The following examples of ternary algebras were considered :

a) F being a symmetric bilinear form over the field K, the ternary operation is defined by : $(x, y, z)_\tau = F(x, z)y - F(y, z)x$ and it satisfies the conditions : $(x, y, z)_\tau + (y, z, x)_\tau + (z, x, y)_\tau = 0$ and $(x, x, y)_\tau = 0$ and $(u, v, (x, y, z)_\tau)_\tau = ((u, v, x)_\tau, y, z)_\tau + (x, (u, v, y)_\tau, z)_\tau + (x, y, (u, v, z)_\tau)_\tau$, for any x, y, z, u, v, hence it is a ternary algebra from the family proposed by Yoshiaki Taniguchi [21].

b) B_1, B_2, B_3 being bilinear forms in V, the ternary operation is given by $(x, y, z)_\tau = B_1(y, z)x + B_2(z, x)y + B_3(x, y)z$ and the ternary algebra obtained in this way has interesting properties.

c) In the family of algebras given above, we remark the case $B_1 = -B_2 = B_3 = B$, $B(x, y) = L(x)L(y)$, where L is a linear form over K, when the ternary algebra has some properties of a ternary field (see [13], [18]).

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PROPERTIES OF A NON-ASSOCIATIVE TERNARY STRUCTURE

BY

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Recently, the algebraic structures with n-ary operations, $n \geq 3$, appear more often in specialized papers. Remarkable results have been obtained in the field known under the name of the n-group and n-semigroup theory. All the structures investigated satisfy in one or another form the n-ary associativity.

In [5], a ternary structure has been introduced which is useful for the coordinatization of some projective planes. We shall give here some properties of this algebraic structure.

Definition 1. *The non-empty set M on which the ternary operation (x, y, z) is defined is called the coordinatizing ternary ring, if in M there are two different elements denoted 0 and 1 and the following axioms are fulfilled :*

1. $\forall x, y, z \in M$, $(0, y, z) = z$, $(x, 0, z) = z$, $(x, 1, 0) = x$;
2. $\forall x, y, z \in M$, $((x, 1, y), 1, z) = (x, 1, (y, 1, z))$;
3. $\forall x, y \in M$, $(x, 1, y) = (y, 1, x) = (1, x, y)$;
4. $\forall x, y, z_1, z_2 \in M$, $(x, y, z_1) = (x, y, z_2) \Rightarrow z_1 = z_2$

and analogously for the other places ;

5. $\forall a \in M$, $\exists x \in M$ *so that* $(a, 1, x) = 0$,
6. $\forall x, y, z \in M$, $(x, (y, z, 0), 0) = ((x, y, 0), z, 0)$,
7. $\forall x, y, z \in M$, $((x, 1, y), z, 0) = (x, z, 0), 1, (y, z, 0))$ *and*

$$(x, (y, 1, z), 0) = ((x, y, 0), 1, (x, z, 0)).$$

Remarks. 1 The solution of the equation in axiom 5 is unique. We denote this solution by $-a$ and we call this element the *opposite* of a. 2) If for any $x, y \in M$, $(x, y, 0) = (y, x, 0)$, the coordinatizing ternary ring is called *commutative*. 3) If the equation $(a, x, 0) = 1$ has a solution in M we shall say that a is *invertible* and in this case $a \neq 0$.

Definition 2. *If for any $a \in M$, $a \neq 0$, is invertible, then we say that the coordinatizing ring is a field.*

Proposition 1. *If M is a coordinatizing ternary ring, then M is a binary ring with unity with respect to the operations :* $x+y=(x, 1, y)$, $x\circ y=(x, y, 0)$, *by means of which the ternary composition is expressed by :* $(x, y, z)=x\circ y+z$.

Remarks. 1) The ternary ring just introduced is not associative in a ternary sense and thus it is not a 3-group. 2) Let M be a ternary commutative ring where any $a\neq 0$ is invertible. From $(a, x, 0)=(x, a, 0)$ and from the fact that $(a, x, 0)=1$ has a unique solution, it results that also the equation $(x, a, 0)=1$ has the same solution. The ternary field is then a particular case of the general ternary structure introduced by R. Sandler [4] and used in the coordinatization of a projective plane. This is the reason of the name of such a structure.

