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Reçu : 10.XI.1995

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Tomul XLII, Supliment, s.I.a, Matematică, 1996

ON FUZZY CONTINUITY

BY

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1. Introduction. The concepts of fuzzy almost continuity and fuzzy weakly continuity defined by Azad [3] were introduced by Yalvac [8] by using the concept of quasi-coincidence. Also Allam and Zahran [1] introduced and studied the concept of fuzzy δ continuity by using the concept of quasi-coincidence.

The purpose of this paper is to investigate the interrelation among these concepts. In Section 3 we show that both fuzzy continuity and fuzzy δ-continuity (resp. fuzzy weakly continuity and fuzzy almost continuity Type II) are independent. Also we prove that fuzzy almost continuity implies fuzzy θ-continuity. In Section 4 we investigate some properties of δ-closure and θ closure.

2. Preliminaries. Throughout this paper X and Y mean fuzzy topological spaces (fts, s, for short). The value of a fuzzy set A at the element x of X will be denoted by $A(x)$, $P \in A$ means that a fuzzy point P takes its non zero value in $(0,1]$ at the support x_p and $p(x_p) \leq A(x_p)$ [5]. The definitions of fuzzy sets, fts, s and other concepts can be found in [1,9]. The words "fuzzy", "quasi", "neighborhood" will be abbreviated as "f", "q" and "nbd" respectively.

Definition 2.1. *Let A be a fuzzy set of a fts X, then*
(i) A is called f. regular open [3] if $A = (\overline{A})^\circ$.
(ii) A is called f. semiopen [3] if there exists a f. open set V such that $V \leq A \leq \overline{V}$. The family of all f. regular open sets will be denoted by $RO(X)$.

Communicated at the 25-th National Conference on Geometry and Topology, September 18-23, 1995, Iaşi, România

Definition 2.2. *[5] A f. point p is q coincident with a f. set A or pqA iff $P(x_p) + A(x_p) > 1$. A f. set A is q coincident with a f. set B or AqB if there exists $x \in X$ such that $A(x) + B(x) > 1$. A f. set A of a fts X is q–nbd of a fuzzy point p if there exists a f. open set B such that $pqB < A$.*

Theorem 2.3. *[5] Let A and B be f. sets of X. $A \leq B$ if and only if A and B' are not q coincident denoted by $A\not{q}B$ (where B' is the complement of B).*

The statement of the following theorem have been stated in [4, 8].

Theorem 2.4. *Let $f: X \to Y$ be a function and let A and B be f. sets of a fts X and a fts Y respectively, then.*
(i) $(f(A))' \leq f(A')$, $(f^{-1}(B))' = (f^{-1}(B))'$
(ii) $A \leq f^{-1}(f(A))$, $f(F^{-1}(B)) \leq B$
(iii) If f is one–to–one, then $f^{-1}(f(A)) = A$.
(iv) If f is onto, then $f(f^{-1}(B)) = B$
(v) If f is one–to one and onto, then $(f(A))' = f(A')$.

Let f be a function from X into Y. If p is a f. point in X, $f(p)$ is a f. point in Y and supp $(f(p)) = f(x_p)$, $f(p)(f(x_p)) = P(x_p)$. If p is a f. point in Y, then $f^{-1}(p)$ need not be a f. point X. If f is one–to one and P is a f. point in $F(X)$, then $f^{-1}(p)$ is a f.point in X, supp $f^{-1}(p) = f^{-1}(y_p)$ and $f^{-1}(p)(f^{-1}(y_p)) = P(y_p)$ [8].

Theorem 2.5. *[5] Let $f: X \to Y$ be a function and p be a f. point in X, then*
(i) If $B \leq Y$ and $f(p)qB$, then $pqf^{-1}(B)$.
(ii) If $A \leq X$ and pqA, then $f(p)qf(A)$.

