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FIELDS OF MEROMORPHIC FUNCTIONS

BY

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The aim of this note is to give a rough sketch of how the Poincaré-Matsuda classification of differential function fields free of movable singularities [2] may be extended to higher dimensions. Proofs will be given elsewhere. Here we shall only make a brief discussion of our method which is entirely different from Matsuda's.

Start with finitely generated field extensions $\mathbb{C} \subset K \subset L$ where $\mathbb{C}$ is the complex field and K, L are fields of meromorphic functions in some domain in $\mathbb{C}^r$ such that $(\partial/\partial w_i)(K) \subset K$ and $(\partial/\partial w_i)(L) \subset L$ for $1 \leqslant i \leqslant r (w_1, \ldots, w_r$ being coordinates on $\mathbb{C}^r)$. One says [2] that L/K is free of movable singularities if there exists a smooth projective K-scheme V with function field L all of whose local rings are globally invariated by $\partial/\partial w_1, \ldots, \partial/\partial w_r$; this concept comes from differential equations and is quite classical going back to Fuchs, Poincaré and Painlevé.

One is interested in classifying all differential function fields L/K free of movable singularities. From function-theoretic point of view nothing changes if we allow the replacing of L/K by an extension L^*/K^* with L^*/L and K^*/K finite extensions. The main effect of Matsuda's classification [2] may be summarized as follows:

Theorem A. *If* tr. deg.$_K$ $L=1$ *and* L/K *is free of movable singularities, then there exist* L^* *and* K^* *as above such that one of the following situations occurs:*

1) L^*/K^* *is purely transcendental;*

2) L^* *is generated over* K^* *by functions of the form* $P(G)$ *where* P *are elliptic functions and* G *are primitive over* K^*.

Here are our main results:

Theorem B. *If* tr. deg.$_K$ $L=2$ *and* L/K *is free of movable singularities, then there exist* L^* *and* K^* *as above such that one of the following situations occurs:*

1) L^*/K^* *is purely transcendental;*

2) *There exists an intermediate differential field* $K^* \subset R^* \subset L^*$ *such that* L^*/R^* *is as in Theorem A, 1) and* R^*/K^* *is as in Theorem A, 2);*

3) L^* *is generated over* K^* *by functions of the form* $P(G_1, G_2)$ *with* P *abelian and* G_1, G_2 *primitive.*

Theorem C. *If* tr. deg.$_K$ $L=3$ *and* L/K *is free of movable singularities and has Kodaira dimension zero, then there exist* L^* *and* K^* *as above such that* L^* *is generated over* K^* *by functions of the form* $P(G_1, G_2, G_3)$ *with* P *abelian and* G_1, G_2, G_3 *primitive.*

In fact we conjecture that the Kodaira dimension of a differential function field free of movable singularities is always $\leqslant 0$.

A few words about the method. First one constructs a smooth projective morphism $f: X \to S$ whose generic fibre is V/K. Two situations may occur. The first one (and this is the happy one) is when a coarse moduli space M exists for the fibres of f. In this case we use Kodaira-Spencer deformation theory to prove that the classifying map $S \to M$ has a point as its image. The second situation is when V is either ruled or it has irregularity $q \geqslant 1$. A geometric analysis of the Albanese map will lead us again to the answer.

REFERENCES

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2. Matsuda M. — *First order algebraic differential equations*. Springer, 1980.

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