Proposition 2. *Any commutative ternary field is a pseudo-planary ring in Sandler's sense.*

Some properties of these rings :

I. The associativity can be replaced by introducing a ternary sum : $\overset{n}{\underset{i=1}{ST}} a_i=(\dots((a_1, 1, a_2), 1, a_3), \dots, 1, a_n)$ and a ternary product : $\overset{n}{\underset{i=1}{PT}} a_i= (\dots, ((a_1, a_2, 0), a_3, 0), \dots, a_n, 0)$. By complete induction we can prove that for $1\leqslant m\leqslant n-1$, $\overset{n}{\underset{i=1}{ST}} a_i=((\overset{m}{\underset{i=1}{ST}} a_i), 1, (\overset{n}{\underset{i=m+1}{ST}} a_i))$ and $\overset{n}{\underset{i=1}{PT}} a_i=((\overset{m}{\underset{i=1}{PT}} a_i), (\overset{n}{\underset{i=m+1}{PT}} a_i), 0)$, for any $a_i\in M$, $i=\overline{1, n}$. We can introduce also the concepts of multiple and power for an element $a\in M$. We obtain :

$$na=\overset{n}{\underset{i=1}{ST}} a=(\dots((a, 1, a), 1, a)\dots, 1, a),$$

$$an=\overset{n}{\underset{i=1}{PT}} a=(\dots((a, a, 0), a, 0)\dots, a, 0).$$

The calculus with multiples and powers is similar to the usual one.

II. For the ternary rings we can define : subring, ideal, factor ring.

Definition 3. *The subset $I\neq\emptyset$, $I\subseteq M$ is called a ternary subring in M, if I is a ternary ring with respect to the induced operation.*

Proposition 3. *I is a subring iff $1\in I$ and for any $a, b\in I$* $(a, 1, -b)\in I$, $(a, b, 0)\in I$.

Because the intersection of ternary subrings is a ternary subring too, we can introduce the notion of subring generated by a set.

Given the subring $I\subseteq M$, let us define the relation : $\forall a, b\in M$, $aRb \Leftrightarrow (a, 1, -b)\in I$.

We show that R is an equivalence on M and thus it determines a decomposition of the initial ring in classes.

The set of classes will be denoted by M/I and we shall call it the *factor set* determined by the subring I.

If for $a\in M$ we denote by C_a the class of equivalence of a, then $C_a= =(I, 1, a)$.

The problem of ternary structuralization of the set M/I is not possible unless : 1. $\forall a, b\in M$, $\forall i\in I$, $(a, i, b)\in I$, 2. $\forall a\in M$, $\forall i\in I$, $(i, b, 0)\in I$. These conditions can be written as follows

$$(a, I, b)\subseteq I,\ \forall a, b\in M,\ (I, b, 0)\subseteq I,\ \forall b\in M.$$

Such a subring will be called *ternary ideal* and the corresponding factor ternary ring of M with respect to the ideal I.

III. In the ternary ring we can also put the problem of divisibility.

Definition 4. *We say that a divides b and we shall write a/b, if $c\in M$ exists so that $(a, c, 0)=b$.* This relation has the following properties :

1. $$a/a,\ \forall a\in M.$$

Indeed from the axiom 1 we have $(a, 1, 0)=a$, thus $c=1$.

2. $$\text{If } a/b \text{ and } b/c\Rightarrow a/c.$$

From the definition it follows that there exist $u, v\in M$ so that $(a, u, 0)= =b$, $(b, v, 0)=c$. Replacing b in the second relation and having in view the axiom 6 we obtain : $((a, u, 0), v, 0)=c$, $(a, (u, 0, v), 0)=c$. But $(u, 0, v)\in \in M$ thus $(u, 0, v)=w$ and $(a, w, 0)=c$ that is a/c.

3. If a/b and b/a there exists $u, v\in M$ so that $(u, 0, v)=1$ and $(v, 0, u)= =1$. Indeed there exists $u, v\in M$ so that : $b=(a, u, 0)$ and $a=(b, v, 0)$. Substituting successively a and b, we obtain $((b, v, 0), u, 0)=b$ and $((a, u, 0), v, 0)=a$.

Using the axioms 6 and 1, we have : $(b, (v, 0, u), 0)=b$ and $(a, (u, 0, v), 0)=a$ and thus $(u, 0, v)=1$ and $(v, 0, u)=1$.

This property suggests the introduction of the association relation in divisibility $a\sim b$ if a/b and b/a.

4. If a/b and $c\in M$, then $a/(b, c, a)$,

5. If a/b and a/c, then $a/(b, 1, c)$,

If M is a commutative ternary ring then if a/b then $(a, c, 0)/(b, c, 0)$ and if $c\neq 0$ $(a, c, 0)/(b, c, 0)$, then a/b.

If we call *zero divisors* the elements $a\neq 0$, $b\neq 0$ for which $(a, b, 0)=0$, then we remark that the ternary ring offers the possibility of solving the problems of divisibility in a similar way as for the binary integral domains.

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