**Proposition 2.1.** ([2]). *Let $B$ be a Banach manifold, $\{U_a\}_{a\in\mathcal{A}}$ an open covering of $B$, $\boldsymbol{E} = \varprojlim \boldsymbol{E}_i$ a Fréchet space and*

$$\{T^*_{ab} : U_a \cap U_b \longrightarrow H^\circ(\boldsymbol{E})\}_{a,b\in\mathcal{A}}$$

*a family of smooth mappings such that $T^*_{ab}(x) \circ T^*_{bc}(x) = T^*_{ac}(x), T^*_{aa}(x) = (id_{\boldsymbol{E}_i})_{i\in \boldsymbol{N}}$. Then, there exists a (uniquely determined) Fréchet-vector bundle $E$, which is projective limit of Banach vector bundles, with generalized transition functions $\{T^*_{ab}\}_{a,b\in\mathcal{A}}$ and classical (in Lang's sense) transition functions $\{\varepsilon \circ T^*_{ab}\}_{a,b\in\mathcal{A}}$.*

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# SOME REMARKS ON METRIC STRUCTURE IN THE GENERALIZED FINSLER SPACE

BY

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**1. Introduction.** Let $\xi = (E, \pi, M)$ be a $C^\infty$ vector bundle with an $n$-dimensional base manifold $M$, type fibre $F = R^m$ and a total space of dimension $(n+m)$. In some local chart the point $u \in E$ has the coordinates

$$(x^1, \dots, x^n, y^1, \dots, y^m) = (x^i, y^a) = (x, y)$$

$$i, j, h, k, l = 1, \dots, n \qquad a, b, c, d, e, f = 1, \dots, m.$$

The allowable coordinate transformations $(x, y) \to (x', y')$ are given by

$$x^{i'} = x^{i'}(x^1, \dots, x^n) \qquad y^{a'} = M^{a'}_a(x) y^a. \tag{1.1}$$

where

$$\operatorname{rank}[\partial x^{i'}/\partial x^i] = n \qquad \operatorname{rank}[M^{a'}_a] = m.$$

The inverse transformation $(x', y') \to (x, y)$ exists and is determined by

$$x^i = x^i(x^{1'}, \dots, x^{n'}) \qquad y^a = M^a_{a'}(x') y^{a'}. \tag{1.2}$$

The usual relations

$$\begin{aligned} &M^{a'}_a M^a_{b'} = \delta^{a'}_{b'} \qquad B^{i'}_j B^j_{j'} = \delta^{i'}_{j'} \qquad B^{i'}_j = \partial x^{i'}/\partial x^j \\ &M^{a'}_b M^a_{a'} = \delta^a_b \qquad B^i_{k'} B^{k'}_j = \delta^i_j \qquad B^j_{j'} = \partial x^j/\partial x^{j'} \end{aligned} \tag{1.3}$$

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are valid. The tangent space of $E$, $T(E)$ is spanned by $\overline{B} = \{\partial_i, \partial_a\}$, where

$$\partial_i = \partial/\partial x^i \qquad \partial_a = \partial/\partial y^a.$$

With respect to (1.1) and (1.2) they have the following law of transformation:
(1.4)

$$\begin{bmatrix} \partial_i \\ \partial_a \end{bmatrix} = J \begin{bmatrix} \partial_{i'} \\ \partial_{a'} \end{bmatrix} \qquad J = \frac{D(x', y')}{D(x, y)} = \begin{bmatrix} B_i^{i'} & M_{bi}^{a'} y^b \\ 0 & M_a^{a'} \end{bmatrix} \qquad M_{bi}^{a'} = \partial_i M_b^{a'}.$$

If $M$ is paracompact, then there exists a family of functions $N_i^a(x, y)$, obeying the following law of transformations:

$$N_i^a(x, y) = N_{i'}^{a'}(x', y') B_i^{i'} M_{a'}^a - M_{b' i'}^a y^{b'} B_i^{i'}. \tag{1.5}$$

