local R-morphism and $\hat{f}: \hat{A} \to \hat{B}/b\hat{B}$ *the map induced by* f. *Then* f *is surjective whenever* $\hat{f}$ *is surjective.*

Proof. Let f be given by $Y \to h = (h_1, \ldots, h_N) \in (\underline{m}, Z)B^N$. Let $\hat{f}$ be surjective. Then there exists some formal power series $\hat{t} = (\hat{t}_1, \ldots, \hat{t}_r)$ from $\hat{A}$ such that $\hat{t}_i(h) \equiv Z_i \bmod b\hat{B}$; $i = 1, \ldots, r$. By Taylor's formula there exist some formal power series $\hat{s}_{ij}$ from $\hat{R}[[Y, Z]]$ such that $-\hat{t}_i + \hat{t}_i(h) = \sum_{j=1}^{N} \hat{s}_{ij}(Y_j - h_j)$, $i = 1, \ldots, r$. Thus $\hat{f}$ is surjective iff the system of polynomial equations

$$(**) \qquad T_i + \sum_{j=1}^{N} S_{ij}(Y_j - h_j) = Z_i + \sum_{i=1}^{k} b_i U_k, \quad i = 1, \ldots, r,$$

has in $\hat{R}[[Y, Z]]$ the formal solution $(\hat{t}, \hat{s}, \hat{u})$ where $\hat{t}_i \in \hat{R}[[Y]]$, $\hat{s}_{ij}$, $\hat{u}_k \in \hat{R}[[Y, Z]]$. By Theorem (1.3) there exists a solution (t, s, u) of $(**)$ in the henselization $R\langle Y, Z\rangle$ of $R[Y, Z]$ at $(\underline{m}, Y, Z)$ such that $t_i \in A$. Substituting Y by h we get $t_i(h) \equiv Z_i \bmod bB$, $i = 1, \ldots, r$ which says that f is also surjective.

(2.3) **Remark**. A nice geometric application of Theorem (1.0) is given in Theorem (3.8) from [A_2]. There it is shown that isolated singularities are in fact algebraic.

REFERENCES

[A_1] Artin, M. — *On the solution of analytic equations*, Inv. math., 5 (1968), 277–291.

[A_2] Artin, M. — *Algebraic approximation of structures over complete local rings*, Publ. Math. IHES, 36 (1969), 23–58.

[A_3] Artin, M. — *Construction techniques for algebraic spaces*, Actes Congrès Intern. Math., tome 1 (1970), 419–423.

[B] Becker, J. — *A counterexample to Artin Approximation with respect to subrings*, Math. Ann. 230, (1977), 195–196.

[Br] Brown, M. L. — *Artin's approximation property*, Thesis, Cambridge, Peterhouse, August 1979.

[CP] Cipu, M. and Popescu, D. — *Some extensions of Néron's p-desingularization and approximation*, Rev. Roum. Math. Pures et Appl., t. XXIV, 10(1981), 1299–1304.

[DL] Denef, J. and Lipshitz, L. — *Ultraproducts and approximation in local rings II*, Math. Ann. 253 (1980), 1–28.

[G] Gabrielov, A. M. — *The formal relations between analytic functions* (Russian), Funktional, Anal. i Priložen. 5 (1971), no 4, 64–65.

[K] Kurke, H., Mostowski, T., Pfister, G., Popescu, D. and Roczen, M. — *Die Approximationseigenschaft lokaler Ringe*, Lect. Notes in Math. 634, Springer-Verlag, 1978.

[L] Lipman, J. — *Rational singularities with applications to algebraic surfaces and unique factorization*, Publ. Math. IHES 36 (1969), 195–279.

[PP] Pfister, G. and Popescu, D. — *Die strenge Approximationseigenschaft lokaler Ringe*. Invent. Math. 30 (1975), 145–174.

[P_1] Popescu D. — *Algebraically pure morphisms*, Rev. Roum. Math. Pures et Appl., t. 24, 6(1979), 947–977.

[P_2] Popescu, D. — *General Néron desingularization and approximation II*, INCREST Prepr. 49/1983.

