$$\alpha \iint\limits_{\mathcal{D}} \varphi_{\rho,\rho\lambda}\psi_\lambda d\tau = \alpha \int\limits_{\Gamma} \varphi_{\rho,\rho}\psi_\lambda n_\lambda d\tau - \alpha \iint\limits_{\mathcal{D}} \varphi_{\rho,\rho}\psi_{\lambda,\lambda} d\tau \tag{4.10}$$

$$\mu \iint\limits_{\mathcal{D}} \varphi_{\rho,\rho\lambda}\psi_\lambda d\tau = \gamma \int\limits_{\Gamma} \varphi_{\rho,\lambda}\psi_\lambda n_\rho ds - \gamma \iint\limits_{\mathcal{D}} \varphi_{\rho,\lambda}\psi_{\lambda,\rho} d\tau. \tag{4.11}$$

$$\beta \iint\limits_{\mathcal{D}} \psi_\lambda \Delta\varphi_\lambda d\tau = \beta \int\limits_{\Gamma} \varphi_{\lambda\rho}\psi_\lambda n_\rho ds - \beta \iint\limits_{\mathcal{D}} \varphi_{\lambda,\rho}\psi_{\lambda,\rho} d\tau. \tag{4.12}$$

On porte les relations (4.8)–(4.12) dans (4.5) et on trouve :

$$\iint\limits_{\mathcal{D}} \bar{V}\mathcal{L}\bar{W} d\tau = 2\iint\limits_{\mathcal{D}} \mathcal{D}(\bar{V},\bar{W}) d\tau - \int\limits_{\Gamma} \mathcal{D}(\bar{V},\bar{W})\, ds \tag{4.13}$$

qui est la première formule de Betti.

Si le vecteur $\bar{V}$ coïncide avec $\bar{W}$, alors on trouve la deuxième formule formule de Betti :

$$\iint\limits_{\mathcal{D}} \bar{W}\mathcal{L}\bar{W} d\tau = 2\iint\limits_{\mathcal{D}} \rho\psi(\bar{W}) d - \int\limits_{\Gamma} \mathcal{D}(W, \bar{W}) ds \tag{4.14}$$

La troisième formule de Betti est donné par :

$$\iint\limits_{\mathcal{D}} (\bar{V}\mathcal{L}\bar{W} - \bar{W}\mathcal{L}\bar{V}) d\tau = \int\limits_{\Gamma} [\mathcal{D}(\bar{V}, \bar{W}) - \mathcal{D}(\bar{W}, \bar{V})] ds. \tag{4.15}$$

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Analele ştiinţifice ale Universităţii „Al. I. Cuza" din Iaşi
Supliment la tomul XXVII, s. I a, 1981

TORSION PROBLEM IN THE THEORY OF NON-SIMPLE ELASTIC MATERIALS

BY

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To the memory of Professor Mendel Haimovici

1. Introduction. The importance of Saint-Venant's problem requires no emphasis. Indeed, a comprehensive bibliography of the vast and varied literature to which the work contained in [1] has given impetus would multiply the length of this study.

The elastic energy of non-simple media depends on the space derivatives of the displacement vector up to the n-th order, $n > 1$. In the case that $n = 2$, we speak of the first strain-gradient theory (see [2], [3]).

This paper is concerned with the torsion problem of elastic cylinders in the first strain-gradient theory. We prove that this problem requires the solution of a special boundary value problem for a fourth order elliptic equation in two independent variables. We are led to a new torsional rigidity and prove that it is positive. For simplicity, we restricted our attention to homogeneous and isotropic bodies. Following [4]–[6] we can extend the method in order to study the case of inhomogeneous and anisotropic cylinders.

The results are used to study the torsion of a circular cylinder.

2. Basic Equations. Throughout this paper a rectangular Cartesian coordinate system $Ox_i (i = 1, 2, 3)$ is used. We shall employ the usual summation and differentiation conventions : Greek subscripts are understood to range over the integers (1, 2), whereas Latin subscripts (unless otherwise specified) to the range (1, 2, 3) ; summation over repeated subscripts is implied and subscripts preceded by a comma denote partial differentiation with respect to the corresponding Cartesian coordinate.