These functions are called the coefficients of non-linear connection. By using them, we can transform the basis $\overline{B} = \{\partial_i, \partial_a\}$, whose elements, under the coordinate transformation (1.1) and (1.2) do not transform as tensors (see (1.4)), into the adapted basis $B = \{\delta_i, \partial_a\}$, whose elements have this property. $\delta_i$ is defined by

$$\delta_i = \partial_i - N_i^a \partial_a. \tag{1.6}$$

It is easy to prove, from (1.4) and (1.5), that

$$\delta_{i'} = \delta_i B_{i'}^i. \tag{1.7}$$

Any vector field $X$ defined on $E$ can be expressed in the basis $B$ in the following form

$$X = X^i \delta_i + X^a \partial_a, \qquad X^i = X^{i'} B_{i'}^i, \qquad X^a = X^{a'} M_{a'}^a. \tag{1.8}$$

We call $X^a \partial_a$ the vertical vector field and $X^i \delta_i$ the horizontal vector field of $T(E)$, respectively. We shall denote the subspace of $T(E)$ spanned by $\{\delta_i\}$ by $T_H(E)$ and the subspace spanned by $\{\partial_a\}$ by $T_V(E)$. So we have

$$T(E) = T_H(E) \oplus T_V(E), \quad \dim T_H(E) = n, \quad \dim T_V(E) = m.$$

Let us consider $T^*(E)$, the dual tangent space of $E$. The natural basis in $T^*(E)$ is $\overline{B}^* = \{dx^1, \dots, dx^n, dy^1, \dots, dy^m\} = \{dx^i, dy^a\}$. From (1.1) we have

$$\text{(a)} \quad dx^{i'} = B_i^{i'} dx^i \qquad \text{(b)} \quad dy^{a'} = M_{ai}^{a'} y^a dx^i + M_a^{a'} dy^a. \tag{1.9}$$



It is obvious that $dy^a$ $a = 1, \dots, m$ do not transform as tensors, so we introduce a new basis

$$B^* = \{dx^i, \delta y^a\}$$

of $T^*(E)$, where

$$\delta y^a = dy^a + N_i^a(x, y) dx^i. \tag{1.10}$$

According to the coordinate transformation (1.1) and (1.2) the bases $B^*$ and $B^{*'} = \{dx^{i'}, \delta y^{a'}\}$ are related by (1.9a) and

$$\delta y^a = M_{a'}^a(x') \delta y^{a'} \qquad \delta y^{a'} = M_a^{a'}(x) \delta y^a. \tag{1.11}$$

If $B^*$ and $B^{*'}$ are two bases of $T^*(E)$ related by (1.9a) and (1.11), then any 1-form $w \in T^*(E)$ satisfies the relations

$$w = w_i dx^i + w_a \delta y^a = w_{i'} dx^{i'} + w_{a'} \delta y^{a'}.$$

where

$$w_{i'} = w_i B_{i'}^i \qquad w_{a'} = w_a M_{a'}^a.$$

Let us consider $T^*(E) \otimes T^*(E)$. In this space the metric tensor $G$ expressed in the basis $B^*$ has the form

$$G = g_{ij} dx^i \otimes dx^j + g_{ib} dx^i \otimes \delta y^b + g_{aj} \delta y^a \otimes dx^j + g_{ab} \delta y^a \otimes \delta y^b. \tag{1.12}$$

We shall suppose, that $G$ has the rank $n + m$ and that, it is symmetric and positively define, i.e. it is a Riemannian metric on $E$. With respect to the coordinate transformations (1.1) and (1.2) the coordinates of the metric tensor $G$ transform in the following way

$$\begin{aligned} g_{i'j'} &= g_{ij} B_{i'}^i B_{j'}^j \qquad & g_{i'b'} &= g_{ib} B_{i'}^i M_{b'}^b \\ g_{a'j'} &= g_{aj} M_{a'}^a B_{j'}^j \qquad & g_{a'b'} &= g_{ab} M_{a'}^a M_{b'}^b \end{aligned}. \tag{1.13}$$

