Corollary 2.5. *If $(R_i)_{i\in I}$ is an arbitrary collection of semiprime rings, then the ring $\prod_{i\in I}(R_iC_i)$ is centrally closed, where C_i is the extended centroid of R_i.*

Corollary 2.6. *Let $(R_i)_{i=\overline{1,n}}$ be a finite set of semiprime rings, $R=\bigoplus_{i=1}^{n}R_i$, C the extended centroid of R and C_i the extended centroid of R_i, $i=\overline{1,n}$, then : i). $\bigoplus_{i=1}^{n}C_i\simeq C$; ii). If R_i is centrally closed for each $i=\overline{1,n}$, then R is centrally closed ; iii). $\bigoplus_{i=1}^{n}R_iC_i$ is centrally closed.*

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EXTENSIONS OF LEFT NEAR-RINGS

BY

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In the theory of groups, the problem of constructing a group E such that it has a normal subgroup A' isomorphic to a given group A and its factor-goup E/A' is isomorphic to another given group B has known a nice development, since, in 1926, Schreier [12] had written his paper. Schreier's extensions of groups inspired the same constructions for rings (Hall [5], Everett [3], Szendrei [14], Rédei [11]), for universal algebras (Zitomirskii [17]) and monoids (Strecker [13], van Buggenhout [16], Coudron [2], Hancock [6], Inasaridze [7]). If A is an abelian group, then the extensions of A by B can be described in terms of the cohomology theory and this solution, although not exhaustive, is more illuminating. Lausch [8, 9] used the cohomology theory in order to build the extensions of a ring by a distributively generated near-ring.

In this paper, we include a brief outline of the general extension problem for zero-symmetric left near-rings by using Schreier's idea. Recall some definitions and notations in the near-ring theory. For other informations, see Pilz [10].

Definition 1. *A* **left near-ring** *is a triple $(N, +,\cdot)$, where N is a nonvoid set, „+" and „·" are binary operations such that : 1. $(N, +)$ is a group (not necessarily abelian); 2. $(N, \cdot)$ is a semigroup; 3. for all $x, y, z \in N$, $x\cdot(y+z)=$ $=x\cdot y+x\cdot z$.*

We consider only the left near-rings for which $0\cdot x=0$, for all $x\in N$, i.e. the *zero-symmetric left near-rings*. Such a near-ring is, for example, the set of affine transformations of a linear space, with respect to the pointwise addition and the mappings composition.

Definition 2. *A nonvoid subset I of a left near-ring N is called an ideal of N if: $(I,+)$ is a normal subgroup of $(N, +)$, $N.I\subseteq I$ and for all $x,y\in N$ and $i\in I$, $(i+x)\cdot y-x\cdot y\in I$.*

We note that each ideal I is also a subnear-ring.

Definition 3. *Let N and N' be two left near-rings. A mapping $\varphi : N\to N'$ is a near-ring morphism, if $\varphi(x+y)=\varphi(x)+\varphi(y)$, $\varphi(x\,.\,y)=\varphi(x)\,.\,\varphi(y)$, for all $x, y\in N$.*

The kernel of φ, Ker $\varphi=\{x\in N/\varphi(x)=0'\in N'\}$ is an ideal of N. Given the ideal I of N, we may talk about the factor near-ring N/I. The notion of exact sequence is similar with the same notion for groups.

Definition 4. *Let N, A, E be three left near-rings. We call E the extension of A by N, if there exists the exact sequence of near-rings and morphisms of near-rings :*

$$0 \to A \xrightarrow{\Phi} E \xrightarrow{\Psi} N \to 0,$$

i.e. *A is isomorphic to an ideal A' of E and N is isomorphic to E/A'.*

Definition 5. *Two extensions E and E' of the near-ring A by the near-ring N are called equivalent, if there exists a near-ring isomorphism $\Lambda : E \to E'$, such that the diagrams :*

$$\begin{array}{ccccccccc} 0 & \to & A & \xrightarrow{\Phi} & E & \xrightarrow{\Psi} & N & \to & 0 \\ & & \downarrow{\scriptstyle 1_A} & & \downarrow{\scriptstyle \Lambda} & & \downarrow{\scriptstyle 1_N} & & \\ 0 & \to & A & \xrightarrow{\Phi'} & E' & \xrightarrow{\Psi'} & N & \to & 0 \end{array}$$

are commutative.

Now we give the construction of the extension.

