The components of the vector $\mathbf{u}^{(j)}$ are $u_k^{(j)} = \mathrm{Re}[u_k^{*(j)} e^{-i\omega t}]$. The solution (4.20) can be used in order to obtain the representation of Somigliana type.

In the case of elastostatics the fundamental solution was established in [9].

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ON CERTAIN ASPECTS FOR A SIMPLE RANDOM WALK WITH A REFLECTING AND A RANDOM ABSORBING BARRIER

BY

P.-C. G. VASSILIOU

In the present we study the probability distribution of the maximum distance upwards run by the barrier in a simple random walk with a reflecting and a random absorbing barrier. A theorem is provided where the above-probability distribution is expressed in terms of the probabilities of absorption and non-absorption of two other interesting random walk processes. The first one is the probability of the process being in a non-absorbing state at time n in a simple random walk between an absorbing and a reflecting barrier. The second is the probability of absorption at time n in a simple random walk process in the presence of a reflecting, a random absorbing and a non-random absorbing barrier.

1. Introduction. Certain aspects of a simple random walk with a reflecting and a random absorbing barrier are presented in the present paper. A stochastic process $\{X_n\}$, $n \geq 0$ is a simple random walk if the state space of the process is the integers and the process executes independent steps $+1, 0, -1$ with stationary (constant) probabilities. The random absorbing barrier is assumed to undergo a simple random walk $\{Y_n\}$, $n \geq 0$ which is independent of $\{X_n\}$. In Vassiliou and Karakostas (1978) for the first time the idea of the random absorbing barrier was introduced. This generalization introduces a certain element of realism into the random walk process. A simple application of the idea of the random absorbing barrier to an insurance problem could be found in Vassiliou (1980a). The absorption probabilities of the simple random walk process between an absorbing (non-random) and a reflecting barrier were given in a correct form by Blassi (1976). This was done for the special case when the steps of the process are $(-1, 1)$. Vassiliou (1980b) studied the probability distribution of the absorption probabilities in a simple random walk between a random absorbing barrier and a reflecting barrier. This was done in two ways: (i) difference equations were derived and an algorithm for their numerical solution was given and (ii) these absorption probabilities were expressed in terms of other known probabilities.

In the present, in section 2, we study the probability distribution of the maximum distance upwards run by the barrier at time n. The probability distributions of the maximums are interesting aspects of any stochastic process in probability theory (see Cox and Miller 1972, p. 39). A theorem is provided where the wanted probability distribution is expressed in terms of the probabilities of absorption and non-absorption of two other interesting random walk processes. Such relations have found a wide range of applications in probability theory. Here among other applications they could lead to analytic expressions for the required probability distribution. This is so because the other two probability distributions are given in the present in closed analytic form.

The first probability distribution is the probability of the process being in a non-absorbing state at time n in a simple random walk between an absorbing (non-random) and a reflecting barrier. This is studied in section 3, where an appropriate system of difference equations is established. With the use of probability generating functions and properties of matrices and determinants, we develop an interesting methodology for solving such systems of difference equations. The probability distribution studied in this section apart from the close connection with section 2 has an interest of its own a portion of which lies in the area of actuarial studies (see for example Furst (1957)).

The second probability distribution is the probability of absorption at time n in a simple random walk process in the presence of a reflecting barrier a random absorbing barrier and a non-random absorbing barrier. The random absorbing barrier performs a simple random walk $\{Y_n\}$, $n \geqq 0$ independent of $\{X_n\}$. Note that the non-random barrier works as an absorbing one for both the random barrier and the process. When any kind of absorption occurs then the process stops. In section 4 we study the probability of absorption of the random barrier by the non-random barrier at time n. This is done by prooving a theorem where the required probability distribution is expressed in terms of known probabilities of other random walk processes. The problem studied in section 4 has certainly an interest of its own, apart from the close connection with the problem studied in section 2. Moreover it is hoped that the results of the present paper, a part from serving the present purpose, are of general interest in the present kind of problems and in the methodology of random walk process.