Theorem 2.6. *[5] Let A be a f. set of X and p be a f. point, $p \in \overline{A}$ if and only if for each q – nbd B of p, BqA.*

Definition 2.7. *An f.set A of an fts X is said to be Rq – nbd of an f. point p if and only if there exists an f. set $B \in RO(X)$ such that $pqB \leq A$.*

3. Functions and fuzzy topological spaces.

Definition 3.1. *A function $f: X \to Y$ is said to be f. δ. continuous [1] (res. f. almost continuous [8], f.θ continuous [6, 7], f.weakly θ continuous [1], f. weakly continuous [8] if for each f. point P in X and each f. open q.nbV of $f(p)$, there is a f. open q – nb U of P such that $f(\overline{U})^\circ \leq (\overline{V})^\circ$ (rep. $f(U) \leq (\overline{V})^\circ$, $f(\overline{U}) \leq \overline{V}$, $f(\overline{U})^\circ) \leq \overline{V}$, $f(U) \leq \overline{V}$).*

Theorem 3.2. *Every f. almost continuous is f.θ. continuous.*

Proof. Let P be a f. point in X and V be a *q.nbd* of $f(P)$. Since f is f. almost continuous, then $f(U) \leq \overline{V}^\circ \leq \overline{V}$. So $f(U) \not{q}(\overline{V})'$. Now let a f. point $r \in X$, $r \in \overline{U}$ and $f(r) \in \overline{V}$, there exists a f. open q – *nbd* W of r such that $f(W) \leq (\overline{\overline{V}'})^\circ == \overline{V}' \Rightarrow f(W) \not{q}\overline{V}$. Since $r \in \overline{U}$, then WqU, $f(U)q(\overline{V})'$. This contradiction implies to $f(\overline{U}) \leq \overline{V}$. So f is f. θ–continuous.

The converse of the above theorem need not true as shown by the following example.

Example 3.3. Let $X = \{x, y\}$ and let A and B be two f. sets of X such that $A = (.3, 0)$, $B = (.2, 0)$. Consider the f. topologies $\tau_1 = \{X, 0, (.3, 0)\}$, $\tau_2 = \{X, 0, (.2, 0)\}$. The identify function $f: (X, \tau_1) \to (X, \tau_2)$ is a f. θ. continuous but not f. almost continuous. For let P be a f. point in X and V any fuzzy open q – *nbd* of $f(P)$.

(i) if $V = B$, Put $U = A$, $f(\overline{U}) = f(.7, 1) \leq \overline{V} = (.8, 1)$. Then f.$\theta$. continuous.

(ii) if $V = X$, then the prove is obvious. $P = (.9, 0)$, it is clear that B is a f. open q – *nbd* of $f(p)$ and the only f. open *q.nbd's* of p are A and X, but $f(A) \not\leq \overline{B}^\circ$ and $f(X) \not\leq \overline{B}^\circ$. Therefore f is not f. almost continuous.

Remark 3.4. from Definitions 3.1 and Theorem 3.2 one may notice that: f.δ continuous $\Rightarrow$ f. almost continuous $\Rightarrow$ f.θ–continuous $\Rightarrow$ f. weakly θ continuous $\Rightarrow$ f. weak continuous. But the converse need not be true in general as shown by Examples 4.2 [8], 3.3, 4.3 [2], 3.23 [1] respectively.

Theorem 3.5. *[8] Let $f: X \to Y$ be a function from a fts X into a fts Y, then the following statement are equivalent:*
(i) f is f. weakly continuous.
(ii) $\overline{f^{-1}(F)^\circ} \leq f^{-1}(F)$, for each f. closed set F of Y.
(iii) For each f. point p in X and each f. open set V of Y with $f(p)qr$, $f^{-1}(\overline{V})$ is a q – nb of P.

Definition 3.6. *A function $f: X \to Y$ is said to be f. almost continuous type II ($f.a.C.II$) if for each f. point P in X and each f. open set V with $f(P)qV$, there exists a f. open set U with PqU, such that $f(U) \leq (f^{-1}(V))$.*

Lemma 3.7. *[8] Let A be a f. set of X such that $A(x) = t \neq 0$, for each $x \in X$. If for any r which satisfies $0 < r < t$, we choose the f. point P_{1-r}, then pqA.*

Theorem 3.8. *If $f: X \to Y$ be a function, then the following statements are equivalent.*

(i) f is $f.a.c.H$.

(ii) For each f. point p in X and each f. open $q-nbd$ V of $f(p)$, there exists a f. open $q-nbd$ U of P such that $U \leq \overline{f^{-1}(V)}$.

(iii) $f^{-1}(V) = \overline{(f^{-1}(V))}^{\circ}$ for each f. open set V in X.

Proof. One may notice that if P be a f. point in X and U be a f. set in X, then $P_r q U$ if and only if $P_{1-r} \in V$.

