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SUR UN PROBLÈME D'ÉLASTICITÉ PLANE EN POSITION DÉFORMÉE

PAR

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Dédié à la mémoire du Professeur Mendel Haimovici

Dans cet ouvrage on considère un calcul qui se propose, d'une côté-répondre aux critiques liées aux, équations de l'élasticité en position initiale (non déformée) — et de l'autre côté, éviter les difficultés résultées des équations eu positions déformée.

On propose un calcul en position presque déformée — un calcul du second ordre, qui met en évidence une liaison aveel es tensions qui proviennent d'un calcul en position non déformée-calcul du premier ordre.

On considérera une position provisoirement voisine eventuellement celle déformée, qui sera écrite en fonction des coordonnées initiales (x, y).

Soit en ce sens un rectangle défini par les dimensions (dx, dy) et une position quelconque voisine possible (fig. 1).

Les éléments qui caractérisent la position voisine seront notés par un astérisque. En négligeant pourtant quelques termes on a

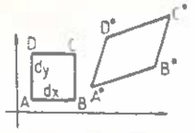

Fig. 1

$$\overline{A^*B^*} = \overline{D^*C^*}\left(\left(1+\frac{\partial u^*}{\partial x}\right)dx,\ \frac{\partial v^*}{\partial x}dx\right) \tag{1}$$

$$\overline{A^*D^*} = \overline{B^*C^*}\left(\frac{\partial u^*}{\partial y}dy,\ \left(1+\frac{\partial v^*}{\partial y}\right)dy\right).$$

Soit p^*_{xx}, p^*_{xy}, p^*_{yy} les efforts sur des surfaces du parallélogramme $A^*B^*C^*D^*$. On considère aussi une hypothèse sur la nature de la conservation de l'intensité de la force massique autant en intensité qu'en direction. En vertu de cette hypothése on a

$$XdA = X^*dA^*,\ YdA = Y^*dA^*$$

Une première forme des équations d'équilibre en position voisine est

$$\frac{\partial p^*_{xx}}{dx}|\overline{B^*C^*}| + \frac{\partial p^*_{xy}}{\partial y}dy|\overline{A^*B^*}| + Xdxdy = 0$$

$$\frac{\partial p^*_{yx}}{\partial x}dx|\overline{B^*C^*}| + \frac{\partial p^*_{yy}}{\partial y}dy|\overline{A^*B^*}| + Ydxdy = 0 \tag{2}$$

On tient compte que les formules (1) conduisent à

$$|\overline{A^*B^*}| = \sqrt{\left(1+\frac{\partial u^*}{\partial x}\right)^2 + \left(\frac{\partial v^*}{\partial x}\right)^2}\,dx \cong \left(1+\frac{\partial u^*}{\partial x}\right)dx,\quad |\overline{B^*C^*}| = \left(1+\frac{\partial v^*}{\partial y}\right)dy.$$

Après des simplifications on est conduit aux équations d'équilibre

$$\frac{\partial p^*_{xx}}{\partial x}\left(1+\frac{\partial v^*}{\partial y}\right)+\frac{\partial p^*_{xy}}{\partial y}\left(1+\frac{\partial u^*}{\partial x}\right)+X=0,\quad \frac{\partial p^*_{yx}}{\partial x}\left(1+\frac{\partial v^*}{\partial y}\right)+\frac{\partial p^*_{yy}}{\partial y}\left(1+\frac{\partial u^*}{\partial x}\right)+Y=0,$$

ou

$$(3)\qquad \begin{aligned} &\frac{\partial p^*_{xx}}{\partial x}+\frac{\partial p^*_{xy}}{\partial y}+\frac{\partial p^*_{xx}}{\partial x}\frac{\partial v^*}{\partial y}+\frac{\partial p^*_{xy}}{\partial y}\frac{\partial u^*}{\partial x}+X=0,\\ &\frac{\partial p^*_{yx}}{\partial x}+\frac{\partial p^*_{yy}}{\partial y}+\frac{\partial p^*_{yx}}{\partial x}\frac{\partial v^*}{\partial y}+\frac{\partial p^*_{yy}}{\partial y}\frac{\partial u^*}{\partial x}+Y=0,\end{aligned}$$