We shall define the covariant coordinates of the vector $X = X^i \delta_i + X^a \partial_a$ by

$$X_i = g_{ij} X^j + g_{ib} X^b \qquad X_a = g_{aj} X^j + g_{ab} X^b.$$

**Theorem 1.1.** *If the Riemannian metric tensor $G(x, y)$ is given on $E$, then exists one and only one nonlinear connection $N$, such that $T_H(E)$ is orthogonal to $T_V(E)$ with respect to $G$. The connection coefficients $N_i^a(x, y)$*

*of $N$ are determined by (1.17) (they are functions only of $G$) and with respect to the coordinate transformations (1.1) and (1.2) they satisfy the relation (1.5).*

**Proof.** *Let us suppose, that the metric tensor $G$ in the basis $\overline{B}^* = \{dx^i, dy^a\}$ of $T^*(E)$ is given by*

$$G = \overline{g}_{ij}dx^i \otimes dx^j + \overline{g}_{ib}dx^i \otimes dy^b + \overline{g}_{aj}dy^a \otimes dx^j + \overline{g}_{ab}dy^a \otimes dy^b. \tag{1.14}$$

*Substituting $dy^a = \delta y^a - N_i^a(x,y)dx^i$ from (1.10) into (1.14) we obtain the expression for the metric tensor in the basis $B^*$. $N_i^a(x,y)$ are arbitrary functions, which satisfy the transformation law (1.5). The comparation of the obtained formula with (1.12) gives*

$$(1.15)\quad \begin{aligned} &(a)\quad g_{ij} = \overline{g}_{ij} - \overline{g}_{ib}N_j^b\overline{g}_{aj}N_i^a + \overline{g}_{ab}N_i^aN_j^b \\ &(b)\quad g_{ib} = \overline{g}_{ib} - \overline{g}_{ab}N_i^a \\ &(c)\quad g_{aj} = \overline{g}_{aj} - \overline{g}_{ab}N_j^b \\ &(d)\quad g_{ab} = \overline{g}_{ab}. \end{aligned}$$

By the assumption of Theorem 1.1 the matrix

$$\begin{bmatrix} g_{ij} & g_{ib} \\ g_{aj} & g_{ab} \end{bmatrix}$$

is symmetric, positively definite, with rank $n+m$. From (1.15) it is obvious, that for the arbitrary nonlinear connection $N_i^a$ and the metric $G$ given by (1.12) $T_H(E)$ is not necessarily orthogonal to $T_V(E)$. It will be iff $g_{ib} = 0$, $\forall i = 1,\dots,n$, $\forall b = 1,\dots,m$, i.e.

$$\overline{g}_{ib} - \overline{g}_{ab}N_i^a = 0 \Rightarrow \overline{g}_{ib} - g_{ab}N_i^a = 0. \tag{1.16}$$

From (1.16) we obtain

$$N_i^c = \overline{g}_{ib}g^{bc}, \tag{1.17}$$

where $[g^{ab}]$ is the inverse matrix of $[g_{ab}] = [\overline{g}_{ab}]$. (1.17) determines $N_i^c$ as function of the metric tensor. To prove, that so obtained non-linear connection satisfies (1.5), we write $G$ in the basis $\{dx^{i'}, dy^{a'}\}$ in the form:

$$G = \overline{g}_{i'j'}dx^{i'} \otimes dx^{j'} + \overline{g}_{i'b'}dx^{i'} \otimes dy^{b'} + \overline{g}_{a'j'}dy^{a'} \otimes dx^{j'} + \overline{g}_{a'b'}dy^{a'} \otimes dy^{b'}. \tag{1.18}$$



Substituting (1.9a) and (1.9b) into (1.18) and comparing the obtained expression with (1.14), we get

$$(1.19)\quad \begin{aligned} &(a)\quad \overline{g}_{ib} = \overline{g}_{i'b'}B_i^{i'}M_b^{b'} + \overline{g}_{a'b'}M_{ai}^{a'}y^aM_b^{b'} \\ &(b)\quad \overline{g}_{ab} = \overline{g}_{a'b'}M_a^{a'}M_b^{b'} \Leftrightarrow g_{ab} = g_{a'b'}M_a^{a'}M_b^{b'}. \end{aligned}$$

