varnothing$, then $\mathcal{C}(R(X, Y)) = \{T_>(C) \mid C \in \mathcal{C}_2^+(Z)\} \cup \{T_<(C) \mid C \in \mathcal{C}_2^-(Z)\} \cup \{R(X, Y) - \bigcup \{T_>(C) \mid C \in \mathcal{C}_2^+(Z)\} - \bigcup \{T_<(C) \mid C \in \mathcal{C}_2^-(Z)\}\}$ and $\#(\mathcal{C}(R(X, Y))) = \#(\mathcal{C}_2^+(Z)) + \#(\mathcal{C}_2^-(Z)) + 1$;*

(b) *in all other cases, $R(X, Y)$ is connected.*

Proof. First note that, if $C \in \mathcal{C}_2^+(Z)$, then $T_>(C)$ is closed and open in $R(X, Y)$; therefore, $T_>(C)$ is a component of $R(X, Y)$. Similarly, for each $C \in \mathcal{C}_2^-(Z)$, $T_<(C)$ is a component of $R(X, Y)$. The finiteness of $\mathcal{C}(Z)$ then implies that

$$V = R(X, Y) - \bigcup \{T_>(C) \mid C \in \mathcal{C}_2^+(Z)\} - \bigcup \{T_<(C) \mid C \in \mathcal{C}_2^-(Z)\}$$

is also open and closed in $R(X, Y)$. It still remains to show that V is connected. Denote

$$U_C = R(X, Y_C) \cup R(X_C, Y)$$

for each $C \in \mathcal{C}(Z)$ and

$$V_C = \begin{cases} U_C, & \text{for } C \in \mathcal{C}_1(Z), \\ U_C - T_>(C), & \text{for } C \in \mathcal{C}_2^+(Z), \\ U_C - T_<(C), & \text{for } C \in \mathcal{C}_2^-(Z). \end{cases}$$

Note that, for $C, C' \in \mathcal{C}(Z)$, $C \neq C'$, we have

$$U_C \cap U_{C'} \supset (X_C \times Y_{C'}) \cup (X_{C'} \times Y_C)$$

and

$$U_C \cap R(C', C') = \varnothing.$$

Therefore, $V_C \cap V_{C'} = U_C \cap U_{C'} \neq \varnothing$, for each $(C, C') \in \mathcal{C}(Z) \times \mathcal{C}(Z)$ with $C \neq C'$. On the other hand, the same argument shows that

$$V = \bigcup \{U_C \mid C \in \mathcal{C}(Z)\} - \bigcup \{T_>(C) \mid C \in \mathcal{C}_2^+(Z)\} - \bigcup \{T_<(C) \mid C \in \mathcal{C}_2^-(Z)\} = \bigcup \{V_C \mid C \in \mathcal{C}(Z)\}.$$

By Lemma 2.17, each V_C, $C \in \mathcal{C}(Z)$, is connected. It follows that V is also connected.

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POLYNOMIAL SOLUTIONS IN THE AXIAL COORDINATE FOR MICROPOLAR BEAMS AND THE BUCKLING EFFECT

BY

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To the memory of Professor Mendel Haimovici

We consider a micropolar beam limited by two planes $x_3 = 0$, $x_3 = h$ $(h > 0)$ and by a cylindrical surface F. We will denote by V the domain occupied by the beam and also the volume of this domain; S will be the domain and also the area of a cross-section of the beam. The boundary of S will be denoted by Γ.

We will take the axes x_1, x_2, x_3 such that

$$\int_S x_\alpha d\sigma = 0, \qquad \int_S x_1 x_2 d\sigma = 0 \qquad (\alpha = 1, 2).$$

We consider first a material with the following constitutive equations [3], [6]

$$\begin{cases} t_{ij} = \lambda e_{rr}\delta_{ij} + (\mu + \varkappa)\, e_{ij} + \mu e_{ji}, \\ m_{ij} = \alpha\varphi_{r,r}\delta_{ij} + \beta\varphi_{i,j} + \gamma\varphi_{j,i}, \\ e_{ij} = u_{j,i} + \varepsilon_{ijm}\varphi_m, \end{cases} \tag{1}$$

where t_{ij} are the stress tensor components, m_{ij} the couple-stress tensor components, u_i the displacement components, φ_i the rotation vector components; δ_{ij} and ε_{ijm} are the Kronecker's delta and alternating symbol; λ, μ, $\varkappa$, α, β, γ are constants which characterize the properties of the material.

We use the summation convention over repeated indices. The index j after comma indicates partial differentation with respect to x_j. All Latin indices take the values, 1, 2, 3 and all Greek indices take the values 1, 2.

1. Formulation of the problem. We consider that the lateral surface of the beam is free from tractions and couple-stresses. The body forces and the body couples are absent. The beam is loaded only on the ends in such a way that it is in equilibrium.

Our purpose, for such a beam, is to look for the most general solution which is polynominal in x_3 and to determine the end tractions and couple stresses which are compatible with it. Then, the problem is defined by the equations

$$t_{ji,j} = 0, \tag{2}$$

$$m_{ji,j} + \varepsilon_{ipq}t_{pq} = 0 \quad \text{in } V; \tag{3}$$

$$t_{\omega i}n_\omega = 0, \tag{4}$$

(5) $$m_{\omega i}\, n_\omega = 0 \quad \text{on } F\,;$$

(6) $$u_i = \sum_{k=0}^{n} U_i^{(k)}(x_1, x_2)\, x_3^k,\qquad \varphi_i = \sum_{k=0}^{n} \Phi_i^{(k)}(x_1, x_2)\, x_3^k,$$

where n $(n_1, n_2, 0)$ is the unit vector along the outer normal to F. The functions $U_i^{(k)}(x_1, x_2)$, $\Phi_i^{(k)}(x_1, x_2)$ and the integer n must be determined.

2. **General equations.** In order to solve this problem we will use up the same technics of calculus as in [1]. From (1) and (6) we obtain

(7) $$\begin{aligned}
&t_{\rho\rho} = \sum t_{\rho\rho}^{(k)} x_3^k + \lambda \sum k U_3^{(k)} x_3^{k-1} \qquad \text{(not summed)},\\
&t_{33} = \sum t_{33}^{(k)} x_3^k + (\lambda + 2\mu + \varkappa) \sum k U_3^{(k)} x_3^{k-1},\\
&t_{\omega\rho} = \sum t_{\omega\rho}^{(k)} x_3^k \qquad (\omega \neq \rho),\\
&t_{\rho 3} = (\mu + \varkappa) \sum \left[U_{3,\rho}^{(k)} - (-1)^\rho \Phi_\nu^{(k)}\right] x_3^k + \mu\left[\sum k U_\rho^{(k)} x_3^{k-1} + (-1)^\rho \Phi_\nu^{(k)} x_3^k\right],\\
&t_{3\rho} = \mu \sum \left[U_{3,\rho}^{(k)} - (-1)^\rho \Phi_\nu^{(k)}\right] x_3^k + (\mu + \varkappa)\left[\sum k U_\rho^{(k)} x_3^{k-1} +\right.\\
&\left.+ (-1)^\rho \Phi_\nu^{(k)} x_3^k\right] \quad (\nu \neq \rho),
\end{aligned}$$