If E is an extension of A by N, we identify A to $\Phi(A)$, then we choose a transversal of the set of classes of E/A, such that this set corresponds to the isomorphism $N \simeq E/A$, i.e. there exist the mappings $m : N \times N \to A$, $t : N \times N \to A$, for which $x_\alpha + x_\beta = x_{\alpha+\beta} + m(\alpha, \beta)$, $x_\alpha . x_\beta = x_{\alpha.\beta} + t(\alpha, \beta)$, for any $\alpha, \beta \in N$.

Obviously, $x_\omega = m(\omega, \omega)$ and $t(\omega, \omega) = - m(\omega, \omega) + (m(\omega, \omega))^2$, for the neutral element ω with respect to the addition of N.

Denote $a^\alpha = -x_\alpha + a + x_\alpha$, $a^b = -b + a + b$, for $\alpha \in N$, $a, b \in A$, $x_\alpha \in S$, $S = \{x_\alpha \in E / \alpha \in N\}$. Hence we have a mapping $\theta : N \to \mathrm{Aut}\,(A, +)$, defined by $\theta(\alpha) = \theta_\alpha$, where $\theta_\alpha(a) = a^\alpha$, for any $\alpha \in N$ and $a \in A$. The normality of $(A, +)$ in $(E, +)$ implies :

(I) $(a^\alpha)^\beta = (a^{\alpha+\beta})^{m(\alpha, \beta)}$, for any $\alpha, \beta \in N$, $a \in A$.

The associativity and the left distributivity laws imply the existence of the mappings $s : N \times N \times A \to A$, $w : N \times A \times A \to A$, which satisfy the identities :

(II) $m(\alpha, \beta+\gamma) + m(\beta, \gamma) = m(\alpha+\beta, \gamma) + (m(\alpha, \beta))^\gamma$,

(III) $a \cdot b = w(\alpha, a, b) - m(\omega, \omega) + a$,

(IV) $s(\alpha\beta, \gamma, s(\alpha, \beta, a) + w(\alpha, b, a)) + w(\alpha, \beta, c, s(\alpha, \beta, a) + w(\alpha, b, a)) =$
$= s(\alpha, \beta\gamma, a) + w(\alpha, s(\beta, \gamma, b), a) + w(\alpha, w(\beta, c, b), a)$,

(V) $s(\alpha, \beta+\gamma, a) + w(\alpha, m(\beta, \gamma) + b^\gamma + c, a) = m(\alpha\beta, \alpha\gamma) + (s(\alpha, \beta, a))^{\alpha\gamma} +$
$+ (w(\alpha, \beta, a))^{\alpha\gamma} + s(\alpha, \gamma, a) + w(\alpha, c, a)$,

for all $\alpha, \beta, \gamma \in N$, $a, b, c \in A$. We have also $t(\alpha, \beta) = s(\alpha, \beta, 0)$, for all $\alpha, \beta \in N$.

Conversely, given two left near-rings A and N and the mappings m, θ, s, w which satisfy the conditions (I) — (V), then the set $E = N \times A$, endowed with the binary operations :

$$(\alpha, a) + (\beta, b) = (\alpha + \beta,\ m(\alpha, \beta) + a^\beta + b),$$

$$(\alpha, a) \, . \, (\beta, b) = (\alpha\beta,\ s(\alpha, \beta, a) + w(\alpha, b, a)),$$

is an extension of A by N.

This is the general method to obtain such extensions. We find also the conditions for the equivalence of two extensions of A by N.

In order to obtain the split extension of A by N, we have to take $m(\alpha, \beta) = 0$, for any $\alpha, \beta \in N$. The relations (I) — (V) become simpler if we take $m(\alpha, \omega) = m(\omega, \alpha) = 0$, for any $\alpha \in N$ and similar conditions for s and w (*normalised sets of factors*).

The case when N and E are distributive near-rings and A is a zero near-ring is also interesting. Here we replace s and w by two simpler mappings $u : N \times A \to A$, $v : A \times N \to A$, such that $s(\alpha, \beta, a) = t(\alpha, \beta) + v(a, \beta)$, $w(\alpha, b.a) = u(\alpha, b)$, for any $\alpha, \beta \in N$, $a, b \in A$.

In spite of the analogy with the case of rings (see Everett [3]), the relations and the calculations remain more complicated.

In the case when A is a ring and N is an A-module, the extension is the abstract affine near-ring of Gonshor [4] and in the case when A is a left near-ring and N is a left A-group, in some additional conditions, the extension is the infra-near-ring of affine type (Ştefănescu [15]).

The calculations can be done without great dificulties, although they ask an elaborated machinery.

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