En exprimant les composants p^*_{xx}, ... sur les surfaces du parallélogramme par les composants σ^*_{xx}, ..., on obtient

$$p^*_{xx}=\sigma^*_{xx}-\sigma^*_{xy}\frac{\partial u^*}{\partial y},\quad p^*_{xy}=\sigma^*_{xy}-\sigma^*_{xx}\frac{\partial v^*}{\partial x};$$

$$p^*_{yx}=\sigma^*_{yx}-\sigma^*_{yy}\frac{\partial u^*}{\partial y},\quad p^*_{yy}=\sigma^*_{yy}-\sigma^*_{yx}\frac{\partial v^*}{\partial x}.$$

Effections la décompositions des efforts σ^* sous la forme $\sigma^*=\sigma^0+\sigma^{**}$, dans laquelle σ^0 signifie l'état obtenu par le calcul du premier ordre et σ^{**} signifie la variation du premier ordre parue à la suite de la considérations des équations en positions voisine. En négligeant de nouveau les produits infiniment petits on obtient :

$$(4)\quad p^*_{xx}=\sigma^*_{xx}-\sigma^*_{xy}\frac{\partial u^*}{\partial y},\ p^*_{xy}=\sigma^*_{xy}-\sigma^*_{xx}\frac{\partial v^*}{\partial x},\ p^*_{yx}=\sigma^*_{yx}-\sigma^*_{yy}\frac{\partial u^*}{\partial y},\ p^*_{yy}=\sigma^*_{yy}-\sigma^*_{yx}\frac{\partial v^*}{\partial x},$$

$$\frac{\partial p^*_{xx}}{\partial x}\frac{\partial v^*}{\partial y}=\frac{\partial v^*}{\partial y}\frac{\partial}{\partial x}\left(\sigma^*_{xx}+\sigma^{**}_{xx}-\sigma^*_{xy}\frac{\partial u^*}{\partial y}\right)\cong\frac{\partial\sigma^*_{xx}}{\partial x}\frac{\partial v^*}{\partial y},$$

$$\frac{\partial p^*_{xy}}{\partial y}\frac{\partial u^*}{\partial x}\cong\frac{\partial\sigma^*_{xy}}{\partial y}\frac{\partial u^*}{\partial x},\quad \frac{\partial p^*_{yx}}{\partial x}\frac{\partial v^*}{\partial y}\cong\frac{\partial\sigma^*_{xy}}{\partial x}\frac{\partial v^*}{\partial x},\quad \frac{\partial p^*_{yy}}{\partial y}\frac{\partial u^*}{\partial x}\cong\frac{\partial\sigma^*_{yy}}{\partial y}\frac{\partial u^*}{\partial x}.$$

On associe le groupe de relations physiques

$$\sigma^*_{xx}=\frac{E}{1-\mu^2}\left(\frac{\partial u^*}{\partial x}+\mu\frac{\partial v^*}{\partial y}\right);\qquad \sigma^*_{yy}=\frac{E}{1-\mu^2}\left(\frac{\partial v^*}{\partial y}+\mu\frac{\partial u^*}{\partial x}\right),$$

$$\sigma^*_{xy}=\sigma^*_{yx}=\frac{E}{2(1+\mu)}\left(\frac{\partial u^*}{\partial y}+\frac{\partial v^*}{\partial x}\right)$$

et on remplace ces éléments en (4). On a

$$p^*_{xx}=\frac{E}{1-\mu^2}\left(\frac{\partial u^*}{\partial x}+\mu\frac{\partial v^*}{\partial y}\right)-\sigma^*_{xy}\frac{\partial u^*}{\partial y},\quad p^*_{yx}=\frac{E}{2(1+\mu)}\left(\frac{\partial u^*}{\partial y}+\frac{\partial v^*}{\partial x}\right)-\sigma^*_{yy}\frac{\partial u^*}{\partial y},$$

$$p^*_{xy}=\frac{E}{2(1+\mu)}\left(\frac{\partial u^*}{\partial y}+\frac{\partial v^*}{\partial x}\right)-\sigma^*_{xx}\frac{\partial v^*}{\partial x},\quad p_{yy}=\frac{E}{1-\mu^2}\left(\frac{\partial v^*}{\partial y}+\mu\frac{\partial u^*}{\partial x}\right)-\sigma^*_{yx}\frac{\partial v^*}{\partial x},$$