We consider a body which occupies the bounded region B with Lipschitz boundary ∂B consisting of a finite number of smooth surfaces. Let Γ_α be intersections of two adjoined smooth surfaces and $\Gamma = \bigcup \Gamma_\alpha$.

The basic equations of the equilibrium theory, in the absence of body forces, are [3].

— the equations of equilibrium

$$\tau_{ji,j} - \mu_{sji,sj} = 0, \tag{2.1}$$

— the constitutive equations

$$\tau_{ij} = \lambda e_{rr}\delta_{ij} + 2\mu e_{ij},$$

$$\mu_{ijk} = \frac{1}{2}\alpha_1(\varkappa_{rri}\delta_{jk} + 2\varkappa_{krr}\delta_{ij} + \varkappa_{rrj}\delta_{ik}) + \alpha_2(\varkappa_{irr}\delta_{jk} + \varkappa_{jrr}\delta_{ik}) + 2\alpha_3\varkappa_{rrk}\delta_{ij} + \tag{2.2}$$
$$+ 2\alpha_4\varkappa_{ijk} + \alpha_5(\varkappa_{kij} + \varkappa_{kji}),$$

— the geometrical equations

$$e_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}), \quad \varkappa_{ijk} = u_{k,ij}. \tag{2.3}$$

In these equations we have used the following notations: τ_{ij} — the stress tensor, μ_{ijk} — the couple-stress tensor, e_{ij}, $\varkappa_{ijk}$ — the strain tensors, u_i — the components of the displacement vector, δ_{ij} — Kronecker's delta, λ, μ, $\alpha_s(s=\overline{1,5})$ — the elastic coefficients.

In the case of the traction problem we have the following boundary conditions [2], [3]

$$P_i \equiv [\tau_{ji} - \mu_{sji,s} + (\mu_{pji}n_p n_r - \mu_{pri}n_p n_j)_{,r}]\, n_j = \tilde{P}_i,$$

$$R_i \equiv \mu_{rsi} n_r n_s = \tilde{R}_i \quad \text{on} \quad \partial B \setminus \Gamma, \qquad Q_i \equiv \langle \mu_{pji} n_p \gamma_{ij} \rangle = \tilde{Q}_i \quad \text{on} \quad \Gamma, \tag{2.4}$$

where we have used the notations: n_i — the components of the outward unit normal to ∂B, $\langle\ \rangle$ denotes the difference of limits from both sides of Γ_α, $\gamma_{ij} = \varepsilon_{jpq} s_p n_q$, s_i — the components of the unit tangent vector to Γ_α, ε_{ijk} — the alternating symbol, $\tilde{P}_i$, $\tilde{R}_i$ and $\tilde{Q}_i$ — prescribed functions, piecewise regular on $\partial B \setminus \Gamma$ and Γ, respectively.

We assume that the potential energy density is a positive definite quadratic form. The necessary and sufficient conditions for the existence of a solution are [7]

$$\int_{\partial B} \tilde{P}_i da + \int_{\Gamma} \tilde{Q}_i ds = 0, \quad \int_{\partial B} \varepsilon_{ijk}(x_j P_k + n_j \tilde{R}_k) da + \int_{\Gamma} \varepsilon_{ijk} x_j \tilde{Q}_k ds = 0. \tag{2.5}$$

3. Statement of the Problem. Throughout the following sections B denotes the interior of a right cylinder of length l with the open cross-section Σ and lateral boundary S. The rectangular coordinate frame is chosen such that the x_3-axis is parallel to the generators of B and x_1Ox_2-plane contains one of terminal cross-sections. We denote by Σ_1 and Σ_2, respectively, the cross-section located at $x_3=0$ and $x_3=l$. We denote by L the boundary of the generic cross-section Σ. In view of the foregoing agreements, we have

$$B = \{x \mid (x_1, x_2) \in \Sigma,\ 0 < x_3 < l\}, \quad S = \{x \mid (x_1, x_2) \in L,\ 0 \leqslant x_3 \leqslant l\},$$
$$\Sigma_1 = \{x \mid (x_1, x_2) \in \overline{\Sigma},\ x_3 = 0\}, \quad \Sigma_2 = \{x \mid (x_1, x_2) \in \overline{\Sigma},\ x_3 = l\},$$
$$\Gamma_1 = \{x \mid (x_1, x_2) \in L,\ x_3 = 0\}, \quad \Gamma_2 = \{x \mid (x_1, x_2) \in L,\ x_3 = l\}.$$