and

(8) $$\begin{aligned}
&m_{\rho\rho} = (\alpha + \beta + \gamma)\, \Phi_{\rho,\rho}^{(k)} x_3^k + \alpha\left[\sum \Phi_{\nu,\nu}^{k} x_3^k + \sum k \Phi_3^{(k)} x_3^{k-1}\right]\\
&\text{(not summed, } \nu \neq \rho),\\
&m_{33} = (\alpha + \beta + \gamma) \sum k \Phi_3^{(k)} x_3^{k-1} + \alpha \sum \Phi_{\eta,\eta}^{(k)} x_3^k,\\
&m_{\omega\rho} = \sum \left(\beta \Phi_{\omega,\rho}^{(k)} + \gamma \Phi_{\rho,\omega}^{(k)}\right) x_3^k, \quad (\omega \neq \rho),\\
&m_{\rho 3} = \sum m_{\rho 3}^{(k)} x_3^k + \beta \sum k \Phi_\rho^{(k)} x_3^{k-1},\\
&m_{3\rho} = \sum m_{3\rho}^{(k)} x_3^k + \gamma \sum k \Phi_\rho^{(k)} x_3^{k-1},
\end{aligned}$$

where $t_{\omega\rho}^{(k)}$, $t_{33}^{(k)}$ and $m_{\rho 3}^{(k)}$, $m_{\rho 3}^{(k)}$ are the components of the stress and the components of the couple-stress of plane strain problem defined by the displacement $(U_1^{(k)}, U_2^{(k)}, 0)$ and the rotation $(0, 0, \Phi_3^{(k)})$. The equilibrium equations (2), (3) and the boundary conditions (4), (5) will be satisfied if and only if, for arbitrary k, we have

(9a) $$\begin{aligned}
&t_{\omega\rho,\omega}^{(k)} + (k+1)\left[(\lambda + \mu)\, U_{3,\rho}^{(k+1)} + (-1)^\rho \varkappa \Phi_\nu^{(k+1)}\right] +\\
&+ (k+1)(k+2)(\mu + \varkappa)\, U_\rho^{(k+2)} = 0 \quad (\nu \neq \rho),\\
&m_{\omega 3,\omega}^{(k)} + t_{12}^{(k)} - t_{21}^{(k)} + (k+1)(k+2)(\alpha + \beta)\, \Phi_{\omega,\omega}^{(k+1)} +\\
&+ (k+1)(k+2)(\alpha + \beta + \gamma)\, \Phi_3^{(k+2)} = 0 \quad \text{in } S\,;
\end{aligned}$$

(9b) $$\begin{cases} t_{\omega\rho}^{(k)} n_\omega = -(k+1)\lambda U_3^{(k+1)} n_\rho,\\ m_{\omega 3}^{(k)} n_\omega = (k+1)\beta \Phi_\omega^{(k+1)} n_\omega \quad \text{on } \Gamma\,; \end{cases}$$

and

(10a) $$\begin{aligned}
&\Delta_1(U_3^{(k)}, \Phi_1^{(k)}, \Phi_2^{(k)}) = -(k+1)\left(\mu U_{\omega,\omega}^{(k+1)} + t_{33}^{(k+1)}\right) -\\
&\quad -(k+1)(k+2)(\lambda + 2\mu + \varkappa)\, U_3^{(k+2)},\\
&\Delta_2^{(\rho)}(U_3^{(k)}, \Phi_1^{(k)}, \Phi_2^{(k)}) = -(k+1)\left[m_{3\rho}^{(k+1)} + \alpha \Phi_{3,\rho}^{(k+1)} - (-1)^\rho \varkappa U_\nu^{(k+1)}\right] -\\
&\quad -(k+1)(k+2)\, \gamma \Phi_\rho^{(k+2)} \quad \text{in } S \ (\nu \neq \rho),
\end{aligned}$$

(10b) $$\begin{aligned}
&D_1(U_3^{(k)}, \Phi_1^{(k)}, \Phi_2^{(k)}) = -(k+1)\, \mu U_\omega^{(k+1)} n_\omega,\\
&D_2^{(\rho)}(U_3^{(k)}, \Phi_1^{(k)}, \Phi_2^{(k)}) = -(k+1)\, \alpha \Phi_3^{(k+1)} n_\rho \quad \text{on } \Gamma,
\end{aligned}$$

where

$$\begin{aligned}
&\Delta_1(U_3, \Phi_1, \Phi_2) = (\mu + \varkappa)\, U_{3,\omega\omega} + \varkappa(\Phi_{2,1} - \Phi_{1,2}),\\
&\Delta_2^{(1)}(U_3, \Phi_1, \Phi_2) = (\alpha + \beta + \gamma)\, \Phi_{1,11} + \gamma \Phi_{1,22} + (\alpha + \beta)\, \Phi_{2,12} + \varkappa(U_{3,2} - 2\Phi_1)\\
&\Delta_2^{(2)}(U_3, \Phi_1, \Phi_2) = \gamma \Phi_{2,11} + (\alpha + \beta + \gamma)\, \Phi_{2,22} + (\alpha + \beta)\, \Phi_{1,12} - \varkappa(U_{3,1} + 2\Phi_2),\\
&D_1(U_3, \Phi_1, \Phi_2) = (\mu + \varkappa)\, U_{3,\omega} n_\omega + \varkappa(\Phi_2 n_1 - \Phi_1 n_2),\\
&D_2^{(1)}(U_3, \Phi_1, \Phi_2) = \left[(\alpha + \beta + \gamma)\, \Phi_{1,1} + \alpha \Phi_{2,2}\right] n_1 + \left[\gamma \Phi_{1,2} + \beta \Phi_{2,1}\right] n_2,\\
&D_2^{(2)}(U_3, \Phi_1, \Phi_2) = \left[\beta \Phi_{1,2} + \gamma \Phi_{2,1}\right] n_1 + \left[(\alpha + \beta + \gamma)\, \Phi_{2,2} + \alpha \Phi_{1,1}\right] n_2.
\end{aligned}$$

The problem defined by (9*a*), (9*b*) is a plane problem with body forces and body couples.