2. Setting definitions and results. Suppose that $\{X_n\}$, $n \geqq 0$, is a random walk on the integers, with independent steps each of magnitude $+1$ with probability p_{1c}, or 0 with probability p_{0c} or -1 with probability p_{-1c} and let $X_0 = j$ and $p_{1c}+p_{0c}+p_{-1c}=1$. Suppose also that a random absorbing barrier A is initially at $a > j$ and that it undergoes a random walk $\{Y_n\}$, $n \geqq 0$ with steps 1 with probability p_{1a}, or 0 with probability p_{0a} or -1 with probability p_{-1c} and let $p_{1a}+p_{0a}+p_{-1a}=1$. We assume that the two simple random walks are independent and we define by $D_n = Y_n - X_n$, $n=1, 2, \ldots$ Suppose also that at state 0 there is a reflecting barrier and that when $X_n=0$ then $P(X_{n+1}=0)=p_{1c}$ and $P(X_{n+1}=1)=1-p_{1c}$, $n=1, 2, \ldots$

We define by $U_n^b = \max\limits_{0 \leqq r \leqq n} Y_r$. One of the problems interesting us in the present is the evaluation of the probability distribution $P(U_n^b < m)$. Let us define

$$q^{(n)}(r, r_n, j, m) = P\{D_1>0, D_2>0, \ldots, D_n=r_n>0, Y_1<m, Y_2<m, \ldots, Y_n = m \mid D_0=r, X_0=j\},$$

where $n, r, r_n = 1, 2, \ldots$

Since the required probability distribution is the probability for the barrier not reaching state m before n we have that

$$P\{U_n^b < m\} = \sum_{s=n+1}^{\infty} \sum_{r_s=1}^{m} q^{(s)}(r, r_s, j, m).$$

Consider the following probabilities

$$f_{**}^{(n)}(a, m) = P\{0<Y_1<m, 0<Y_2<m, \ldots, 0<Y_n=m \mid Y_0=a\} \tag{1}$$

$$p_*^{(n)}(j, k) = P\{0 \leqq X_1 < m-1,\ 0 \leqq X_2 < m-1, \ldots, 0 \leqq X_n = k < m-1 \mid X_0=j\}. \tag{2}$$

Probability (1) could be considered as the probability of absorption at m at time n in a simple random walk process $\{X_n\}$ in the presence of two non-random absorbing barriers at states 0 and m. This is a well known probability and could be found in closed analytic form in Cox and Miller (1972). Probability (2) could be considered as the probability of being in an intermediate state k in a simple random walk process between a non-random absorbing barrier at state $m-1$ and a reflecting barrier at state 0. This probability is unknown and we will provide a method of evaluating it in closed analytic form in the next section. Define the following quantity which as we shall see later is a probability

$$U(s, i) = q^{(s)}(r, 0, j, i)\, p_*^{(n-s)}(i, m-r_n)\, f_{**}^{(n-s)}(i, m) + q^{(s)}(r, -1, j, i)\, p_*^{(n-s)}(i, m-r_n)\, f_{**}^{(n-s)}(i-1, m)$$

and also let

$n_1 = [1/2(a+n-m)]$ where by $[a]$ we mean the integer part of a;
$n_2 = [1/2(n+m-r_n-j)]$; $n_3 = [n - r_n/2]$; $r_1 = \max(0, a-s)$;
$r_2 = \min\{m-1, s+j\}$; $r_3 = \max\{0, m+s-n\}$; $r_4 = \min\{m-1, n+m-s-r_n\}$.