[R] Rotthaus, C — *Potenzreihenerweiterung und formale Fasern in lokalen Ringen mit Approximationseigenschaft.*, Manuscripta Math. 42 (1983), 53–65.

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Analele ştiinţifice ale Universităţii „Al. I. Cuza" din Iaşi
Tomul XXXI, s. I a, 1985 (supliment)

A NULLSTELLENSATZ OVER SOME HENSELIAN VALUED FIELDS

BY

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A basic tool of algebraic geometry is Hilbert's Nullstellensatz. It has been observed by A. Robinson [9] that this classical statement is equivalent in model theoretic terms with the fact that an algebraically closed field is existentially complete in every field extension. In the last time the methods of model theoretic algebra were successfully applied in order to get suitable Nullstellensätze over various fields.

The aim of the present work is to apply the general model theoretic approach from [1] to the case of a base field K of characteristic zero equipped with a proper Henselian valuation such that the residue field $k = k_v$ and the value group $G = G_v$ satisfy some supplementary conditions. We consider two types of restrictions on k. First we assume that k is a pseudo-real closed (prc) field [8] of characteristic zero such that $-1 \notin k^2$; in particular we consider the case when k is a formally real prc field. Next we assume that k is a finite field. As for the value group G we assume that the following finiteness conditions are satisfied.

Given a prime p and a natural number r, let us denote by $\Delta_{p,r} = \Delta_{p,r}(G)$ the maximal convex subgroup U of G such that $\dim U/pU \leqslant r$. Consider the next restrictions on G:

i) For every convex subgroup U of G such that G/U is not divisible, U/pU are finite for all primes p.

ii) The chain of the convex subgroups $\Delta_{p,r}$ for all p, r is finite.

The regular ordered groups [10], [11] satisfy the conditions i) and ii) above.

Given such a base field K and an ideal $\mathfrak{a}$ of the polynomial ring $K[z]$, $z = (z_1, \ldots, z_n)$, we consider the set $V_K(\mathfrak{a}) = \{a \in K^n \mid f(a) = 0$ for all $f \in \mathfrak{a}\}$ of K-rational zeros of $\mathfrak{a}$ and the corresponding ideal of $K[z]$

$$I\, V_K(\mathfrak{a}) = \{f \in K[z] \mid f = 0 \text{ on } V_K(\mathfrak{a})\}.$$

We explicitely construct an overring H of $K[z]$ contained in $F = K(z)$ in such a way that for every ideal $\mathfrak{a}$ of $K[z]$, $IV_K(\mathfrak{a}) = K[z] \cap \sqrt{\mathfrak{a}}$.

The ring H turns out to be the intersection of the valuation rings corresponding to the K — rational places of F/K lying over the polynomial ring $K[z]$. In other words, H is the holomorphy ring of certain subspace of the Riemann space of all places of F/K which are rational over K.

The details and the proofs are given in [4].

As remarkable special cases of the present general approach we mention the following ones.

1) K is $\mathcal{P}$-adically closed in Kochen-Roquette sense [7].

2) K is a Becker's generalized real closed field ; thus we get an alternative to the Nullstellensatz over such fields given in [5].

3) k is real closed and G is a Z-group ; in this case we get an alternative to the Nullstellensätze from [6] and [3].

4) K is equipped with an algebraically maximal t-ordering [2].