3. **The solution of the problem.** Now, let us solve the above problems for $k = n, n-1, n-2\ldots$ In the following we will denote by $A_\omega^{(s)}$, C_s, τ_s arbitrary constants

$$k = n$$

The equations (9*a*)—(10*b*), in this case are

$$\begin{aligned}
&t_{\omega\rho,\omega}^{(n)} = 0,\\
&m_{\omega 3,\omega}^{(n)} + t_{12}^{(n)} - t_{21}^{(n)} = 0 \quad \text{in } S,\\
&t_{\omega\rho}^{(n)} n_\omega = 0, \quad m_{\omega 3}^{(n)} n_\omega = 0 \quad \text{on } \Gamma
\end{aligned}$$

and

$$\begin{aligned}
&\Delta_1(U_3^{(n)}, \Phi_1^{(n)}, \Phi_2^{(n)}) = 0,\\
&\Delta_2^{(\rho)}(U_3^{(n)}, \Phi_1^{(n)}, \Phi_2^{(n)}) = 0 \quad \text{in } S,\\
&D_1(U_3^{(n)}, \Phi_1^{(n)}, \Phi_2^{(n)}) = 0, \quad D_2^{(\rho)}(U_3^{(n)}, \Phi_1^{(n)}, \Phi_2^{(n)}) = 0 \quad \text{on } \Gamma.
\end{aligned}$$

It results

(11) $$\begin{aligned}
&U_\rho^{(n)} = -A_\rho^{(n)} + (-1)^\rho \tau_n x_\nu, \quad U_3^{(n)} = C_n,\\
&\Phi_\rho^{(n)} = 0, \quad \Phi_3^{(n)} = \tau_n \quad (\nu \neq \rho).
\end{aligned}$$

$$k = n - 1$$

The corresponding equations are

$$\begin{aligned}
&t_{\omega\rho,\omega}^{(n-1)} = 0, \quad m_{\omega 3,\omega}^{(n-1)} + t_{12}^{(n-1)} - t_{21}^{(n-1)} = 0 \quad \text{in } S,\\
&t_{\omega\rho}^{(n-1)} n_\omega = -n C_n \lambda n_\rho, \quad m_{\omega 3}^{(n-1)} n_\omega = 0 \quad \text{on } \Gamma
\end{aligned}$$

and

$$\begin{aligned}
&\Delta_1(U_3^{(n-1)}, \Phi_1^{(n-1)}, \Phi_2^{(n-1)}) = 0,\\
&\Delta_2^{(\rho)}(U_3^{(n-1)}, \Phi_1^{(n-1)}, \Phi_2^{(n-1)}) = n\varkappa\left[(-1)^\rho A_\nu^{(n)} + \tau_n x_\rho\right] \quad \text{in } S.\\
&D_1(U_3^{(n-1)}, \Phi_1^{(n-1)}, \Phi_2^{(n-1)}) = n\mu\left[A_\omega^{(n)} n_\omega + \tau_n(x_2 n_1 - x_1 n_2)\right],\\
&D_2^{(\rho)}(U_3^{(n-1)}, \Phi_1^{(n-1)}, \Phi_3^{(n-1)}) = -n\alpha \tau_n n_\rho \quad \text{on } \Gamma.
\end{aligned}$$

Let Ω, θ_1, θ_2 be the solution of the following problem

$$\begin{aligned} &\Delta_1(\Omega, \theta_1, \theta_2) = 0, \\ &\Delta_2^{(\rho)}(\Omega, \theta_1, \theta_2) = \varkappa x_\rho \quad \text{in } S, \\ &D_1(\Omega, \theta_1, \theta_2) = \mu(x_2 n_1 - x_1 n_2), \\ &D_2^{(\rho)}(\Omega, \theta_1, \theta_2) = -\alpha n_\rho \quad \text{on } \Gamma, \end{aligned}$$

which is studied in [4].

Then we have

$$U_\rho^{(n-1)} = -\frac{n\lambda}{2\lambda + 2\mu + \varkappa} C_n x_\rho - A_\rho^{(n-1)} + (-1)^\rho \tau_n x_\nu,$$

$$(12) \qquad \begin{aligned} &U_3^{(n-1)} = n(A_\omega^{(n)} x_\omega + \tau_n \Omega) + C_{n-1}, \\ &\Phi_\rho^{(n-1)} = n[-(-1)^\rho A_\nu^{(n)} + \tau_n \theta_\rho], \\ &\Phi_3^{(n-1)} = \tau_{n-1} \quad (\nu \neq \rho) \end{aligned}$$

and

$$t_{33}^{(n-1)} = \frac{2n\lambda^2}{2\lambda + 2\mu + \varkappa} C_n, \quad m_{3\rho}^{(n-1)} = 0.$$

$$k = n - 2$$

From (9a)-(10b) we obtain

$$(13) \qquad \begin{aligned} &t_{\omega\rho,\omega}^{(n-2)} + n(n-1)\{\lambda A_\rho^{(n)} + \tau_n[(\lambda + \mu)\Omega_{,\rho} + (-1)^\rho \theta_\nu + (-1)^\rho(\mu + \varkappa)x_\nu]\} = 0, \\ &m_{\omega 3}^{(n-2)} + t_{12}^{(n-2)} - t_{21}^{(n-2)} + n(n-1)\tau_n[(\alpha + \beta)\theta_{\omega,\omega} + \alpha + \beta + \gamma] = 0 \quad \text{in } S, \\ &t_{\omega\rho}^{(n-2)} n_\omega = -(n-1)\lambda[n(A_\omega^{(n)} x_\omega + \tau_n \Omega) + C_{n-1}]n_\rho, \\ &m_{\omega 3}^{(n-2)} n_\omega = -n(n-1)\beta[A_2^{(n)} n_1 - A_1^{(n)} n_2 + \tau_n \theta_\omega n_\omega] \quad \text{on } \Gamma \end{aligned}$$

and

$$(14) \qquad \begin{aligned} &\Delta_1(U_3^{(n-2)}, \Phi_1^{(n-2)}, \Phi_2^{(n-2)}) = n(n-1)\left[2\mu + \varkappa + \frac{\lambda\varkappa}{2\lambda + 2\mu + \varkappa}\right] C_n, \\ &\Delta_2^{(\rho)}(U_3^{(n-2)}, \Phi_1^{(n-2)}, \Phi_2^{(n-2)}) = (n-1)\left[(-1)^\rho n C_n \frac{\lambda\varkappa x_\nu}{2\lambda + 2\mu + \varkappa} + \right. \\ &\qquad \left. + (-1)^\rho \varkappa A_\nu^{(n-1)} + \tau_{n-1} x^\rho)\right] \quad \text{in } S, \\ &D_1(U_3^{(n-2)}, \Phi_1^{(n-2)}, \Phi_2^{(n-2)}) = (n-1)\mu\left[\frac{n\lambda C_n}{2\lambda + 2\mu + \varkappa} x_\omega n_\omega + A_\omega^{(n-1)} n_\omega + \right. \\ &\qquad \left. + \tau_{n-1}(x_2 n_1 - x_1 n_2)\right], \\ &D_2^{(\rho)}(U_3^{(n-2)}, \Phi_1^{(n-2)}, \Phi_2^{(n-2)}) = -(n-1)\alpha\tau_{n-1} n_\rho \quad \text{on } \Gamma. \end{aligned}$$