Then the following theorem is true

Theorem 1. *Let* $n^* = r/2$ if r *even and* $n^* = (r+1)/2$ if r *odd then*

$$q^{(n)}(r, r_n, j, m) = p_*^{(n)}(j, m-r_n)\, f_{**}^{(n)}(a, m) - \sum_{s=n^*}^{n_1} \sum_{i=r_1}^{r_2} U(s, i) - \sum_{s=n_1+1}^{n_2} \sum_{i=r_1}^{r_2} U(s, i) - \sum_{s=n_2+1}^{n_3} \sum_{i=r_3}^{r_4} U(s, i). \tag{3}$$

Proof. Consider the probability

$$q_*^{(n)}(r, r_n, j, m) = P\{0<Y_1<m, 0<Y_2<m, \ldots, 0<Y_n=m,\ 0 \leqq X_1 < m,\ 0 \leqq X_2 < m, \ldots, 0 < X_n = m-r_n \mid X_0=j, Y_0=a\}. \tag{4}$$

Then, since the two random walks $\{Y_n\}$, $\{X_n\}$ are independent from (1), (2) and (4) we get that

$$q_*^{(n)}(r, r_n, j, m) = p_*^{(n)}(j, m-r_n)\, f_{**}^{(n)}(a, m).$$

Define the event

$$E(s, i) = P\{D_1>0, D_2>0, \ldots, D_s=0 \text{ or } -1,\ Y_1<m, \ldots, Y_s<m,\ X_1 \geqq 0, X_2 \geqq 0, \ldots, X_s = i,\ 0<Y_{s+1}<m,\ 0<Y_{s+2}<m, \ldots, 0<Y_n=m,\ 0 \leqq X_{s+1}<m,\ 0 \leqq X_{s+2}<m, \ldots, 0 \leqq X_n = m-r_n\}.$$

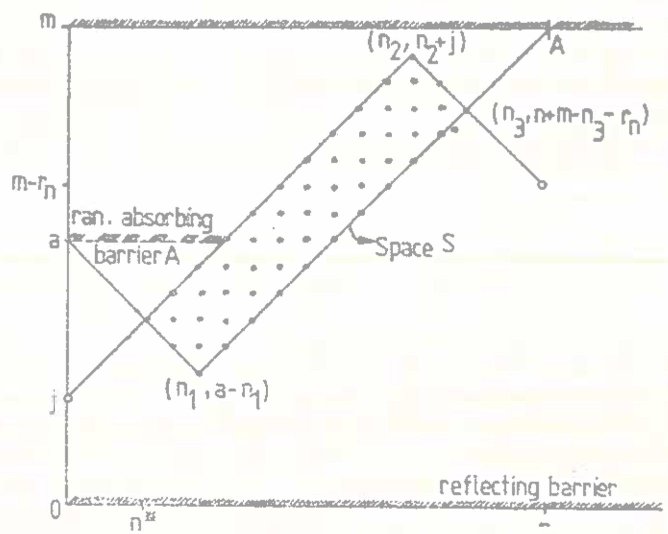
Then we could proove that

$$P\{E(s, i) \mid X_0=j,\ Y_0=a\} = q^{(s)}(r, 0, j, i)\, p_*^{(n-s)}(i, m-r_n)\, f_{**}^{(n-s)}(i, m) + q^{(s)}(r, -1, j, i)\, p_*^{(n-s)}(i, m-r_n)\, f_{**}^{(n-s)}(i-1, m) = U(s, i).$$

Comparing the probabilities $q^{(n)}(r, r_n, j, n)$ and $q_*^{(n)}(r, r_n, j, n)$ we get that

(5) $$q^{(n)}(r, r_n, j, n) = q_*^{(n)}(r, r_n, j, n) - P\{\bigcup_{s,i} E(s, i)\},$$

where with the symbol $\bigcup_{s,i}$ we mean the union of all the events $E(s, i)$ for all the possible values of the parameters s, i. Now consider the space S as the intersection of the parallelogram with vertices (n^*, n^*+j), $(n_1, a-n_1)$, (n_2, n_2+j), $(n_3, n+m-n_3-r_n)$ and the space $S_0=\{(x, y): x\geq 0, y<m\}$, and the mesh of unit size on S (see figure 1); then the coordinates (x, y) of the points of the mesh determine all the possible values of the parameters (s, i) in equation (5). Since all the events $E(s, i)$ for the various values of (s, i) are mutually exclusive events, we get (3).