We assume that the domain Σ is C^∞ — smooth [8].

The cylinder is supposed to be free from lateral loading, so that the conditions on the lateral surface are

$$P_i = 0, \quad R_i = 0 \quad \text{on} \quad S. \tag{3.1}$$

We assume that the load on the cylinder is distributed over its ends, Σ_1 and Σ_2, in a way which fulfills the equilibrium conditions of a rigid body. Let the loading applied on Σ_1 be statically equivalent to the force $\mathbf{F}=0$ and the moment $\mathbf{M}(0, 0, M)$. Thus, for $x_3=0$ we have the following conditions

$$\int_{\Sigma_1} P_\alpha da + \int_{\Gamma_1} Q_\alpha ds = 0, \tag{3.2}$$

$$\int_{\Sigma_1} P_3 da + \int_{\Gamma_1} Q_3 ds = 0, \tag{3.3}$$

$$\int_{\Sigma_1} (x_\alpha P_3 + R_\alpha) da + \int_{\Gamma_1} x_\alpha Q_3 ds = 0, \tag{3.4}$$

$$\int_{\Sigma_1} \varepsilon_{\alpha\beta3} x_\alpha P_\beta da + \int_{\Gamma_1} \varepsilon_{\alpha\beta3} x_\alpha Q_\beta ds = M. \tag{3.5}$$

For $x_3 = l$ we have the conditions

$$\int_{\Sigma_2} P_\alpha da + \int_{\Gamma_2} Q_\alpha ds = 0, \tag{3.6}$$

$$\int_{\Sigma_2} P_3 da + \int_{\Gamma_2} Q_3 ds = 0, \tag{3.7}$$

$$\int_{\Sigma_2} (x_\alpha P_3 - R_\alpha) da + \int_{\Gamma_2} x_\alpha Q_3 ds = 0, \tag{3.8}$$

$$\int_{\Sigma_2} \varepsilon_{\alpha\beta3} x_\alpha P_\beta da + \int_{\Gamma_2} \varepsilon_{\alpha\beta3} x_\alpha Q_\beta ds = -M. \tag{3.9}$$

Let us note that from (2.4) we obtain

$$\begin{aligned} &P_i = -\tau_{3i} + 2\mu_{\alpha3i,\alpha} + \mu_{33i,3}, \quad R_i = \mu_{33i} \quad \text{on} \quad \Sigma_1, \\ &P_i = \tau_{3i} - 2\mu_{\alpha3i,\alpha} - \mu_{33i,3}, \quad R_i = \mu_{33i} \quad \text{on} \quad \Sigma_2, \\ &Q_i = -2\mu_{\alpha3i} n_\alpha \quad \text{on} \quad \Gamma_1, \quad Q_i = 2\mu_{\alpha3i} n_\alpha \quad \text{on} \quad \Gamma_2, \end{aligned} \tag{3.10}$$

where $(n_1, n_2, 0)$ are the direction cosines of the exterior normal to S.

The torsion problem consists in the solving of the equations (2.1)–(2.3) with the conditions (3.1)–(3.9).

4. Solution of the Problem. We try to solve the torsion problem assuming that

$$u_\alpha = \tau \varepsilon_{\beta\alpha3} x_\beta x_3, \quad u_3 = \tau \varphi(x_1, x_2), \tag{4.1}$$

where the function φ and the constant τ will be specified in the following.