In the equations (13) are included three plane problems. Denoting by $\tau_{\omega\rho}^{(i)}$ and $c_{\rho 3}^{(i)}$ the stresses and the couple-stresses of these problems, the mentioned problems are defined by

$$(15) \qquad \begin{aligned} &\tau_{\omega\rho,\omega}^{(\nu)} + \lambda\delta_{\rho\nu} = 0, \\ &c_{\omega 3,\omega}^{(\nu)} + \tau_{12}^{(\nu)} - \tau_{21}^{(\nu)} = 0 \quad \text{in } S, \\ &\tau_{\omega\rho}^{(\nu)} n_\omega = -\lambda x_\nu n_\rho, \quad c_{\omega 3}^{(\nu)} n_\omega = -(-1)^\nu \beta n_\delta \ (\delta \neq \nu) \quad \text{on } \Gamma; \end{aligned}$$

$$(16) \qquad \left\{ \begin{aligned} &\tau_{\omega\rho,\omega}^{(3)} + (\lambda + \mu)\Omega_{,\rho} + (-1)^\rho \varkappa \theta_\nu + (-1)^\rho(\mu + \varkappa)x_\nu = 0 \quad (\nu \neq \rho), \\ &c_{\omega 3,\omega}^{(3)} + \tau_{12}^{(3)} - \tau_{21}^{(3)} + (\alpha + \beta)\theta_{\omega,\omega} + \alpha + \beta + \gamma = 0 \quad \text{in } S, \\ &\tau_{\omega\rho}^{(3)} n_\omega = -\lambda\Omega n_\rho, \ c_{\omega 3}^{(3)} n_\omega = -\beta\theta_\omega n_\omega \quad \text{on } \Gamma. \end{aligned} \right.$$

The first two problems (15) have solutions and in the following we will denote by $u_1^{(\rho)}$ $u_2^{(\rho)}$ and $\varphi_3^{(\rho)}$ the displacements and the rotations of these problems.

The resultant moment of the loads in the problem (16) is equal to

$$D = \int_S [\mu(x_1\Omega_{,2} - x_2\Omega_{,1}) + \varkappa x_\omega \theta_\omega + (\mu + \varkappa) x_\omega x_\omega + \alpha\theta_{\omega,\omega} + \alpha + \beta + \gamma] d\tau$$

and it is known that $D > 0$ [4], therefore the problem (16) has no solution.

Then, the necessary and sufficient conditions for the existence of the solution for (13) require that $\tau_n = 0$ and for the problem (14) that $C_n = 0$.

These conditions being fulfiled the solutions of our problems are given by

$$(17) \qquad \begin{aligned} &U_\rho^{(n-2)} = n(n-1) A_\omega^{(n)} u_\rho^{(\omega)} - (n-1)C_{n-1} \frac{\lambda x_\rho}{2\lambda + 2\mu + \varkappa} - \\ &\qquad - A_\rho^{(n-2)} + (-1)^\rho \tau_{n-2} x_\nu, \\ &U_3^{(n-2)} = (n-1)(A_\omega^{(n-1)} x_\omega + \tau_{n-1}\Omega) + C_{n-2}, \\ &\Phi_\rho^{(n-2)} = (n-1)[-(-1)^\rho A_\nu^{(n-1)} + \tau_{n-1}\theta_\rho], \\ &\Phi_3^{(n-2)} = n(n-1)A_\omega^{(n)} \varphi_3^{(\omega)} + \tau_{n-2} \quad (\nu \neq \rho), \end{aligned}$$

and after some calculus we have

$$t_{33}^{(n-2)} = (n-1)A_\rho^{(n)} u_{\omega,\omega}^{(\rho)} - (n-1) C_{n-1} \frac{2\lambda^2}{2\lambda + 2\mu + \varkappa},$$

$$m_{3\rho}^{(n-2)} = n(n-1)\beta A_\omega^{(n)} \varphi_{3,\rho}^{(\omega)}.$$

$$k = n - 3$$

The corresponding equations are given by

$$(18) \qquad \begin{aligned} &t_{\omega\rho,\omega}^{(n-3)} + (n-1)(n-2)\{\lambda A_\rho^{(n-1)} + \tau_{n-1}[(\lambda + \mu)\Omega_{,\rho} + (-1)^\rho \varkappa \theta_\nu + \\ &\qquad + (-1)^\rho(\mu + \varkappa)x_\nu]\} = 0, \\ &m_{\omega 3,\omega}^{(n-3)} + t_{12}^{(n-3)} - t_{21}^{(n-3)} + (n-1)(n-2)\tau_{n-1}[(\alpha + \beta)\theta_{\omega,\omega} + \alpha + \beta + \gamma] = 0 \\ &\qquad \text{in } S, \\ &t_{\omega\rho}^{(n-3)} n_\omega = -(n-2)\lambda[(n-1)(A_\omega^{(n-1)} x_\omega + \tau_{n-1}\Omega) + C_{n-2}] n_\rho, \\ &m_{\omega 3}^{(n-3)} n_\omega = -(n-1)(n-2)\beta[(A_2^{(n-1)} + \tau_{n-1}\theta_1)n_1 + (-A_1^{(n-1)} + \\ &\qquad + \tau_{n-1}\theta_2)n_2] \quad \text{on } \Gamma, \end{aligned}$$

and

$$(19) \qquad \begin{aligned} &\Delta_1(U_3^{(n-3)}, \Phi_1^{(n-3)}, \Phi_2^{(n-3)}) = -(n-1)(n-2)\Big\{n[(\lambda + \mu)A_\rho^{(n)} u_{\omega,\omega}^{(\rho)} + \\ &\qquad + \Big(\lambda + 2\mu + \varkappa) A_\omega^{(n)} x_\omega] - (2\mu + \varkappa + \frac{\lambda\varkappa}{2\lambda + 2\mu + \varkappa}\Big) C_{n-1}\Big\}, \end{aligned}$$

$$\Delta_2^{(\rho)}(U_3^{(n-3)}, \Phi_1^{(n-3)}, \Phi_2^{(n-3)}) = -(n-1)(n-2)\Big\{ n[A_\omega^{(n)}(\alpha+\beta)\varphi_{3,\rho}^{(\omega)} +$$

$$+(-1)^\rho \varkappa A_\omega^{(n)} u_\nu^{(\omega)} - (-1)^\rho \gamma A_\nu^{(n)}] - (-1)^\rho C_{n-1}\frac{\lambda\varkappa}{2\lambda+2\mu+\varkappa}x_\nu\Big\} +$$

$$+\varkappa[(-1)^\rho A_\nu^{(n-2)} + \tau_{n-2}x_\rho] \quad \text{in } S,$$

(19) $$D_1(U_3^{(n-3)}, \Phi_1^{(n-3)}, \Phi_2^{(n-3)}) = -n(n-1)(n-2)A_\rho^{(n)}u_\omega^{(\rho)}n_\omega + (n-1)(n-2)\times$$

$$\times C_{n-1}\frac{\lambda\mu}{2\lambda+2\mu+\varkappa}x_\omega n_\omega +$$

$$+(n-2)\mu[A_\omega^{(n-2)}n_\omega + \tau_{n-2}(x_2n_1 - x_1n_2)],$$

$$D_2^{(\rho)}(U_3^{(n-3)}, \Phi_1^{(n-3)}, \Phi_2^{(n-3)}) = -n(n-1)(n-2)\alpha A_\omega^{(n)}\varphi_3^{(\omega)}n_\rho -$$

$$-(n-2)\,\alpha\tau_{n-2}n_\rho \quad \text{on } \Gamma.$$

The conditions for the existence of solutions of these problems require that $\tau_{n-1}=0$, $C_{n-1}=0$, the other constants being arbitrary.