En remplaçant dans des équations d'équilibre et en identifiant la position voisine en position déformée on obtient

$$\frac{E}{1-\mu^2}\left(\frac{\partial^2 u}{\partial x^2}+\mu\frac{\partial^2 v}{\partial x\partial y}\right)+\frac{E}{2(1+\mu)}\left(\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 v}{\partial x\partial y}\right)+X=$$

$$=\frac{\partial\sigma^*_{xy}}{\partial x}\frac{\partial u}{\partial y}+\sigma^*_{xy}\frac{\partial^2 u}{\partial x\partial y}+\frac{\partial\sigma^*_{xx}}{\partial y}\frac{\partial v}{\partial x}+\sigma^*_{xx}\frac{\partial^2 v}{\partial x\partial y}-\frac{\partial\sigma^*_{xx}}{\partial x}\frac{\partial v}{\partial y}-\frac{\partial\sigma^*_{xy}}{\partial y}\frac{\partial u}{\partial x},$$

$$\frac{E}{1-\mu^2}\left(\frac{\partial^2 v}{\partial y^2}+\mu\frac{\partial^2 u}{\partial x\partial y}\right)+\frac{E}{2(1+\mu)}\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 v}{\partial x\partial y}\right)+Y=$$

$$=\frac{\partial\sigma^*_{yy}}{\partial x}\frac{\partial u}{\partial y}+\sigma^*_{yy}\frac{\partial^2 u}{\partial x\partial y}+\frac{\partial\sigma^*_{yx}}{\partial y}\frac{\partial v}{\partial x}+\sigma^*_{yx}\frac{\partial^2 v}{\partial x\partial y}-\frac{\partial\sigma^*_{xy}}{\partial x}\frac{\partial v}{\partial y}-\frac{\partial\sigma^*_{yy}}{\partial y}\frac{\partial u}{\partial x}$$

qui constituent les équations de type Lamé.

Le présence des termes qui proviennent du calcul du premier ordre suggère un calcul à petits pas — un calcul par approximations succesives.

En ce qui concerne les conditions à la limite, on peut considérer un élément marginal pour lequel on a

$$p^*_{n^*x^*}|\overline{A^*B^*}|=p^*_{xx}|\overline{O^*B^*}|+p^*_{xy}|\overline{O^*A^*}|,\quad p^*_{n^*y}|\overline{A^*B^*}|=p^*_{yx}|\overline{O^*B^*}|+p^*_{yy}|\overline{O^*A^*}|.$$

En tenant compte des évaluations antérieures on a

$$p_{nx}=\left[\frac{E}{1-\mu^2}\left(\frac{\partial u}{\partial x}+\mu\frac{\partial v}{\partial y}\right)-\sigma^*_{xy}\frac{\partial u}{\partial y}+\sigma^*_{xx}\frac{\partial v}{\partial y}\right]\cos(n,x)+$$

$$+\left[\frac{E}{2(1+\mu)}\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right)-\sigma^*_{xx}\frac{\partial v}{\partial x}+\sigma^*_{xy}\frac{\partial u}{\partial x}\right]\cos(n,y),$$

$$p_{ny}=\left[\frac{E}{1-\mu^2}\left(\frac{\partial v}{\partial y}+\mu\frac{\partial u}{\partial x}\right)-\sigma^*_{xy}\frac{\partial v}{\partial x}+\sigma^*_{yy}\frac{\partial u}{\partial x}\right]\cos(n,y)+$$

$$+\left[\frac{E}{2(1+\mu)}\left(\frac{\partial u}{\partial v}+\frac{\partial v}{\partial x}\right)-\sigma^*_{yy}\frac{\partial u}{\partial y}+\sigma^*_{xy}\frac{\partial v}{\partial y}\right]\cos(n,x).$$

Exemple. Soit un parallélépipède régulier de grande longueur contraint à une pression uniforme p et appuyé sur une base élastique et lisse. Avec hypothèses ennoncées, on a :

$$(h,y)=-p,\qquad u(0,y)=0,\qquad (0,y)=0;$$

on demande l'état élastique. On peut voir immédiatement qu'en remplaçant ces valeurs dans les équations (5) on obtient

$$\frac{E}{1-\mu^2}\left(\frac{\partial^2 u}{\partial x^2}+\mu\frac{\partial^2 v}{\partial x\partial y}\right)+\frac{E}{2(1+\mu)}\left(\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 v}{\partial x\partial y}\right)=-p\frac{\partial^2 v}{\partial x\partial y},$$

$$\frac{E}{1-\mu^2}\left(\frac{\partial^2 v}{\partial y^2}+\mu\frac{\partial^2 u}{\partial x\partial y}\right)+\frac{E}{2(1+\mu)}\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 u}{\partial x\partial y}\right)=0.$$