(i) $\to$ (ii) Let P_r be a f. point in X and V be a f. open $q-nbd$ of $f(p)$. If $r < 1$, then $f(P_{1-r}) \in V$ and by (i) there exists a f. open set U in X with $P_{1-r} \in U$ such that $f(U) \leq \overline{(f^{-1}(V))}$. Hence there exists a f. open $q-nbd$ U of P_r such that $f(U) \leq \overline{(f^{-1}(V))}$. If $r = 1, V(f(x)) = t > 0$. Let P be a f. point with support $f(x)$ and $P(f(x)) = 1-s$, $0 < s < t$. By Lemma 3.7, PqV. Then $f(P_s) \in V$. From (i) there exists a f. open set U in X with $P_s \in U$ and $f(u) \leq \overline{(f^{-1}(V))}$. Hence $P_{1-s} q U$ and $f(U) < \overline{(f^{-1}(V))}$. Therefore $P_r q U$ and $f(u) \leq \overline{(f^{-1}(V))}$.

(ii) $\to$ (iii) Let P_r be a f. point in X and V be a f. open set in Y, $P_r \in f^{-1}(V)$, then $f(P_r) \in V$ and $f(P_{1-r}) q V$. By (ii) there is a f. open $q-nbd$ U of P_{1-r} such that $u \leq \overline{(f^{-1}(v))}$. Since $P_r \in U \leq \overline{(f^{-1}(v))}$. Then $P \in \overline{(f^{-1}(v))}^{\circ}$. Therefore $f^{-1}(V) \leq \overline{(f^{-1}(V))}^{\circ}$.

(iii) $\to$ (i) Let P be a f. point in X and V be a f. open set in Y with $f(P) \in V$, then $P \in f^{-1}(V)$. By (iii) $P \in \overline{(f^{-1}(V))}^{\circ}$ and therefore there exists an f. open set U in X such that $P \in U \leq \overline{(f^{-1}(V))}$. Hence f is $f.a.c.H$.

Remark 3.9. The concepts of f. continuity and f. δ-continuity are independent of each other as shown by the following two examples.

Example 3.10. Let $X = \{x, y, z\}$ and let A and B be f. sets X defined as follows:

$A(X) = (.2, .2, .2)$,
$B(X) = (.1, .1, .1)$.

Consider the fuzzy topologies $\tau_1 = \{X, O, A, B\}$ and $\tau_2 = \{X, O, B\}$ and the function $f: (X, \tau_1) \to (X, \tau_2)$ defined by $f(x) = x$, $f(y) = y$ and $f(z) = z$.

The function f is fuzzy continuity but not $f.\delta$ continuity, since for a fuzzy point $P = (.05, 0, 0)$ $f(P) = P \in B = (.1, .1, .1) \in \tau_2$ and every f. open set U such that $P \in U$, $f(\overline{U})^{\circ} = (.2, .2, .2) \not\leq \overline{V}^{\circ} = (.1, .1, .1)$.

Example 3.11. Let $X = \{x, y, z\}$, $Y = \{a, b\}$ and $f: X \to Y$ be a function defined by $f(x) = a$, $f(y) = f(z) = b$. Consider the fuzzy topologies

$\tau_1 = \{X, 0, (.9, .8, .8), (.4, .5, .5)\}$ and $\tau_2 = \{Y, O, (.9, .8,), (.6, .4)\}$. It is clear that f is f. δ continuous since for any f. point P, $f(p) \in Y$, we have

(i) if $f(p) \in (.9, .8) \in \tau_2$, there is a $U = (.9, .8, .8) \in \tau_1, P \in U$ such that $f(\overline{U})^{\circ}) = Y \leq \overline{(.9, .8)}^{\circ} = Y$

(ii) if $f(P) \in (.6, .4) \in \tau_2$, there is an $U = (.9, .8, .8) \in \tau_1, P \in U$ such that $f(\overline{U})^{\circ} = Y \leq \overline{(.6, .4)} = Y$. But f is not f. continuous, since $(.6, .4) \in \tau_2$ and $f^{-1}(.6, .4) = (.6, .4, .4) \in \tau_1$.

Remark 3.12. The concepts of f. weakly continuous and $f.a.c.H$. are independent of each other as shown by the following two examples.