At the last, we obtain

$$U_\rho^{(n-3)} = (n-1)(n-2)A_\omega^{(n-1)}u_\rho^{(\omega)} - (n-2)C_{n-2}\frac{\lambda x_\rho}{2\lambda+2\mu+\varkappa} - A_\rho^{(n-3)} +$$

$$+(-1)^\rho\tau_{n-3}x_\nu,$$

(20) $$U_3^{(n-3)} = n(n-1)(n-2)A_\omega^{(n)}\chi_\omega + (n-2)(A_\omega^{(n-1)}x_\omega + \tau_{n-2}\Omega) + C_{n-3},$$

$$\Phi_\rho^{(n-3)} = n(n-1)(n-2)A_\omega^{(n)}\psi_\rho^{(\omega)} + (n-2)[-(-1)^\rho A_\nu^{(n-2)} + \tau_{n-2}\theta_\rho],$$

$$\Phi_3^{(n-3)} = (n-1)(n-2)A_\omega^{(n-1)}\varphi_3^{(\omega)} + \tau_{n-3} \quad (\nu\neq\rho),$$

where χ_ρ, $\psi_1^{(\rho)}$, $\psi_2^{(\rho)}$ are defined by the following boundary value problems

$$\Delta_1(\chi_\rho, \psi_1^{(\rho)}, \psi_2^{(\rho)}) = -(\lambda+\mu)u_{\omega,\omega}^{(\rho)} - (\lambda+2\mu+\varkappa)x_\rho,$$

$$\Delta_2^{(\nu)}(\chi_\rho, \psi_1^{(\rho)}, \psi_2^{(\rho)}) = -(\alpha+\beta)\varphi_{3,\nu}^{(\rho)} - (-1)^\nu\varkappa u_\delta^{(\rho)} + (-1)^\nu\gamma\delta_{\rho\delta} \;(\delta\neq\nu) \text{ in } S,$$

$$D_1(\chi_\rho, \psi_1^{(\rho)}, \psi_2^{(\rho)}) = -\mu u_\omega^{(\rho)}n_\omega,$$

$$D_2^{(\nu)}(\chi_\rho, \psi_1^{(\rho)}, \psi_2^{(\rho)}) = -\alpha\varphi_3^{(\rho)}n_\nu \quad \text{on } \Gamma.$$

$$k = n-4$$

From (9a) — (10b) we have

$$t_{\omega\rho,\omega}^{(n-4)} + n(n-1)(n-2)(n-3)A_\omega^{(n)}[\lambda+\mu)\chi_{\omega,\rho} + (-1)^\rho\varkappa\psi_\nu^{(\omega)} + (\mu+\varkappa)u_\rho^{(\omega)}] +$$

$$+(n-2)(n-3)\{\lambda A_\rho^{(n-2)} + \tau_{n-2}[(\lambda+\mu)\Omega_{,\rho} + (-1)^\rho\varkappa\theta_\nu +$$

$$+(-1)^\rho(\mu+\varkappa)x_\nu]\} = 0 \quad (\nu\neq\rho),$$

$$m_{\omega3,\omega}^{(n-4)} + t_{12}^{(n-4)} - t_{21}^{(n-4)} + n(n-1)(n-2)(n-3)A_\omega^{(n)}[(\alpha+\beta)\psi_{\rho,\rho}^{(\omega)} +$$

$$+(\alpha+\beta+\gamma)\Phi_3^{(\omega)}] + (n-2)(n-3)\tau_{n-2}[(\alpha+\beta)\theta_{\omega,\omega} + \alpha+\beta+\gamma] = 0 \quad \text{in } S,$$

$$t_{\omega\rho}^{(n-4)}n_\omega = -\lambda\{(n-2)(n-3)[n(n-1)A_\omega^{(n)}\chi_\omega - A_\omega^{(n-2)}x_\omega + \tau_{n-2}\Omega - C_{n-3}]\}n_\rho,$$

$$m_{\omega3}^{(n-4)}n_\omega = (n-2)(n-3)\beta[n(n-1)A_\rho^{(n)}\psi_\omega^{(\rho)}n_\omega + \tau_{n-2}\theta_\omega n_\omega +$$

$$+A_1^{(n-2)}n_2 - A_2^{(n-2)}n_1] \quad \text{on } \Gamma;$$

. . . .

Because

$$L_{11}L_{22} - L_{12}^2 = L_{11}L_{22} - L_{21}^2 \neq 0,$$

(21) $$L_{\omega\rho} = \int_S [\mu\chi_{\omega,\rho} + (-1)^\rho\varkappa\psi_\nu^{(\omega)} + (\mu+\varkappa)u_\rho^{(\omega)}]d\sigma \quad (\nu\neq\rho),$$

and the rigidity of torsion $D > 0$ [4], we obtain

$$A_\omega^{(n)} = 0, \quad \tau_{n-2} = 0 \quad (n \geqslant 4).$$

Taking into account the above results we find that in our problem the displacements and the rotations, up to a rigid motion, are given by

$$u_1 = \left[a_3\left(u_1^{(1)} - \frac{1}{6}x_3\right) + b_3u_1^{(2)}\right]x_3 - \tau x_2x_3 +$$

$$+\left[a_2\left(u_1^{(1)} - \frac{1}{2}x_3^2\right) + b_2u_1^{(2)}\right] - c_2\frac{\lambda x_1}{2\lambda+2\mu+\varkappa},$$

$$u_2 = \left[a_3u_2^{(1)} + b_3\left(u_2^{(2)} - \frac{1}{6}x_3^2\right)\right]x_3 + \tau x_1x_3 +$$

(22) $$+\left[a_2u_2^{(1)} + b_2\left(u_2^{(2)} - \frac{1}{2}x_3^2\right)\right] - c_2\frac{\lambda x_2}{2\lambda+2\mu+\varkappa},$$

$$u_3 = \frac{1}{2}(a_3x_1 + b_3x_2)x_3^2 + (a_2x_1 + b_2x_2 + c_2)x_3 + a_3\chi_1 + b_3\chi_2 + \tau\Omega,$$

$$\varphi_1 = a_3\psi_1^{(1)} + b_3\left(\psi_1^{(2)} + \frac{1}{2}x_3^2\right) + b_2x_3 + \tau\theta_1,$$

$$\varphi_2 = a_3\left(\psi_2^{(1)} - \frac{1}{2}x_3^2\right) + b_3\psi_2^{(2)} - a_2x_3 + \tau\theta_2,$$

$$\varphi_3 = (a_3\varphi_3^{(1)} + b_3\varphi_3^{(2)})x_3 + \tau x_3 + a_2\varphi_3^{(1)} + b_2\varphi_3^{(2)},$$

where

$$a_3 = 6A_1^{(3)},\ b_3 = 6A_2^{(3)},\ a_2 = 2A_1^{(2)},\ b_2 = 2A_2^{(2)},$$

$$a_1 = A_1^{(1)},\ b_1 = A_2^{(1)},\ c_2 = C_1,\ \tau = \tau_1,$$

are arbitrary constants.