Les conditions à la limite sont

$$(6_1)\qquad u=0,\quad \sigma_{xy}=0 \text{ sur } x=0 \qquad (6_2)\qquad \sigma_{xx}=-p,\quad \sigma_{xy}=0 \text{ sur } x=h$$

$$(6_3)\qquad \sigma_{xy}=\sigma_{yy}=0 \text{ sur } y=\pm b$$

Des relations (6) on déduit les conditions à la limite :

$$u=0,$$

$$\frac{E}{1-\mu^2}\left(\frac{\partial v}{\partial y}+\mu\frac{\partial u}{\partial x}\right)=0,\quad \text{sur } x=0,\quad n(-1,0),\qquad \frac{E}{1-\mu^2}\left[\frac{\partial u}{\partial x}+\mu\frac{\partial v}{\partial y}-p\frac{\partial v}{\partial y}\right]=-p,$$

$$\frac{E}{1-\mu^2}\left(\frac{\partial u}{\partial y}+\mu\frac{\partial u}{\partial x}\right)=0 \text{ sur } x=0,\ n(1,0),\qquad \frac{E}{2(1+\mu)}\left(\frac{\partial u}{\partial y}+\mu\frac{\partial v}{\partial x}\right)=0,$$

$$\frac{E}{2(1+\mu)}\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right)+p\frac{\partial v}{\partial x}=0 \text{ sur } y=b,\ n(0,1),\qquad \frac{E}{2(1+\mu)}\left(\frac{\partial u}{\partial y}+\mu\frac{\partial v}{\partial x}\right)=0,$$

$$\frac{E}{2(1+\mu)}\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right)+p\frac{\partial v}{\partial x}=0 \text{ sur } y=-b,\ n(0,-1)$$

Evidemment on doit avoir $u(x,-y)=u(x,y)$, $v(x,-y)=-v(x,y)$. En cherchant des solutions de la forme

$$u=a_{11}x^2+a_{22}y^2+2a_{13}x,\quad v=2b_{12}xy+2b_{23}y,$$

on déduit

$$u=-\frac{2p}{E}x,\quad v=\frac{2\mu p}{E}y$$

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MOMENTUM CONNECTIONS, APPLICATIONS IN MECHANICS AND FIELD THEORY

BY

M. MIȘICU

To the memory of Professor Mendel Haimovici

The below exposed attempt to develop a theory of momentum connections is performed in view of an advanced consistency between the structure of material objects or of physical fields and the geometrical description via the momentum representation of the fundamental characteristics. The respective elements are inspirated by a previously formulated higher order momentum mechanics as well as from a procedure of generating „structural" connections. The results allow subsequent developments concerning the path dynamics, the continuum and structural mechanics as well as the field theory, the gravity, relativity, cosmology, etc. The rather expositive here presented elements are substantiated by a more comprehensive treatment, actually in print.

I.N-dimensional spaces with „primitive" fiber and representative M-spaces. We list below the basic elements which occur in the subsequent definitions of momentum components of different kinds leading to similar covariant transformations laws.

1) The set X_N of elements in a one-to-one bi-continual correspondence with the value of N variables or coordinates $x=(x_1, x_2, \ldots, x_N)$ which stand for the coordinates of the points $P\in X_N$ in direct and converse correspondence with the values x. We assume also the on-to-one bi-continual correspondences $x^I=F^I(y)$ between two arbitrary systems of variables x, y assigned to the spaces X_N, Y_N. If the variables are regarded as cartezian coordinates of an euclidian space E_N it follows that between the corresponding points $P(x)\in X_N$, $Q(y)\in E_N$ holds true an on-to-one bi-continual correspondence, X_N being the topological representation of E_N.

2) The euclidian space E_P (differential fiber) with coordinates $(dx^1, \ldots, dx^N)$ at $P\in X_N$.

3) The differential affinity (differomorphism) at P: $dx^I=A^I_i dx^i$ induced by the coordinate transformation $x^I=x^I(x^i)$, $(A^I_i=\partial x^I/\partial x^i)_P$).

4) The set Ξ_N of elements assigned to each point $P(x)\in X_N$ in a one-to-one bi-continual correspondence with the values of N variables $\xi=(\xi^1, \ldots, \xi^N)$ with origin at P so that to any point $\Pi\in\Xi_N$ for $P\in X_N$ should correspond a set of values ξ and conversely. We say that Ξ_N is the „primitive" fiber at P. As in 1) we consider the transformation $\xi^I=\Phi^I_P(\eta)$ between the values of coordinates of the point $\Pi(\xi)\in\Xi_N$ at $P\in X_N$ and $\eta(\eta)\in\mathcal{Y}_N$ at $Q\in Y_N$ which introduces the N-dimensional space $\mathcal{Y}_N$ with