Example 3.13. Let $X = \{x, y, z\}, Y = \{a, b\}$
Consider the fuzzy topologies

$$\tau_1 = \{X, O, (.8, .9, .8)\},$$
$$\tau_2 = \{Y, O, (.4, .6), (.5, .4), (.5, .6), (.4, .4)\}.$$

The function $f: (X, \tau_1) \to (Y, \tau_2)$ defined by $f(x) = a$ and $f(y) = f(z) = b$ is $f.a.c.H.$, since for each $V \in \tau_2, U \leq \overline{f^{-1}(V)} = X$, for each $U \in \tau_1$.

But f is not f. weakly continuous, since for a f. point $P = (.7, 0, 0)$, $f(P) q (.4, .6) \in \tau_2$, but $f^{-1}\overline{(.4, .6)} = (.5, .6, .6)$ is not $q.nbd$ of P.

Example 3.14. Let $X = \{x, y, z\}, Y = \{a, b\}$ and $f: X \to Y$ is defined by $f(x) = a$ and $f(y) = f(z) = b$.
Consider the fuzzy topologies

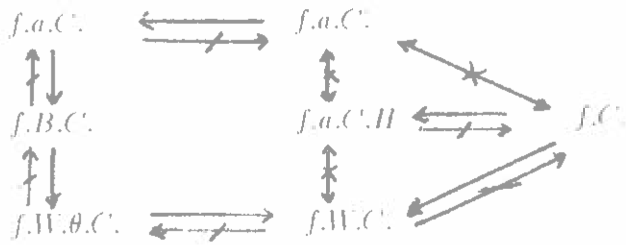
$$\tau_1 = \{X, O, (.9, .8, .8), (.6, .7, .5), (.4, .5, .5)\}$$
$$\tau_2 = \{Y, O, (.9, .8), (.6, .5)\}$$

The only f. closed sets in Y are Y, $(.1, .2)$, and $(.4, .5)$. $\overline{(f^{-1}(Y^{\circ}))} = X \leq$ $\leq f^{-1}(Y) = X$, $\overline{(f^{-1}(.1, .2)^{\circ})} = 0 \leq f^{-1}(.1, .2) =$ $= (.1, .2, .2) \overline{(f^{-1}(.4, .5)^{\circ})} = 0 \leq f^{-1}(.4, .5) = (.4, .5, .5)$. Therefore f is f. weakly continuous. But f is not $f.a.c.H$, since for a f. point $P = (.5, O, O) \in X$, $f(P) = (.5, O) \in (.5, .5)$. The only f. open sets in X containing P are $(.9, .8, .8)$ and $(.6, .7, .5)$, but $(.9, .8, .8), (.6, .7, .5) \not\leq \overline{f^{-1}(V)} = (.6, .5, .5)$. So f is not $f.a.c.H$.

Remark 3.16. It is clear that each f. continuous is $f.a.c.H$. but the converse need not be true in general since the function f in Example 3.13 is $f.a.c.H$ but not f. continuous.

Remark 3.17. The concepts of $f.\delta$. continuous and $f.a.c.H$ are independent of each other since a function f in Example 3.11. is $f.\delta$. continuous but not $f.a.c.H$. Also a function f in Example 3.12 is $f.a.c.H$ but not $f.\delta$. continuous.

From the above discussion we obtain following diagram, where $A \not\Rightarrow B$ indicates that A does not necessarily imply B and $A \not\Leftrightarrow B$ indicates that A and B are independent of each other. Also in the following diagram we have abbreviated as follows: $W =$ weakly; $C =$ continuous; $f =$ fuzzy; $a =$ almost.



4. δ– Closure and θ–closure of a fuzzy set.

Definition 4.1. *[1] A f. point P is said to be a f. δ cluster point of a f. set A if and only if each $Rq-nbd$ of P is q coincident with A. The union of all f. δ cluster points of A is defined to be a δ- -closure of A and denoted by $[A]_\delta$. A f. set A is f.δ closed if and only if $A = [A]_\delta$ and the complement of a f.δ closed set is called f.δ open.*

Definition 4.2. *[7] A fuzzy point p is said to be a f.θ cluster point of a set A if and only if for every f. open $q-nbd$ U of P, $\overline{U}$ is q coincident with A. The set of all f. θ cluster of A and denoted by $[A]_\theta$. A f. set A is θ- closed if and only if $A = [A]_\theta$ and the complement of a f. θ closed set is f. θ open.*

Remark 4.3. It is easy to see that $\overline{A} \leq [A]_\delta \leq [A]_\theta$ for any f. set A in a fts X. For any f. open set A, $\overline{A} = [A]_\delta = [A]_\theta$ [7].