The stresses and the couple stresses corresponding to (22) are given by

$$t_{\rho\rho} = (a_3\tau_{\rho\rho}^{(1)} + b_3\tau_{\rho\rho}^{(2)})x_3 + a_2\tau_{\rho\rho}^{(1)} + b_2\tau_{\rho\rho}^{(2)} + \lambda[(a_3x_1 + b_3x_2)\,x_3 + a_2x_1 + b_2x_2]$$

(not summed),

$$t_{33} = [a_3(\tau_{33}^{(1)} + \lambda x_1) + b_3(\tau_{33}^{(2)} + \lambda x_2)]x_3 + a_2(\tau_{33}^{(1)} + \lambda x_1) + b_2(\tau_{33}^{(2)} +$$

$$+\lambda x_2) + c_2\frac{(2\mu+\varkappa)(3\lambda+2\mu+\varkappa)}{2\lambda+2\mu+\varkappa},$$

$$t_{\omega\rho}=(a_3\tau^{(1)}_{\omega\rho}+b_3\tau^{(2)}_{\omega\rho})x_3+a_2\tau^{(1)}_{\omega\rho}+b_2\tau^{(2)}_{\omega\rho}\quad(\omega\neq\rho),$$
$$t_{\rho3}=a_3[(\mu+\varkappa)\chi_{1,\rho}-(-1)^\rho\varkappa\psi^{(1)}_\nu+\mu u^{(1)}_\rho]+b_3[(\mu+\varkappa)\chi_{2,\rho}-(-1)^\rho\varkappa\psi^{(2)}_\nu+$$
$$+\mu u^{(2)}_\rho]+\tau[(\mu+\varkappa)\Omega_{,\rho}-(-1)^\rho\varkappa\theta_\nu+(-1)^\rho\mu x_\nu],$$
$$t_{3\rho}=a_3[\mu\chi_{1,\rho}+(-1)^\rho\varkappa\psi^{(1)}_\nu+(\mu+\varkappa)u^{(1)}_\rho]+b_3[\mu\chi_{2,\rho}+(-1)^\rho\varkappa\psi^{(2)}_\nu+$$
$$+(\mu+\varkappa)u^{(2)}_\rho]+\tau[\mu\Omega_{,\rho}+(-1)^\rho\varkappa\theta_\nu+(-1)^\rho(\mu+\varkappa)x_\nu]\quad(\nu\neq\rho),$$

and

$$m_{\rho\rho}=a_3[(\alpha+\beta+\gamma)\psi^{(1)}_{\rho,\rho}+\alpha(\psi^{(1)}_{\rho,\rho}+\varphi^{(1)}_3)]+b_3[(\alpha+\beta+\gamma)\psi^{(2)}_{\rho,\rho}+\alpha(\psi^{(2)}_{\rho,\rho}+\varphi^{(2)}_3)]+$$
$$+\tau[(\alpha+\beta+\gamma)\theta_{\rho,\rho}+\alpha(\theta_{\nu,\nu}+1)]\quad(\nu\neq\rho,\ \textit{not summed}),$$
$$m_{33}=a_3[(\alpha+\beta+\gamma)\varphi^{(1)}_3+\alpha\psi^{(1)}_{\omega,\omega}]+b_3[(\alpha+\beta+\gamma)\psi^{(2)}_3+\alpha\psi^{(2)}_{\omega,\omega}]+$$
$$+\tau[\alpha\theta_{\omega,\omega}+\alpha+\beta+\gamma],$$
$$m_{\omega\rho}=a_3[\beta\psi^{(1)}_{\omega,\rho}+\gamma\psi^{(1)}_{\rho,\omega}]+b_3[\beta\psi^{(2)}_{\omega,\rho}+\gamma\psi^{(2)}_{\rho,\omega}]+\tau[\beta\theta_{\omega,\rho}+\gamma\theta_{\rho,\omega}]\quad(\rho\neq\omega),$$
$$m_{\rho3}=a_3(c^{(1)}_{\rho3}-\beta\delta_{\rho2})x_3+b_3(c^{(2)}_{\rho3}+\beta\delta_{\rho1})x_3+a_2(c^{(1)}_{\rho3}-\beta\delta_{\rho2})+b_2(c^{(2)}_{\rho3}+\beta\delta_{\rho1}),$$
$$m_{3\rho}=a_3(c^{(1)}_{3\rho}-\gamma\delta_{\rho2})x_3+b_3(c^{(2)}_{3\rho}+\gamma\delta_{\rho1})x_3+a_2(c^{(1)}_{3\rho}-\gamma\delta_{\rho2})+b_2(c^{(2)}_{3\rho}+\gamma\delta_{\rho1}).$$

In order to demonstrate (21) let us consider $a_3=b_3=c_2=\tau=0$, and calculate the energy of deformation in this case. We have

$$W(a_2,b_2)=a_2^2h\int_S[(\tau^{(1)}_{33}+\lambda x_1)x_1-c^{(1)}_{32}+\gamma]d\sigma+$$
$$(23)\qquad +b_2^2h\int_S[(\tau^{(2)}_{33}+\lambda x_2)x_2+c^{(2)}_{31}+\gamma]d\sigma+$$
$$+a_2b_2h\int_S[(\tau^{(1)}_{33}+\lambda x_1)x_2+c^{(1)}_{31}+(\tau^{(2)}_{33}+\lambda x_2)x_1-c^{(2)}_{32}]d\sigma$$

and, from a theorem of reciprocity

$$\int_S[(\tau^{(1)}_{33}+\lambda x_1)x_2+c^{(1)}_{31}]d\sigma=\int_S[(\tau^{(2)}_{33}+\lambda x_2)x_1-c^{(2)}_{32}]d\sigma.$$

On the other hand, from the equilibrium equations we obtain

$$\int_S t_{3\rho}d\sigma=\int_S[t_{33}x_\rho+(-1)^\rho m_{3\nu}]_{,3}\,d\sigma$$

and at last

$$\int_S[\mu\chi_{\omega,\rho}+(-1)^\rho\varkappa\psi^{(\omega)}_\nu+(\mu+\varkappa)u^{(\omega)}_\rho]d\sigma=\int_S[(\tau^{(\omega)}_{33}+\lambda x_\omega)x_\rho+(-1)^\rho c^{(\omega)}_{3\nu}+$$
$$+\gamma\delta_{\rho\omega}]d\sigma\quad(\rho,\omega,\nu=1,2;\ \nu\neq\rho).$$

Because the discriminant of (23) is different from zero it results (21).

4. End conditions and the problem of Saint-Venant. In a problem of continuous media the boundary conditions are described by vector fields, scalar fields etc. Sometimes, under some special circumstances we may replace a vector field by another vector field having the same resultant and

same resultant moment; that is the case in the classical problem of Saint-Venant.