From theorem 1 now it is obvious that in order to evaluate the probability $q^{(n)}(r, r_n, j, m)$ we should evaluate the probabilities $p_*^{(n)}(j, k)$ and $q^{(n)}(r, 0, j, i)$.

3. **Evaluation of the probability distribution** $p_*^{(n)}(j, k)$. In this section certain important aspects of the simple random walk between a reflecting barrier at point 0 and an absorbing barrier at point b are studied. Then if $k<b$ the probability $p_*^{(n)}(j, k)$ as defined in section 3 satisfies the following system of difference equations

$$p_*^{(n)}(j, k) = p p_*^{(n-1)}(j, k-1) + r p_*^{(n-1)}(j, k) + q p_*^{(n-1)}(j, k+1),$$

for $n=1, 2, \ldots, j=0, 1, 2, \ldots, b-1$ and $k=1, 2, \ldots, b-2$.
Also we have that

$$p_*^{(n)}(j, 0) = q p_*^{(n-1)}(j, 1) + (1-q) p_*^{(n-1)}(j, 0),$$
$$p_*^{(n)}(j, b-1) = p p_*^{(n-1)}(j, b-2) + r p_*^{(n-1)}(j, b-1),$$

for $n=1, 2, \ldots, j=0, 1, 2, \ldots, b-1$ with initial and boundary conditions $p^{(0)}(b, b)=1$, $p^{(n)}(b, k)=0$ for $n=1, 2, \ldots$ and $k=0, 1, \ldots, b-1$;

$$p^{(n)}(j, k)=0 \text{ for } n<|j-k|.$$

Let $R(k)$ be the $1\times b$ row vector $[0, 0, \ldots, p, r, q, 0, \ldots, 0]$ where r is at the $k+1$ column, and let $P(n-1)$ be the following column vector

$$P(n-1) = [p_*^{(n-1)}(j, 0), p_*^{(n-1)}(j, 1), \ldots, p_*^{(n-1)}(j, b-1)].$$

Then we have that $p_*^{(n)}(j, k) = R(k)P(n-1)$.
Define the $b\times b$ matrix

$$Q = \begin{bmatrix} 1-q & q & 0 & 0 & \cdot & \cdot & \cdot & 0 \\ p & r & q & 0 & \cdot & \cdot & \cdot & 0 \\ 0 & p & r & q & \cdot & \cdot & \cdot & 0 \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ 0 & 0 & \cdot & \cdot & & & p & r \end{bmatrix}$$

We get

$$p_*^{(n)}(j, k) = R(k)QP(n-2) = R(k)Q^{(n-2)}p(1).$$

If we define by $g(s, k)$ the generating function of the probabilities $p_*^{(n)}(j, k)$, then we get that $g(s, k) = s^2 R(k)(I-sQ)^{-1}P(1)$, where we assume that $\max\{|sp|, |sq|, |sr|\}\leq 1$. Taking into account that $P(1) = [0, 0, \ldots, 0, q, r, p, 0, \ldots, 0]$ where r is at the $j+1$ column we get

$$g(s, k) = \frac{s^2 q}{|I-sQ|}\{qD_1 + rD_{j+1} + pD_{j+2}\},$$

where D_j is the determinant obtained from the determinant $|I-sQ|$ if we replace the j-th row by $R(k)$. Consider the determinant B_n of the form similar to $|I-sQ|$ except that the (1, 1) element is $1-sr$. Then for this determinant the following recurrence relation holds

$$B_n = (1-sr)B_{n-1} - s^2 pq B_{n-2}$$

thus

$$\begin{bmatrix} B_n \\ B_{n-1} \end{bmatrix} = \begin{vmatrix} 1-sr & -s^2pq \\ 1 & 0 \end{vmatrix} \begin{bmatrix} B_{n-1} \\ B_{n-2} \end{bmatrix} = S \begin{bmatrix} B_{n-1} \\ B_{n-2} \end{bmatrix} = S^{n-1} \begin{bmatrix} B_1 \\ B_0 \end{bmatrix},$$