From (2.3) and (4.1) we obtain

$$\begin{aligned} &e_{\alpha\beta} = e_{33} = 0, \quad e_{\alpha3} = \frac{1}{2}\tau(\varphi_{,\alpha} - \varepsilon_{\alpha\beta3} x_\beta), \\ &\varkappa_{\alpha\beta\gamma} = \varkappa_{\alpha33} = \varkappa_{33i} = 0, \quad \varkappa_{3\alpha\beta} = \tau\varepsilon_{\alpha\beta3}, \quad \varkappa_{\alpha\beta3} = \tau\varphi_{,\alpha\beta}. \end{aligned} \tag{4.2}$$

The equations (2.2) lead to

$$\tau_{\alpha\beta} = \tau_{33} = 0, \quad \tau_{\alpha3} = \mu\tau(\varphi_{,\alpha} - \varepsilon_{\alpha\beta3} x_\beta), \quad \mu_{\alpha\beta\gamma} = \mu_{\alpha33} = \mu_{33\alpha} = 0$$

$$\mu_{3\alpha\beta} = \tau\left[\frac{1}{2}\alpha_1\delta_{\alpha\beta}\Delta\varphi + \alpha_5\varphi_{,\alpha\beta} - (\alpha_5 - 2\alpha_4)\varepsilon_{\alpha\beta 3}\right], \tag{4.3}$$

$$\mu_{\alpha\beta 3} = 2\tau(\alpha_3\delta_{\alpha\beta}\Delta\varphi + \alpha_4\varphi_{,\alpha\beta}), \quad \mu_{333} = (\alpha_1 + 2\alpha_3)\tau\Delta\varphi,$$

where $\Delta\varphi = \varphi_{,\alpha\alpha}$.

If we introduce the notation

$$\mathcal{L}\varphi = 2(\alpha_3 + \alpha_4)\Delta\Delta\varphi - \mu\Delta\varphi, \tag{4.4}$$

then the equilibrium equations (2.1) reduce to

$$\mathcal{L}\varphi = 0 \quad \text{on} \quad \Sigma. \tag{4.5}$$

With the help of (2.4) and (4.3), the conditions (3.1) become

$$\mathfrak{M}\varphi = \mu\varepsilon_{\alpha\beta 3}x_\alpha n_\beta, \quad \mathfrak{N}\varphi = 0 \quad \text{on} \quad L, \tag{4.6}$$

where

$$\mathfrak{M}\varphi = -\mu\frac{\partial\varphi}{\partial n} + 2(\alpha_3 + \alpha_4)\frac{\partial}{\partial n}\Delta\varphi + 2\alpha_4\frac{\partial}{\partial s}\frac{\partial^2\varphi}{\partial s\partial n},$$

$$\mathfrak{N}\varphi = -2(\alpha_3 + \alpha_4)\Delta\varphi + 2\alpha_4\frac{\partial^2\varphi}{\partial s^2},$$

$$\frac{\partial\varphi}{\partial n} = \varphi_{,\alpha}n_\alpha, \quad \frac{\partial\varphi}{\partial s} = \varepsilon_{\beta\alpha 3}\varphi_{,\alpha}n_\beta = \varphi_{,\alpha}s_\alpha, \quad \frac{\partial^2\varphi}{\partial s\partial n} = \varphi_{,\alpha\beta}s_\alpha n_\beta, \quad \frac{\partial^2\varphi}{\partial s^2} = \varphi_{,\alpha\beta}s_\alpha s_\beta. \tag{4.7}$$

Let us consider the boundary value problem

$$\mathcal{L}u = f \quad \text{on} \quad \Sigma, \quad \mathfrak{M}u = g, \quad \mathfrak{N}u = h \quad \text{on} \quad L, \tag{4.8}$$

where f, g, h are C^∞-functions. We consider only a „C^∞-theory" but it is possible to get a classical solution of the problem for more general domains and more general assumptions of regularity.