In the micropolar model, the loads are given by two vector fields, namely the tractions vector field and the couple-stresses vector field. Therefore the end conditions for a micropolar beam must be taken in the form

$$(24)\qquad R_i=\int_S t_{3i}\,d\sigma,$$

$$(25)\qquad M_\rho=(-1)^\rho\int_S t_{33}x_\nu d\sigma,\ M_3=\int_S(t_{31}x_2-t_{32}x_1)\,d\sigma,$$

$$(26)\qquad N_i=\int_S m_{3i}\,d\sigma,$$

$$(27)\qquad P_\rho=(-1)^\rho\int_S m_{33}x_\nu d\sigma,\ P_3=\int_S(m_{31}x_2-m_{32}x_1)\,d\sigma\ (\nu\neq\rho)\quad\text{on}\ x_3=0,$$

where R_i, M_i, N_i, P_i are given. Similarly, we define $R^{(h)}_i$, $M^{(h)}_i$, $N^{(h)}_i$ $P^{(h)}_i$ on the end $x_3=h$. The conditions of equilibrium

$$R_i+R^{(h)}_i=0,\quad M_i+M^{(h)}_i+N_i+N^{(h)}_i+\varepsilon_{i\alpha3}hR_\alpha=0$$

must be verified.

We consider $R_i,\ldots,R^{(h)}_i,\ldots$ as given, because we can always calculate them when the loads are given on the ends.

The above end conditions was suggested, in an incomplete manner, in [1].

In a natural extension, to micropolar beams, of the classical problem of Saint-Venant at the equations (1)–(6) we must add the end conditions in the form discussed above.

Because in our general solution (22) we have six arbitrary constants only, it results from this that the Saint-Venant problem does not have a polynomial solution for any situation.

5. End conditions for the uniqueness of polynomial solution. If we look for sufficient end conditions in order to assure the uniqueness of the solution (22) then we have more possibilities.

It seems that the most important are

I. $\mathbf{R}=(R_1,\ R_2,\ R_3)$, $\mathbf{M}=(M_1,\ M_2,\ M_3)$ on $x_3=0$;

II. $\mathbf{R}=(R_1,\ R_2,\ R_3)$, $\mathbf{N}=(N_1,\ N_2,\ N_3)$ on $x_3=0$;

III. $\mathbf{R}=(R_1,\ R_2,\ R_3)$, $\boldsymbol{\mathcal{M}}=(M_1+N_1,\ M_2+N_2,\ M_3+N_3)$ on $x_3=0$.

The last situation was studied in [5]. When the Saint Venant problem has a polynomial solution the three situations, mentioned above, have the same solution.

6. Other models of micropolar beams and the buckling effect. From the above results it appears that the problem of Saint-Venant, for the model (1), has not always a polynomial solution with the end conditions (24)–(27).

Now we look for other micropolar models such that most of the end conditions (24) – (27) are satisfied.

1° If we take in (1) $\varkappa = 0$, then we have a polynomial solution of our problem for arbitrary R_i, M_i, N_i. This means the end conditions (24) – (26) can be fulfiled.

In this case the stress tensor is symmetric

$$t_{ij} = t_{ji}. \tag{28}$$

The model defined by the constitutive equations

$$\begin{cases} t_{ij} = \lambda \varepsilon_{mm} \delta_{ij} + 2\mu \varepsilon_{ij} + \omega \chi_{mm} \delta_{ij} + \pi(\chi_{ij} + \chi_{ji}), \\ m_{ij} = \omega \varepsilon_{mm} \delta_{ij} + 2\pi \varepsilon_{ij} + \alpha \chi_{mm} \delta_{ij} + \beta \chi_{ij} + \gamma \chi_{ji}, \end{cases} \tag{29}$$

where

$$\varepsilon_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}), \quad \chi_{ij} = \varphi_{i,j},$$

has also a symmetric stress tensor and for it our problem has a unique solution for arbitrary R_i, M_i, N_i.

The constitutive equations (29) can be obtained from the model studied in [7].

Let V be the region occupied by the body and $P \subset V$ an arbitrary part of V. From the principle of moment of momentum [2] and the restrictions (28) we derive

$$\frac{d}{dt}\int_P \rho(x)\, J_{ik} \dot{\varphi}_k e_i \, d\tau = \int_P \rho(x)\, \mathbf{M}(x) d\tau + \int_{\delta P} \mathbf{m}\, d\sigma, \tag{30}$$

$$\frac{d}{dt}\int_P (\mathbf{r} \times \rho \mathbf{v})\, d\tau = \int_P (\mathbf{r} \times \rho \mathbf{f})\, d\tau + \int_{\delta P} (\mathbf{r} \times \mathbf{t})\, d\tau, \tag{31}$$

where $\rho(x)$ is the density of the medium, J_{ik} the microinertia tensor, $\mathbf{r}$ is the position of the point $x \in P$ and $\mathbf{v}$ is the velocity of this point ; $\mathbf{M}(x)$ are the external body couples per unit mass, $\mathbf{m}$ is the surface couple stress vector per unit area across the surface ∂P, the boundary of P. The vector field $\mathbf{f}(x)$ represent the external body forces per unit mass and $\mathbf{t}$ are the surface tractions per unit area of ∂P.

Taking (30), (31) in a state of equilibrium and for the entire body V, we have

$$\int_V (\mathbf{r} \times \rho \mathbf{f})\, d\tau + \int_{\delta V} (\mathbf{r} \times \mathbf{t})\, d\sigma = 0, \quad \int_V \mathbf{M} d\tau + \int_{\delta V} \mathbf{m} d\sigma = 0 \tag{32}$$

and (32) are amongst the necessary conditions of equilibrium of the body V. This situation is different from the model (1). As in the other models, the resultant of the external body forces and external surface tractions vanishes in a state of equilibrium.

In this model a neutral motion (without stresses and couple-stresses) is given by

$$u_1 = \omega_3 x_2 - \omega_2 x_3 + a_1, \quad u_2 = \omega_1 x_3 - \omega_3 x_1 + a_2, \quad u_3 = \omega_2 x_1 - \omega_1 x_2 + a_3, \quad \varphi_i = b_i,$$

where ω_i, a_i, b_i are the same in all points of the body.

2°. The coefficients β and γ may depend very closely on the shape of the body. We believe that those coefficients play an important role in the problems of stability, this, for instance, is the case in the model which follows.

In order to derive another model we may impose, in addition to (28), also the restrictions

$$m_{ij} = m_{ji}. \tag{33}$$

With the restriction (28) and (33), from the principle of moment of momentum, we can demonstrate that

$$\int_P (\mathbf{r} \times \rho J_{ik} \dot{\varphi}_k \mathbf{e}_i)\, d\tau = \int_P (\mathbf{r} \times \rho \mathbf{M})\, d\tau + \int_{\delta P} (\mathbf{r} \times \mathbf{m})\, d\sigma$$

and for an equilibrium state we obtain

$$\int_V (\mathbf{r} \times \rho \mathbf{M})\, d\tau + \int_{\delta V} (\mathbf{r} \times \mathbf{m})\, d\sigma = 0 \tag{34}$$

and this condition must be added to the necessary conditions of equilibrium. The conditions (32) are still valid.