Substituting (1.15b) into $g_{ib} = g_{i'b'}B_i^{i'}M_b^{b'}$ (see (1.13)) we get

$$\overline{g}_{ib} - \overline{g}_{ab}N_i^a = (\overline{g}_{i'b'}\overline{g}_{a'b'}N_{i'}^{a'})B_i^{i'}M_b^{b'}.$$

The substitution of (1.19a) and (1.19b) into the above equation gives

$$g_{a'b'}M_b^{b'}(M_{ai}^{a'}y^a - M_a^{a'}N_i^a + N_{i'}^{a'}B_i^{i'}) = 0.$$

Multiplying the above equation first with $M_{c'}^b$ (using (1.3)), then with $g^{c'd'}$ $(g^{c'd'}g_{a'c'} = \delta_{a'}^{d'})$ we obtain the relation

$$M_{ai}^{d'}y^a - M_a^{d'}N_i^a + N_{i'}^{d'}B_i^{i'} = 0,$$

which after multiplication with $M_{d'}^c$ and using the relation

$$M_{a\,i}^{d'}y^aM_{d'}^c = -M_a^{d'}y^aM_{d'\,i'}^cB_i^{i'} = -M_{d'\,i'}^cy^{d'}B_i^{i'} \tag{1.20}$$

takes the form (1.5). ■

**2. The metric tensor expressed by the "metric" function.** Let us suppose, that on the space $E$ the function $F(x,y)$ is defined, which satisfies the following conditions:

a) The function $F(x,y)$ is positively homogeneous of degree 1 in $y$, i.e.

$$F(x,ky) = |k|F(x,y) \quad k \in R \quad k \neq 0$$

b)

$$F(x,y) > 0 \quad \text{for} \quad y \neq 0$$

c) The matrix

$$\overline{G} = \begin{bmatrix} \overline{g}_{ij} & \overline{g}_{ib} \\ \overline{g}_{aj} & \overline{g}_{ab} \end{bmatrix} \tag{2.1}$$

is positively definite, where

$$\overline{g}_{ij} = F^{-2}\partial_i\partial_j L \quad \overline{g}_{ib} = F^{-1}\partial_i\partial_b L$$
$$\overline{g}_{aj} = F^{-1}\partial_a\partial_j L \quad \overline{g}_{ab} = \partial_a\partial_b L \tag{2.2}$$
$$L = 2^{-1}F^2.$$

We want to construct such theory in which the homogeneity condition is important, i.e. all tensor fields defined on $E$ are homogeneous of degree zero in $y$. From (2.2) it is obvious that all elements of the $(n+m)\times(n+m)$ matrix $\overline{G}$ defined by (2.1) are homogeneous of degree zero in $y$. The question is: What conditions should satisfy the function $F(x,y)$ and the coordinate transformations (1.1), so that the elements of the matrix (2.1) are coordinates of the metric tensor $G$ expressed in the basis $\overline{B}^* = \{dx^i, dy^a\}$. From (2.2) it follows that

(2.3)
$$G(x,y) = \overline{g}_{ij}dx^i \otimes dx^j + F^{-1}\overline{g}_{ib}dx^i \otimes dy^b + F^{-1}\overline{g}_{aj}dy^a \otimes dx^j + $$
$$+F^{-2}\overline{g}_{ab}dy^a \otimes dy^b$$

is homogeneous of degree zero in $y$, i.e.

$$G(x,ky) = G(x,y).$$

Using the relations $G(x',y') = G(x,y)$, $F(x',y') = F(x,y)$ and (1.19) we obtain

$$\text{(a)} \quad \overline{g}_{ij} = \overline{g}_{i'j'}B_i^{i'}B_j^{j'} + F^{-1}\overline{g}_{i'b'}B_i^{i'}M_{bj}^{b'}y^b +$$
$$+F^{-1}\overline{g}_{a'j'}M_{ai}^{a'}y^aB_j^{j'} + F^{-2}g_{a'b'}M_{ai}^{a'}y^aM_{bj}^{b'}y^b \tag{2.4}$$
$$\text{(b)} \quad \overline{g}_{ib} = \overline{g}_{i'b'}B_i^{i'}M_b^{b'} + F^{-1}\overline{g}_{a'b'}M_{ai}^{a'}y^aM_b^{b'}$$
$$\text{(c)} \quad \overline{g}_{ab} = \overline{g}_{a'b'}M_a^{a'}M_b^{b'}.$$

On the other hand, from

$$\partial_i L = (\partial_{i'}L)B_i^{i'} + (\partial_{a'}L)M_{ai}^{a'}y^a$$
$$\partial_a L = (\partial_{a'}L)M_a^{a'} \tag{2.5}$$

follows

$$\partial_i\partial_j L = (\partial_{i'}\partial_{j'}L)B_i^{i'}B_j^{j'} + (\partial_{i'}\partial_{a'}L)M_{aj}^{a'}y^aB_i^{i'} +$$
$$+(\partial_{a'}\partial_{j'}L)B_j^{j'}M_{ai}^{a'}y^a + (\partial_{a'}\partial_{b'}L)M_{ai}^{a'}y^aM_{bj}^{b'}y^b + \tag{2.6}$$
$$[(\partial_{i'}L)B_{ij}^{i'} + (\partial_{a'}L)M_{aij}^{a'}y^a]$$



$$\partial_i\partial_a L = (\partial_{i'}\partial_{a'}L)B_i^{i'}M_a^{a'} + (\partial_{b'}\partial_{a'}L)M_{bi}^{b'}y^bM_a^{a'} + (\partial_{b'}L)M_{ai}^{b'} \tag{2.7}$$

$$\partial_a\partial_b L = (\partial_{a'}\partial_{b'}L)M_a^{a'}M_b^{b'}. \tag{2.8}$$

The comparation of (2.2), (2.6), (2.7) and (2.8) with (2.4a,b,c) gives:

**Theorem 2.1.** *The necessary and sufficient conditions, that $\overline{g}_{ij}$, $\overline{g}_{ib}$, $\overline{g}_{aj}$ and $\overline{g}_{ab}$ (defined by (2.2)) are the components of the metric tensor $G$ expressed in the basis $\overline{B}^* = \{dx^i, dy^a\}$, with respect to the coordinate transformation (1.1), are:*

$$(\partial_{i'}L)B_{ij}^{i'} + (\partial_{a'}L)M_{aij}^{a'}y^a = 0 \tag{2.9}$$

$$(\partial_{b'}L)M_{ai}^{b'} = 0 \tag{2.10}$$

**Proof.** *If we substitute (2.9) into (2.6) and multiply the so obtained expression with $F^{-2}$, then use (2.2), we obtain (2.4a). If we substitute (2.10) into (2.7) and multiply the obtained equation with $F^{-1}$, then use (2.2) we obtain (2.4c). Substituting (2.8) in (2.2) we obtain (2.4c). It is clear that for $M_{ai}^{b'} = 0$ and $B_{ij}^{i'} = 0$ the equations (2.9) and (2.10) are satisfied. In this case (1.1) reduces to linear transformation: $x^{i'} = B_i^{i'}x^i$, $y^{a'} = M_a^{a'}y^a$, where $B_i^{i'}$ and $M_a^{a'}$ are constants, and $N_i^a(x,y)$ is arbitrary tensor. In the theory of generalized Finsler space in which the homogeneity condition is valid the non-linear connection $N_i^a(x,y)$ must be homogeneous of degree 1 in $\underline{y}$. From this fact follows, that the basis vectors $\delta y^a$ (defined by (1.10)) $a = \overline{1,m}$ are also homogeneous of degree 1 in $y$. The components of the tensor $G$ in the basis $B^* = \{dx^a, \delta y^a\}$ we obtain in the following way:*

$$\text{(a)} \quad g_{ij} = \overline{g}_{ij} - F^{-1}\overline{g}_{ib}N_j^b - F^{-1}\overline{g}_{aj}N_i^a - F^{-2}\overline{g}_{ab}N_i^aN_j^b$$
$$\text{(b)} \quad g_{aj} = \overline{g}_{aj} - F^{-1}\overline{g}_{ab}N_j^b \tag{2.11}$$
$$\text{(c)} \quad g_{ib} = \overline{g}_{ib} - F^{-1}\overline{g}_{ab}N_i^a$$
$$\text{(d)} \quad g_{ab} = \overline{g}_{ab}.$$

*The functions $g_{ij}(x,y)$, $g_{aj}(x,y)$, $g_{ib}(x,y)$ and $g_{ab}(x,y)$ are homogeneous of degree zero in $y$ and with respect to (1.1) they transform as tensors (see (1.13)). Now we have*

$$G = g_{ij}dx^i \otimes dx^j + F^{-1}g_{ib}dx^i \otimes \delta y^b +$$
$$+F^{-1}g_{aj}\delta y^a \otimes dx^j + F^{-2}g_{ab}\delta y^a \otimes \delta y^b. \tag{2.12}$$

*If* $X^i$, $Y^j$, $X^a$, $Y^b$, $\delta_i$, $F\partial_a$, $F\partial_b$ *are functions of* $(x, y)$ *homogeneous of degree zero in* $y$ *and*

$$X = X^i\delta_i + FX^a\partial_a \qquad Y = Y^j\delta_j + FY^b\partial_b, \tag{2.13}$$

*then* $X$ *and* $Y$ *are vector fields in* $T(E)$ *homogeneous of degree zero in* $y$*. The scalar product of these two vector fields formed with the metric tensor* $G$ *is*

$$< X, Y >= g_{ij}X^iY^j + g_{ib}X^iY^b + g_{aj}X^aY^j + g_{ab}X^aY^b. \tag{2.14}$$

*We want to obtain some non-linear connection expressed by the function* $F(x, y)$*. Under the condition* $(\partial_{b'}L)M^{b'}_{ai} = 0$ *((2.10)) (2.7) takes the form*

$$\partial_i\partial_a L = (\partial_{i'}\partial_{a'}L)B^{i'}_i M^{a'}_a + (\partial_{a'}\partial_{b'}L)M^{b'}_{ci}y^c M^{a'}_a. \tag{2.15}$$

*From (2.11d) and (2.2) we obtain*

$$g_{ab} = \partial_a\partial_b L.$$

*Let us denote by* $[g^{ab}]$ *the inverse matrix of* $[g_{ab}]$*, then (2.15) multiplied by* $g^{ab}$ gives

$$\begin{aligned} g^{ab}(\partial_i\partial_a L) = \; & g^{i'b'}M^a_{e'}M^b_{b'}(\partial_{i'}\partial_{a'}L)B^{i'}_i M^{a'}_a + \\ & + g_{ef}M^e_{b'}M^f_{a'}g^{ab}M^{b'}_{ci}y^c M^{a'}_a. \end{aligned} \tag{2.16}$$

*Using the notation*

$$N^b_i = g^{ab}\partial_i\partial_a L, \tag{2.17}$$

*(1.3) and (2.6) we get*

$$N^b_i = N^{b'}_{i'}B^{i'}_i M^b_{b'} + M^b_{b'}M^{b'}_{ci}y^c.$$

*Using the relation (1.20) we obtain, that* $N^b_i$ *defined by (2.17), under condition (2.10) has the transformation law prescribed by (1.5), so (2.17) gives the nonlinear connection defined by the function* $F$*. It can be seen, that (2.17) corresponds to (1.17). From (2.11)(b) and (2.11)(c) there follows, that for the non-linear connection* $N^b_i$ *determined by (2.17)* $g_{aj} = 0$, $g_{ib} = 0$*, i.e.* $T_H(E)$ *is orthogonal to* $T_V(E)$*. Using the arbitrary non-linear connection*



$N^b_i(x, y)$*, which satisfies (1.5), formulae (2.15a-d) under conditions (2.9) and (2.10) determine the metric structure obtained from the metric function.*

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