We can prove that

$$\int_\Sigma v\mathcal{L}u\, da = \int_L \left(v\mathfrak{M}u + \frac{\partial v}{\partial n}\mathfrak{N}u\right) ds + \int_\Sigma A(u, v)da, \tag{4.9}$$

where

$$A(u,v) = 2\alpha_3\Delta u\Delta v + 2\alpha_4 u_{,\alpha\beta}v_{,\alpha\beta} + \mu u_{,\alpha}v_{,\alpha}. \tag{4.10}$$

From (4.9) we get

$$\int_\Sigma u\mathcal{L}u\, da = \int_L \left(u\mathfrak{M}u + \frac{\partial u}{\partial n}\mathfrak{N}u\right) ds + \int_\Sigma A(u, u)da. \tag{4.11}$$

We have assumed that the potential energy density is a positive definite quadratic form. This fact implies that the form $A(u, u)$ is positive definite. If $f = g = h = 0$, then from (4.11) we obtain $u = c$, where c is an arbitrary constant. Thus, the difference of two solutions of the boundary value problem (4.8) is an arbitrary constant.

Following [7], [8] we obtain

Theorem 4.1. *The necessary and sufficient condition for the existence of a solution of the boundary value problem* (4.8) *is*

$$\int_\Sigma f\, da = \int_L g\, ds. \tag{4.12}$$

Obviously, in the case of the boundary value problem (4.5), (4.6), the condition (4.12) is satisfied. In what follows we assume that the torsion function φ is known.

Let us consider now the conditions on the ends. From (3.10) and (4.3) we conclude that $P_3 = R_\alpha = Q_3 = 0$, so that the conditions (3.3), (3.4), (3.7), (3.8) are identically satisfied. Let us prove that the conditions (3.2) are also satisfied. Using (2.4), (3.10), the last of the equilibrium equations (2.1), the divergence theorem and the boundary conditions (3.1), we can write

$$\int_{\Sigma_1} P_1 da + \int_{\Gamma_1} Q_1 ds = -\int_\Sigma \tau_{31} da = -\int_\Sigma [\tau_{31} + x_1(\tau_{\alpha 3,\alpha} - \mu_{\alpha 33,\alpha 3})] da = -\int_L x_1(\tau_{3\alpha} -$$

$$-\mu_{\alpha 33,3})n_\alpha ds - \int_\Sigma \mu_{133,3} da = -\int_L x_1(\mu_{\rho\nu 3}n_\alpha n_\rho - \mu_{\rho\alpha 3}n_\rho n_\nu)_{,\nu} n_\alpha ds - \int_\Sigma \mu_{133,3} da =$$

$$= -\int_L \mu_{\rho\nu 3}n_\rho n_\nu n_1 ds = -\int_L R_3 n_1 ds = 0.$$

In a similar way we can prove that the conditions $(3.2)_2$, (3.6) are satisfied.

With the help of (3.10), the conditions (3.5) and (3.9) reduce to

$$\int_\Sigma \varepsilon_{3\alpha\beta}(x_\alpha\tau_{3\beta} + 2\mu_{\alpha 3\beta})da = -M. \tag{4.13}$$

From (4.3) and (4.13) we obtain

$$\tau D = -M, \tag{4.14}$$

where

$$D = \int_\Sigma [\mu(\varepsilon_{3\alpha\beta}x_\alpha\varphi_{,\beta} + x_\alpha x_\alpha) + 4(2\alpha_4 - \alpha_5)]da, \tag{4.15}$$

is the torsional rigidity.

Let us prove that $D \neq 0$. Using (2.1)–(2.4), Stokes theorem and integrating by parts we obtain (see e.g. [3], [7])

$$2\int_B W dv = \int_{\partial B} (P_i u_i + R_i N u_i)da + \int_\Gamma Q_i u_i ds, \tag{4.16}$$

where W is the potential energy density and $Nu_i = u_{i,j}n_j$. In the case of the torsion problem we have

$$u_\alpha = 0,\ Nu_3 = 0 \quad \text{on} \quad \Sigma_1,\ u_\alpha = \varepsilon_{3\beta\alpha}\tau l x_\beta,\ Nu_3 = 0 \ \text{on}\ \Sigma_2. \tag{4.17}$$

If we take into account (3.1), (3.10), (4.3) and (4.17), from (4.16) we obtain

$$2\int_B W dv = \tau l \int_\Sigma [x_1\tau_{32} - x_2\tau_{31} + 2(\mu_{132} - \mu_{231})]da = \tau^2 l D, \tag{4.18}$$

so that we conclude that $D > 0$.