where $B_1 = 1-sr$ and $B_0=1$ for convenience.
The characteristic roots of the matrix S are found to be

$$\lambda_1 = 1/2[(1-sr) + \{(1-sr)^2 - 4s^2pq\}^{1/2}],\ \lambda_2 = 1/2[(1-sr) - \{(1-sr)^2 - 4s^2pq\}^{1/2}]$$

and if we define by

$$C = \begin{bmatrix} \lambda_1 & \lambda_2 \\ 1 & 1 \end{bmatrix} \text{ and } C^{-1} = (\lambda_1-\lambda_2)^{-1} \begin{bmatrix} 1 & -\lambda_2 \\ 1 & \lambda_1 \end{bmatrix}$$

we get

$$\begin{bmatrix} B_n \\ B_{n-1} \end{bmatrix} = C \begin{bmatrix} \lambda_1^{n-1} & 0 \\ 0 & \lambda_2^{n-2} \end{bmatrix} C^{-1} \begin{bmatrix} B_1 \\ B_0 \end{bmatrix},$$

and thus $B_n=(\lambda_1^{n+1}-\lambda_2^{n+1})(\lambda_1-\lambda_2)^{-1}$. Hence finally we get

$$|I-sQ|=B_b-spB_{b-1}.$$

Now we have to evaluate the determinants D_j, D_{j+1} and D_{j+2}. In the determinant D_j we multiply the $k+1$ row by θ^{-1} and we add it to the j-th row reducing it to $[0, 0, \ldots, \theta^{-1}, 0, \ldots, 0]$ where θ^{-1} is at the $k+1$ position. Then we have

If $k+1>j$

$$D_j=s^{-1}(-1)^k(-sp)^{k-j-1}\ \{B_{j-1}-spB_{j-2}\}B_{b-(k+1)}.$$

If $k+1<j$

$$D_j=s^{-1}(-1)^k(-sp)^{j-k-1}\{B_{k-1}-spB_{k-2}\}B_{b-j}.$$

If $k+1=j$

$$D_j=s^{-1}(-1)^k\{B_{j-1}-spB_{j-2}\}B_{b-(k+1)}.$$

Thus with no loss of generality we assume that $k+1>j+2$ and we get

$$g(s,k)=\frac{U(s)}{V(s)},$$

where

$$V(s)=B_b-spB_{b-1}=\{(\lambda_1^{b+1}-\lambda_2^{b+1})-sp(\lambda_1^b-\lambda_2^b)\}(\lambda_1-\lambda_2)^{-1}, \text{ and}$$

$$U(s)=sq(-1)^k(-sq)^{k-j-1}(\lambda_1^{b-k+2}-\lambda_2^{b-k+2})(\lambda_1-\lambda_2)^{-2}\{p(\lambda_1^{j+2}-\lambda_2^{j+2})+$$

$$s(\lambda_1^{j+1}-\lambda_2^{j+1})(rp-p^2)+s^2(\lambda_1^j-\lambda_2^j)(qp^2-p^2)-q(sp)^3(\lambda_1^{j-1}-\lambda_2^{j-1})\}.$$

Now let us define by

$$\cos^{-1}a=(1-sk)^{-1}2s\sqrt{pq}$$

then

$$\lambda_{1,2}=(1-sk)(2\cos a)^{-1}e^{\pm ia}=(1-sk)W_{\pm},\ s=(1-sk)(2\cos a\sqrt{pq})^{-1}=(1-sk)Y.$$