REFERENCES

1. Basarab S. — *Towards a general theory of formally $\mathcal{P}$-adic fields*, Manuscripta Math. 30 (1980), 279—327.
2. Basarab Ş. — *On a class of preorderings of higher level*, Manuscripta Math. 37 (1982), 163—210.
3. Basarab S. — *The Nullstellensatz over t-ordered fields: A t-adic analogue of the theory of formally $\mathcal{P}$-adic fields*, Preprint Series in Math. 50 (1980), Bucharest.
4. Basarab S. — *A Nullstellensatz over some Henselian valued fields*, Research report AI-FD046 (1984), Bucharest.
5. Becker E., Jacob B. — *Rational points on algebraic varieties over generalized real closed fields : A model theoretic approach*. To appear in Crelle's Journal.
6. Jacob B. — *A Nullstellensatz for R ((t))*, Comm. Algebra 8 (1980), 1083—1094.
7. Jarden M., Roquette P. — *The Nullstellensatz over $\mathcal{P}$-adically closed fields*, J. Math. Soc., Japan, 23 : 3 (1980) ; 425—460.
8. Prestel A., — *Pseudo-real closed fields*, Lecture Notes Math. 782 (1981), 127—156 Springer.
9. Robinson A. — *Introduction to Model Theory and to the Metamathematics of Algebra*, 1963, North Holland.
10. Robinson A., Zakon E. — *Elementary properties of ordered Abelian groups*, Trans. Amer. Math. Soc. 96 (1960), 222—236.
11. Zakon E. — *Generalized archimedean groups*, Trans. Amer. Math. Soc. 99 (1961), 21—40.

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Analele ştiinţifice ale Universităţii „Al. I. Cuza" din Iaşi
Tomul XXXI, s I a. 1985 (supliment)

VARIETIES OF SMALL DEGREE

BY

PALTIN IONESCU

This is a short report on our recent work [7], to which we refer for complete proofs.

Let $X_d^r \subset \mathbf{P}_{\mathbf{C}}^{r+s}$ be a smooth, connected, nondegenerate complex projective algebraic variety of dimension r and degree d. Hartshorne and Barth-Van de Ven proved that if d is „small" with respect to r, X is a complete intersection. It is conjectured that the same conclusion holds if s is „small" with respect to r. The result of Hartshorne and Barth-Van de Ven would be a consequence of this conjecture, since one has the elementary inequality $d \geqslant s+1$. Our main result is theorem 1 below, which gives a (sharp) bound such that if d is „small" with respect to s, than both the abstract structure and the embedding of X could be determined. Before stating the theorem we give two definitions. Let $X \subset \mathbf{P}^n$ be a smooth, connected, projective variety of dimension $r \geqslant 2$.

Definitions. a) *X is called a scroll if there is a surjective morphism $f : X \to Y$ onto some smooth variety Y of dimension $s>0$, such that the fibres of f are linear varieties of dimension $r-s>0$.*

b) *X is called a hyperquadric fibration if there is a surjective morphism $f : X \to C$ onto a smooth curve C such that the closed fibres of f are hyperquadrics. In this case one can prove that the singular fibres of f are ordinary cones.*

Theorem 1. *Let $X_d^r \subset \mathbf{P}_{\mathbf{C}}^{r+s}$ be as above, $r \geqslant 2$; let H be its general hyperplane section.*

If $d \leqslant 2s+1$, X is one of the following :

a) *A (birationally) ruled surface;*

b) *A scroll over a curve or a (birationally) ruled surface ;*

c) *A hyperquadric fibration ;*

d) *A variety with $\omega_X^{-1} \simeq \mathcal{O}_X(H)^{r-1}$; these are completely classified* (see [8], [4], [5]) ;

e) *$\mathbf{P}^r$, $H \in |\mathcal{O}(1)|$, or Q^r (the hyperquadric), $H \in |\mathcal{O}(1)|$;*

f) *There is a morphism $\sigma : X \to X'$ expressing X as the blowing up of X' with center finitely many points $P_1, \ldots, P_k$ and a divisor H' on X' with $H = \sigma^*(H') - \sigma^{-1}(P_1) - \cdots - \sigma^{-1}(P_k)$ such that :*

$$X' \simeq \mathbf{P}^3,\ H' \in |\mathcal{O}(3)|,\ \text{or}\ X' \simeq Q^3,\ H' \in |\mathcal{O}(2)|,\ \text{or}\ X' \simeq \mathbf{P}^4,$$

$H' \in |\mathcal{O}(2)|$, or X' is a $\mathbf{P}^2$-bundle over a smooth curve such that $\mathcal{O}_X(H')$ induces $\mathcal{O}(2)$ on each fibre :