From (29) we obtain such a model making $\gamma = \beta$. If we take into account that the body couples and the surface couples are modeled by vector fields as the body forces and surface tractions, then the equilibrium conditions (32), (34) which appear in this model are not surprising.

In this new model a neutral motion is given by

$$\mathbf{u} = \boldsymbol{\omega} \times \mathbf{r} + \mathbf{a}, \quad \boldsymbol{\varphi} = \mathbf{w} \times \mathbf{r} + \mathbf{b},$$

where ω_i, a_i, w_i, b_i the components of the vectors $\boldsymbol{\omega}$, $\mathbf{a}$, $\mathbf{w}$, $\mathbf{b}$, are the same in all points of the body.

For this last model new elementary problems appear, for example the bending by axial couples (by m_{33}). The displacements and the rotations corresponding to the bending by axial couples are of the form

$$u_\rho = \frac{1}{2}[(-1)^\nu (a_1^{(\rho)} x_1^2 - a_2^{(\rho)} x_2^2) - a_3^{(\rho)} x_3^2] + a_\rho^{(\nu)} x_1 x_2, \quad u_3 = (a_3^{(1)} x_1 + a_3^{(2)} x_2)\, x_3,$$

$$\varphi_\rho = \frac{1}{2}[(-1)^\nu (b_1^{(\rho)} x_1^2 - b_2^{(\rho)} x_2^2) - b_3^{(\rho)} x_3^2] + b_\rho^{(\nu)} x_1 x_2, \quad \varphi_3 = (b_3^{(1)} x_1 + b_3^{(2)} x_2)\, x_3 \quad (\nu \neq \rho),$$

the constants $a_i^{(\rho)}$, $b_i^{(\rho)}$ being taken such that

$$(\lambda + 2\mu)\, a_1^{(\rho)} + \lambda a_2^{(\rho)} + \lambda a_3^{(\rho)} + (\omega + 2\pi)\, b_1^{(\rho)} + \omega b_2^{(\rho)} + \omega b_3^{(\rho)} = 0,$$

$$\lambda a_1^{(\rho)} + (\lambda + 2\mu)\, a_2^{(\rho)} + \lambda a_3^{(\rho)} + \omega b_1^{(\rho)} + (\omega + 2\pi)\, b_2^{(\rho)} + \omega b_3^{(\rho)} = 0,$$

$$\lambda a_1^{(\rho)} + \lambda a_2^{(\rho)} + (\lambda + 2\mu)\, a_3^{(\rho)} + \omega b_1^{(\rho)} + \omega b_2^{(\rho)} + (\omega + 2\pi)\, b_3^{(\rho)} = 0,$$

$$(\omega + 2\pi)\, a_1^{(\rho)} + \omega a_2^{(\rho)} + \omega a_3^{(\rho)} + (\alpha + 2\beta)\, b_1^{(\rho)} + \alpha b_2^{(\rho)} + \alpha b_3^{(\rho)} = 0,$$

$$\omega a_1^{(\rho)} + (\omega + 2\pi)\, a_2^{(\rho)} + \omega a_3^{(\rho)} + \alpha b_1^{(\rho)} + (\alpha + 2\beta)\, b_2^{(\rho)} + \alpha b_3^{(\rho)} = 0,$$

$$\omega a_1^{(\rho)} + \omega a_2^{(\rho)} + (\omega + 2\pi)\, a_3^{(\rho)} + \alpha b_1^{(\rho)} + \alpha b_2^{(\rho)} + (\alpha + 2\beta)\, b_3^{(\rho)} = H_\rho,$$

where H_ρ are arbitrary constants.

For these displacements and rotations we have $R_i = M_i = N_i = 0$ and consequently they escape to the control of the classical conditions of equilibrium. The rising of these new effects may create the posibility for instability in the behaviour of the beam. They may be amang the sources of the buckling. The problem of Saint-Venant for this model has a unique solution which satisfies the end conditions (24)—(27). It seems that last model may be adequate to study the granular media. In order to derive the general equations of a model we may use the known procedure by starting from a principle of energy. First, we must precize the kinetic energy and the neutral motions of the model.

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CONTINUOUS DEPENDENCE RESULTS IN THERMODYNAMICS OF MATERIALS WITH HEAT CONDUCTION AND VISCOSITY

BY

S. CHIRIŢĂ

To the memory of Professor Mendel Haimovici

1. Introduction. We consider the theory of thermodynamics of elastic materials with heat conduction and viscosity [1, 2]. The main purpose of this paper is to establish the continuous dependence of thermodynamic processes upon initial state and supply terms. The results are local and they are established under certain assumptions on material response. In fact, we prove the continuous dependence results for smooth admissible processes residing in a neighborhood of a positive smooth admissible thermodynamic process. When the class of initial-boundary value problems is restricted to the displacement-boundary value problems, the continuous dependence results are established under a weaker assumption : the smooth admissible processes reside in a neighborhood of a smooth admissible process where a strong ellipticity condition is satisfied.

2. Basic equations. We consider a body that occupies, at time $t=0$, a properly regular region R of Euclidean three-dimensional space $\mathfrak{R}^3$ and is bounded by the piecewise smooth surface ∂R [3]. The configuration of the body at time $t=0$ is taken as the reference configuration. In this paper we use the referential description of motion. We identify the material point X with its position $\mathbf{X}$ in the fixed reference configuration. Let us denote by $X_K (K=1, 2, 3)$ the coordinates of the particle X relative to a fixed system of rectangular Cartesian axes. The coordinates of the particle in the position $\mathbf{x}$ at time t are denoted by $x_i (i=1, 2, 3)$, and

$$\mathbf{x} = \mathbf{x}(\mathbf{X}, t), \qquad (\mathbf{X}, t) \in \bar{R} \times [0, t_0].$$

According to the theory developed in [2], a thermodynamic process in R is described by eight functions, of $\mathbf{X}$ and t, whose values have the following physical interpretations : $\mathbf{x} = \mathbf{x}(\mathbf{X}, t)$ is the spatial position in the motion, $\mathbf{T} = \mathbf{T}(\mathbf{X}, t)$ is the first Piola-Kirchhoff stress tensor, $\mathbf{b} = \mathbf{b}(\mathbf{X}, t)$ is the body force per unit mass, $\psi = \psi(\mathbf{X}, t)$ is the Helmholtz free energy per unit mass, $\mathbf{Q} = \mathbf{Q}(\mathbf{X}, t)$ is the heat flux vector, $r = r(\mathbf{X}, t)$ is the heat supply per unit mass and unit time, $\eta = \eta(\mathbf{X}, t)$ is the specific entropy per unit mass, $\theta = \theta(\mathbf{X}, t)$ is the absolute temperature.

Throughout this paper we shall use the following notations : a superposed dot denotes the material time derivative ; Latin indices have range 1, 2, 3 ; summation over repeated subscripts is implied ; the symbol $|\,.\,|$ denotes a norm either in Euclidean vector space or in a tensor space, while