Then we get for the probability generating function

$$g(s,k)=\frac{U'(a)}{V'(a)},$$

where $V'(a)=\{W_+^{b+1}-W^{b+1}-Y^{-1}p(W_+^b-W_-^b)\}(W_+-W_-)$ and

$$U'(a)=(-1)^k(-p)^{k-j-1}(1-sk)^2Y^{(j-k)}(W_+^{b-k+2}-W_-^{b-k+2})\}p(W_+^{j+2}-W_-^{j+2})$$

$$+Y(W_+^{j+1}-W_-^{j+1})(rp-p^2)+Y^{-2}(W_+^j-W_-^j)(qp^2-p^3)-qp^3Y^{-3}(W_+^{j-1}-W_-^{j-1})\}.$$

If $(p/q)^{1/2}\leq 1+1/b$ then the denominator has $b+1$ distinct roots a_v $(v=1, 2, \ldots, b+1)$. If $(p/q)^{1/2}>1+1/b$ there are only b distinct roots a_v $(v=2, \ldots, b+1)$ that give distinct roots s_v of $V(s)$. The remaining root of $V(s)$ is given by $s_1=k+2(pq)^{1/2}\cosh a_1$ where a_1 is the unique root of the equation $(p/q)^{1/2}=$

$=(\sinh(b+1)a/\sinh ba)$. Thus the probability generating function is written as

$$g(s,k)=\sum_{v=1}^{b+1}\frac{p_v}{(s_v-s)},$$

where

$$p_v=-\frac{U(s_v)}{[\partial V(s)/\partial s]s=s_v}$$

for which explicit expressions could be easily obtained. Then the required probabilities are given by $p_*^{(n)}(j,k)=\sum_{v=1}^{b}\frac{p_v}{s_v^{n-1}}$.

4. **Evaluation of the probability** $q^{(n)}(r,0,j,i)$. In this section we will evaluate the probability distribution $q^{(n)}(r,0,j,k)$ defined previously to be $q^{(n)}(r,0,j,i)=P(D_1>0, D_2>0, \ldots, D_n=0,\ Y_1<m, Y_2<m, \ldots, Y_n=i<m \mid D_0=r, X_0=j)$, $n, r=1, 2, \ldots$

Define the probability distribution

$$p_{**}^{(n)}(j,k)=P(0<Y_1<m, 0<Y_2<m, \ldots, 0<Y_n=k<m \mid Y_0=a).$$

This is the probability of being at state k at time n is a simple random walk process $\{Y_n\}$ in the presence of two non-random absorbing barriers at point 0 and k, which is known. Define also by:

$$p^{(n)}(r,0,j,i)=P(0<Y_1<m, 0<Y_2<m, \ldots, 0<Y_n=i<m,$$

$$X_1<m, X_2<m, \ldots, X_n=i \mid X_0=j, Y_0=a),$$

then, since the two simple random walks $\{Y_n\}$ and $\{X_n\}$ are independent we get

$$p^{(n)}(r,0,j,i)=p_*^{(n)}(j,i)\ p_{**}^{(n)}(a,i).$$

Define by

$$V(c,i)=P\{D_1>0, D_2>0, \ldots, D_s=0\ \tilde{\eta}\ -1,\ Y_1<m,\ Y_2<m, \ldots, Y_s<m,$$

$$X_1\geqq 0,\ X_2\geqq 0, \ldots,$$

$$X_s=c,\ 0<Y_{s+1}<m, \ldots, 0<Y_n<i<m,\ 0\leqq X_{s+1}<m,\ 0\leqq X_{s+2}<m, \ldots,$$

$$0\leqq X_n=1 \mid X_0=j, Y_0=a\}.$$

Then we could proove that

$$V(c,i)=q^{(s)}(r,0,j,c)p_*^{(n-s)}(c,i)+p_{**}^{(n-s)}(c,i)+q^{(s)}(r,-1,j,c)$$

$$p_*^{(n-s)}(c,i)p_{**}^{(n-s)}(c-1,i).$$

Comparing the probabilities $q^{(n)}(r,0,j,i)$ and $p^{(n)}(r,0,j,i)$ we get that

$$q^{(n)}(r,0,j,i)=p^{(n)}(r,0,j,i)-\sum_{c,s}V(c,i),$$

w here all the values (c,i) in the summation are found in an analogous way as that in theorem 1. Thus if we define by

$$n_4=[1/2(a+n-1)],\ n_5=[1/2(n+i-j)],\ c_1=\max\{a-s, 0\},$$
$$c_2=\min\{s+j, m-1\}\ c_3=\max\{0, i+s-n\},\ c_4=\min\{m-1, n+i-s\}.$$