Thus, if the torsion function is known, then the constant τ is determined by (4.14). The torsion problem is solved.

Example. In what follows we use the results established in this section in order to study the torsion of a circular cylinder. We assume that L is a circle of radius a. Axis Ox_3 of the coordinate system is taken along the axis of the cylinder. In this case $n_\alpha = x_\alpha/a$, so that the torsion function satisfies the equation (4.5) and the conditions $\mathfrak{M}\varphi = 0$, $\mathfrak{N}\varphi = 0$ on L. We can take $\varphi = 0$. From (4.15) we obtain the torsional rigidity

$$D = \frac{1}{2}\pi\mu a^4 + 4(2\alpha_4 - \alpha_5)\pi a^2.$$

In the classical theory of elasticity, the torsional rigidity is given by

$$D_c = \frac{1}{2}\pi\mu_c a^4 ,$$

where the subscript c is used to denote quantities in classical theory.

The extension, bending and flexure problem will be presented in another paper.

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Analele ştiinţifice ale Universităţii „Al. I. Cuza" din Iaşi
Supliment la tomul XXVII, s. I a, 1981

SUR L'ESTIMATION DANS LA RÉSOLUTION APPROXIMATIVE D'UN PROBLÈME PARABOLIQUE PAR LA MÉTHODE DES ÉLÉMENTS FINIS

PAR

I. IONESCU et I. MOLNAR

Dédié à la mémoire du Professeur Mendel Haimovici

1. Introduction. Le but de cet article est de présenter une méthode de résolution approximative du problème mixte pour l'équation parabolique la plus simple, ayant les données nulles à la frontière, et aussi d'obtenir une estimation de l'erreur dans la norme de l'espace $L^2(\Omega)$, sous certaines suppositions de régularité pour la condition initiale et pour la frontière du domaine. Le résultat principal de cet ouvrage est le Théorème 4.1., qui établit l'estimation du type $\|u^n - u^n_{kh}\| \leqslant O(h+k)$. La méthode consiste dans la discrétisation relativement à la variable temps, d'où il résulte une suite de problèmes elliptiques correspondants aux temps fixés, qui seront résolus d'une manière récursive par la méthode des éléments finis. Le problème qui conduit à la solution approximative peut être résolu à l'aide du calculateur.

On utilisera les notations suivantes :

$(\cdot\,,\cdot)$-le produit scalaire dans l'espace $L^2(\Omega)$: $(u, v) = \int_\Omega uv dx$

$\|\cdot\|$-la norme dans l'espace $L^2(\Omega)$: $\|v\| = (v, v)^{1/2}$

$H^j(\Omega)$, $j = 1,2$-l'espace de Sobolev, muni de la norme $\|\cdot\|_j$:

$$\|v\|_j = \Big(\sum_{|\alpha| \leqslant j} \|D^\alpha v\|^2\Big)^{1/2}$$

$H^1_0(\Omega)$-l'espace $\overline{C^\infty_0(\Omega)}^{\|\cdot\|_1}$

$$\Delta = \partial^2/\partial x_1^2 + \partial^2/\partial x_2^2$$

$$\nabla = (\partial/\partial x_1,\ \partial/\partial x_2)$$

On note par C une constante positive générique indépendante des h, k, n, g.

On considère pour $u = u(x, t)$ le problème mixte représenté par les conditions suivantes :

$$\begin{cases} \dfrac{\partial u}{\partial t} = \Delta u & (x, t) \in \Omega \times (0, T) \\ u = 0 & (x, t) \in \partial\Omega \times (0, T) \\ u(x, t) = g(x) & x \in \Omega, \qquad t = 0 \end{cases} \tag{1}$$