Definition 4.4. *[7] A f. set A in a fts X is said to be a f.$\delta-nbd$ (f.$\theta-nbd$) of a f. point p if and only if there exists a regularly open $q-nbd$ (fuzzy closed $q-nbd$) V of p such that $V \not{q} A'$.*

Theorem 4.5. *Let f. be a function from a fts X to a fts Y, then the following statements are equivalent:*

(a) f is f. almost continuous
(b) $f(\overline{U}) \leq [f(U)]_\delta$ for every f. set U in X.
(c) The inverse image of every f. δ closed set in Y is f. closed in X.
(d) The inverse image of every f.δ open set in Y is f. open in X.
(e) For each f. point P in X and each f.δ.nbd N of $f(P)$, $f^{-1}(N)$ is a $q-nbd$ of P.

Proof.

(a) $\Rightarrow$ (b) : Let P be a f. point, $P \in \overline{U}$ and let V be a f. open $q-nbd$ of $f(P)$. Then there exists a f. open $q-nbd$ W of P such that $f(W) \leq (\overline{V})^\circ$, but $P \in \overline{U} \Rightarrow WqU \Rightarrow f(W)qf(U) \Rightarrow (\overline{V})^\circ qf(U) \Rightarrow f(P) \in [f(U)]_\delta \Rightarrow P \in f^{-1}[f(U)]_\delta \Rightarrow \overline{U} \leq f^{-1}[f(U)]_\delta$. Therefore $f(\overline{U}) \leq [f(U)]_\delta$.

(b) $\Rightarrow$ (c) Let F be a f.δ-closed set in Y. Then $[F]_\delta = F$ and by (b) $f(\overline{f^{-1}(F)}) \leq [f(f^{-1}(F))] \leq (F)_\delta = F$ which implies $\overline{f^{-1}(F)} \leq f^{-1}(F)$. So $f^{-1}(F)$ is f. closed in X.

(c) $\Rightarrow$ (d) Let M be a f.δ. open in Y, then M' is f. δ-closed. (c) implies $f^{-1}(M')$ in f. closed. But $f^{-1}(M') = (f^{-1}(M))'$. So $f^{-1}(M)$ is f. open.

(d) $\Rightarrow$ (e) Clearly.

(e) $\Rightarrow$ (a) Let P be a f. point in X and V any f. open $q-nbd$ of $f(P)$ in Y, then $(\overline{V})^\circ$ is a $\delta-nbd$ of $f(P)$. (e) implies, $f^{-1}(\overline{V})^\circ$ is a $q.nbd$ of P. Hence there exists a f. open set U with $pqU \leq f^{-1}(\overline{V})^\circ$. So $f(U) \leq (\overline{V})^\circ$. Therefore f is f. almost continuous.

Definition 4.6. *[7] A fts is said to be:*

(a) f. almost regular if for each f. regularly open set V and each f. point pqV, there exists a f. regularly open set U such that $pqU \leq \overline{U} \leq V$.

(b) f. semi regular if for each f. open set V and each f. point pqV, there exists a f. open set U such that $PqU \leq \overline{U}^\circ \leq V$.

Theorem 4.7. *A fts X is f. almost regular if and only if $[A]\delta = [A]_\theta$ for each f. set A in X.*

Proof. Let P be f. point with $P \in [A]_\delta$, there exists a f. regularly open $q-nbd$ V of P with $V \not{q} A$. But X is f. regular, then there exists a f-regularly open set U with $PqU \leq \overline{U} \leq V$ which implies $V \not{q} A \Rightarrow A < V' \leq (\overline{U})' \Rightarrow \overline{U} \not{q} A \Rightarrow P \in [A]_\theta$. So $[A]_\theta \leq [A]_\delta$ but $[A]_\delta \leq [A]_\theta$, then $[A]_\theta 0 = [A]_\delta$.

Conversely, let V be a f. regularly open set and P be a f. point with PqV. Since V' is fuzzy δ-closed in X, $P \in V' = [V']_\delta = [V']_\theta$, then there exists a f. open $q-nbd$ U of P such that $\overline{U} \not{q} V', \overline{U}^\circ \leq V^\circ \leq V$. Put $G = = (\overline{U})^\circ$, then G is f. regularly open set in X such that $PqG \leq \overline{G} \leq \overline{U} \leq V$. Therefore X is f. almost regular.

Lemma 4.8. *$\overline{A} = [A]_\delta$ for any f. semi- open set A in a fts X.*

Proof. Let P be a f. point in X with $P \in [A]_\delta$ such that $P \in \overline{A}$, then there exists a f. open $q-nbd$ V of P with $V \not{q} A$. Since A is f. semi-open, then there exists a f. open set G such that $G \leq A \leq \overline{G}$. Then $V \leq A' \leq G' \Rightarrow \overline{V} \leq \overline{G'} = G' \Rightarrow (\overline{V})^\circ \leq (G')^\circ \leq G' \Rightarrow G \leq ((\overline{V})^\circ)'$. So

Theorem 4.9. *A fts X is f. almost if and only if $\overline{A} = [A]_\delta$ for any f. semi-open set A in X.*

Proof. Clearly from Theorem 4.7. and Lemma 4.8.

Theorem 4.10. *A fts X is f. semi-regular if and only if $\overline{A} = [A]_\delta$.*

Proof. Let P be a fuzzy point with $P \in \overline{A}$, there exists a fuzzy open $q-nbd$ V of P such that VqA. Since X is f-semi-regular, there exists a fuzzy open set U such that $PqU \leq \overline{U}^\circ \leq V$, $V \not{q} A \Rightarrow (\overline{U})^\circ \leq V \leq A' \Rightarrow (\overline{U})^\circ \not{q} A \Rightarrow P \in [A]_\delta \Rightarrow [A]_\delta \leq \overline{A}$, but $\overline{A} \leq [A]_\delta$. Then $\overline{A} = [A]_\delta$.

Conversely, Let P be a fuzzy point and V be a fuzzy open set with PqV. Since $P \in \overline{V'} = [V']_\delta$, then there exists a fuzzy open $q-nbd$ U of P such that $(\overline{V})^\circ \not{q} V'$. Then $(\overline{U})^\circ \leq V$. Therefore X is f. semi-regular.

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Received : 22.IX.1995

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Tomul XLII, Supliment, s.I.a, Matematică, 1996

THE INTEGRABILITY OF ALMOST 2 - π STRUCTURES IN THE BUNDLE OF ACCELERATIONS

BY

MONICA PURCARU AND VICTOR BLĂNUŢĂ

The generalization of almost complex and almost product structures have been made by C.J.Hsu which introduce the notion of $r-\pi$ structure, [5].

In the present paper we consider the group G of transformation of N-linear connections compatible with an almost $2-\pi$ structure given on the bundle of accelerations and prove that it has forty-three invariants, which are d-tensor fields. By means of these invariants, we get the characterizations of the integrability of type I, II, II, IV, or V for the almost $2-\pi$ d-structures. All this integrability theory is based only on geometry of the bundle of accelerations (cf.with Gh.Atanasiu [1] and R.Miron and Gh.Atanasiu [3], [6–9]).

The authors wishes to express their sincere gratitude to Professor Dr.Gh.Atanasiu for the invaluable suggestions and encouragement.

1. Preliminaries. [1]: Let M be a real C^∞-manifold with n-dimensions $(n = 2n')$, let OSC^2M be the osculator bundle of second order (or the bundle of accelerations) and let $N(\underset{(1)}{N}, \underset{(2)}{N})$ be a nonlinear connection on OSC^2M. We denote by $(x^i, y^{(1)i}, y^{(2)i})$ the canonical coordinates on OSC^2M. Then, an adapted basis to the direct sum of the vector spaces

$$T_u(OSC^2M) = N_u(OSC^2M) \oplus V_u(OSC^2M), \forall u = (x, y^{(1)}, y^{(2)}) \in OSC^2M \tag{1.1}$$

is given by $\{\frac{\delta}{\delta x^i}, \frac{\delta}{\delta y^{(1)i}}, \frac{\partial}{\partial y^{(2)i}}\}$, where

Communicated at the 25-th National Conference of Geometry and Topology, September 18-23, 1995, Iaşi, România