We have that

Theorem 2. *Let* $n^*=r/2$, if r *even and* $n^*=(r+2)/2$ if r *odd, then*

a) *If* $2i>a+j$

$$q^{(n)}(r, 0, j, i)=p_*^{(n)}(j, i)p_{**}^{(n)}(a, i)-\sum_{s=n^*}^{n_4}\sum_{c=c_1}^{c_2}V(s, c)-\sum_{s=n_4+1}^{n_5}\sum_{c=c_3}^{c_2}V(s, c)-$$
$$-\sum_{s=n_5+1}^{n-1}\sum_{c=c_3}^{c_4}V(s, c);$$

b) *If* $2i<a+j$

$$q^{(n)}(r, 0, j, i)=p_*^{(n)}(j, i)p_{**}^{(n)}(a, i)-\sum_{s=n^*}^{n_5}\sum_{c=c_1}^{c_2}V(s, c)-\sum_{s=n_5+1}^{n_4}\sum_{c=c_1}^{c_4}V(s, c)-$$
$$-\sum_{s=n_4+1}^{n-1}\sum_{c=c_3}^{c_4}V(s, c).$$

Letting successively $n=n^*, n^*+1,\ldots$ we get from theorem 2 the probabilities $q^{(n)}(r, 0, j, i)$. Note that a similar relation could be obtained for the probability $q^{(n)}(r, -1, j, i)$ and thus all the necessary probabilities have been evaluated.

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THE (m, p)-BARGAINING SET FOR THE THREE PERSON GAMES

BY

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This paper is concerned with a new concept of „stability" of the coalitional structure of a three-person game. Starting from the ideas of a paper written by Aumann and Maschler [1] and considering a game $G=G(n, v)$, we introduce the parameters m_i and $p_i; i=1, 2, 3$, in order to define an m-objection of the player i against the player j and a p-counter-objection of the player j against the player i (see Definitions 1.1 and 1.3). Our parameters correspond to the discrete case, because a player i could change the partners (especially j) provided that in the new coalition he will get with m_i more than in the old coalition (such that his request m_i is satisfied), while the payer j could have a p-counter objection against the player i.

Using the notions of m-objections and p-counter-objections, we define the notion of (m, p)-stability (see Definition 1.6). The set of all (m, p)-stable outcomes, called the (m, p)-bargaining set, is defined in Section 2. We give some criteria that a coalitionally rational payoff configuration be (m, p)-stable in a three-person game.

We do not consider here the case of a n-person game, but the extension for this case is clear, although difficult in writing.

1. The (m, p)-stability. Let $G=(N, v)$ be a cooperative n-person game, where $N=\{1, 2, \ldots, n\}$ is the set of players and $v: \mathcal{P}(N)\setminus\{\varnothing\}\to R$ is the characteristic function of the game.

First we recall some definitions from [1].

We denote the set of all permissible coalitions B of the game by $\mathcal{B}$. A coalition structure is a partition of N. A coalitionally rational payoff configuration (c.r.p.c.) is a pair $(x; C)$, where C is a coalition structure and $x=(x_1, x_2, \ldots, x_n)$, called payoff vector, is assumed to satisfy:

(1.1) $$x_i\geq 0, \text{ for all } i, i\in N,$$

(1.2) $$\sum_{i\in S}x_i=v(S),\quad S\in C.$$

In the following we consider a three-person game, given by: