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SEMI-INVARIANT SUBMANIFOLDS OF A SASAKIAN MANIFOLD

BY

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0. Introduction. The differential geometry of several classes of submanifolds in a Sasakian manifold has been investigated by many people. Interesting results have been obtained by G. Ludden, M. Okumura, K. Yano [3], K. Yano and M. Kon [6] in studying anti-invariant submanifolds of Sasakian manifolds and by M. Kon [4] in studying invariant submanifolds. Moreover, K. Yano and M. Kon have studied generic submanifolds of odd dimensional spheres which are generalizations of anti-invariant submanifolds [7].

Recently in [1] one of the present authors introduced the notion of CR-submanifold in a Kaehler manifold. CR-submanifolds appear as a natural generalization of both complex submanifolds and totally real submanifolds.

The purpose of the present paper is to introduce and study a similar class of submanifolds in a Sasakian manifold. In §1 we introduce the notion of semi-invariant submanifold of a Sasakian manifold and give fundamental formulas for later use. The integrability of all distributions involved in the definition of a semi-invariant submanifold is studied in §2. Finally, we give a complete characterization for totally umbilical semi-invariant submanifolds of a Sasakian manifold.

1. Semi-invariant submanifolds of a Sasakian manifold. First, we recall the definition and some properties of a Sasakian manifold.

Let $\tilde{M}$ be a $(2n+1)$-dimensional differentiable manifold of class C^∞ and F, η, ζ be a tensor field of type (1.1), a 1-form and a vector filed on $\tilde{M}$ respectively, which satisfy

$$F^2=-I+\eta\otimes\xi, \quad F\xi=0, \quad \eta(FX)=0, \quad \eta(\xi)=1 \tag{1.1}$$

for any vector field X on $\tilde{M}$, where I is the identity tensor on $\tilde{M}$.

Then $\tilde{M}$ is called an almost contact manifold and (F, η, ξ) an almost contact structure on $\tilde{M}$.

Now, suppose on $\tilde{M}$ is given a Riemannian metric tensor field g which satisfies the equations

$$g(FX, FY)=g(X, Y)-\eta(X)\eta(Y), \tag{1.2}$$

$$\eta(X)=g(X, \xi), \tag{1.3}$$

for any vector fields X, Y tangent to $\tilde{M}$. Then $\tilde{M}$ is called an almost contact metric manifold. When we have

$$d\eta(X, Y)=g(X, F, Y) \tag{1.4}$$

for any vector fields X, Y tangent to $\tilde{M}$, we say that $\tilde{M}$ is a contact metric manifold. If the Nijenhuis tensor field of F verifies

$$N_F+2d\eta\otimes\xi=0, \tag{1.5}$$

then $\tilde{M}$ is called a normal almost contact manifold. If the contact metric manifold $\tilde{M}$ is normal we say that $\tilde{M}$ ia a Sasakian manifold and (F, η, ξ, g) is called a Sasakian structure on $\tilde{M}$. It is known ([2], p. 73) that an almost contact metric structure is Sasakian if and only if

$$(\tilde{\nabla}_X F)Y=g(X, Y)\xi-\eta(Y)X, \tag{1.6}$$

where $\tilde{\nabla}$ denotes the Riemannian connection of $\tilde{M}$. Also, on a Sasakian manifold we have

$$\tilde{\nabla}_X\xi=-FX \tag{1.7}$$

for all vector field X tangent to $\tilde{M}$.

As we have mentioned in the introduction, anti-invariant submanifolds and invariant submanifolds have been intensively studied and interesting results have been obtained. Nevertheless, it arises the question if there exist some other classes of submanifolds as generalizations of both invariant submanifolds [4] and anti-invariant submanifolds [3], [6]. We shall introduce here such a class of submanifolds in an almost contact metric manifold.

Let M be an m-dimensional Riemannian manifold isometrically immersed in an almost contact metric manifold $\tilde{M}$ such that the structure vector field ξ of $\tilde{M}$ is tangent to M. Denote by TM and $TM^\perp$ respectively the tangent bundle of M and the normal bundle to M. The 1-dimensional distribution on M defined by ξ is denoted by $\{\xi\}$. Suppose there exist on M two differentiable orthogonal distributions D and $D^\perp$ such that the following conditions are fulfilled

(i) $TM=D\oplus D^\perp\oplus\{\xi\}$,

(ii) the distribution D is invariant by F, that is, $F(D_x)=D_x$ for each $x\in M$,

(iii) the distribution $D^\perp$ is anti-invariant by F, that is, $F(D_x^\perp)\subset T_xM^\perp$ for each $x\in M$.

Definition. *A submanifold M of an almost contact metric manifold $\tilde{M}$ endowed with the pair of distributions $(D, D^\perp)$ satisfying conditions (i), (ii) and* (iii) *is called a semi-invariant submanifold of $\tilde{M}$.*

The distributions D and $D^\perp$ are called respectively the *invariant distribution* and the *anti-invariant distribution* of M.

Suppose the dimension of D_x(resp. $D_x^\perp$) be $2p$ (resp. q). Then it is easily seen that when $p=0$ (resp. $q=0$) the semi-invariant submanifold M becomes an anti-invariant submanifold (resp. an invariant submanifold). An example of semi-invariant submanifold in an almost contact metric manifold is given by

Proposition 1.1. *Let M be a hypersurface of an almost contact metric manifold $\tilde{M}$ such that ξ is tangent to M. Then, M is a semi-invariant submanifold of $\tilde{M}$.*

Proof. We define on M the distribution $D^\perp$ by $D^\perp = F(TM^\perp)$. The complementary orthogonal distribution to $D^\perp \oplus \{\xi\}$ in TM is denoted by D. Thus, M endowed with the pair of distributions $(D, D^\perp)$ becomes a semi-invariant submanifold.

Now, suppose R^{2n+1} endowed with its natural Sasakian structure (F, η, ξ, g) ([2], p. 81). If (x^i, y^i, z) is a system of coordinates on R^{2n+1}, then ξ is just $\frac{\partial}{\partial z}$. It is not difficult to see that each hypersurface M of R^{2n+1} which is given locally by

$$x^i = x^i(t^2, \ldots, t^{2n}), \quad y^i = y^i(t^2, \ldots, t^{2n}), \quad z = t^1 + z(t^2, \ldots, t^{2n})$$

is an example of semi-invariant submanifold. In fact, ξ is tangent to M and the assertion follows from Proposition 1.1.

By $\Gamma(TM)$ we mean the module of all differentiable vector fields on M. We say that $X \in \Gamma(TM)$ is a section of D (resp. of $D^\perp$) if $X_x \in D_x$ (resp. $X_x \in D^\perp_x$) for each $x \in M$. Denote by $\Gamma(D)$ (resp. $\Gamma(D^\perp)$) the module of differentiable sections of D (resp. $D^\perp$).

If $\tilde{M}$ is endowed with an almost contact metric structure (F, η, ξ, g) then it is known that $\tilde{M} \times R$ carries an almost Hermitian structure J given by

$$J(X, f\, d/dt) = (FX - f\xi, \ \eta(X)\, d/dt) \,. \tag{1.8}$$

Suppose M is a Riemannian submanifold of $\tilde{M}$ such that ξ is tangent to M. Then, we have

Theorem 1.1. *The submanifold M is a semi-invariant submanifold of $\tilde{M}$ if and only if M is a CR-submanifold of $\tilde{M} \times R$, with D as holomorphic distribution and $\{\xi\} \oplus D^\perp$ as totally real distribution.*

Proof. From (1.8) we have

(1.9) $J(X, 0) = (FX, 0)$ for each vector field X tangent to M such that $\eta(X) = 0$ and

$$J(\xi, 0) = (0, d/dt) \,. \tag{1.10}$$

Now, suppose M is a semi-invariant submanifold of $\tilde{M}$. Then from (1.9) taking into account that D is invariant by F, it follows $J(X, 0) \in \Gamma(D)$, for all $X \in \Gamma(D)$. Also, from (1.9) it follows that $J(Y, 0)$ belongs to the normal bundle to M in $\tilde{M} \times R$ for each $Y \in \Gamma(D^\perp)$. Then, by using (1.10) we obtain that $\{\xi\} \oplus D^\perp$ is going in $TM^\perp$ by J. Conversely, if M is a CR-submanifold of $\tilde{M} \times R$, by (1.9) we obtain that $FX \in \Gamma(D)$ for each $X \in \Gamma(D)$ and $FY \in \Gamma(TM^\perp)$ for each $Y \in \Gamma(D^\perp)$. Hence M is a semi-invariant submanifold of $\tilde{M}$.

In this paper we are dealing only with semi-invariant submanifolds of a Sasakian manifold.

Let M be an m-dimensional Riemannian manifold isometrically immersed in a Sasakian manifold $\tilde{M}$. Then, we have

Proposition 1.2. (K. Yano and M. Kon [6]). *If the structure vector field ξ is normal to M, then M is an anti-invariant submanifold in $\tilde{M}$ and $m \leq n$.*

Let M be a semi-invariant submanifold of a Sasakian manifold $\tilde{M}$. We shall denote by the same symbol g both metrics on M and $\tilde{M}$. The projection morphisms of TM to D and $D^{\perp}$ are denoted respectively by P and Q. Then we have

$$X = PX + QX + \eta(X)\xi \tag{1.11}$$

for all vector fields X tangent to M.

If N is a vector field in the normal bundle $TM^{\perp}$ we put

$$FN = BN + CN \tag{1.12}$$

where $BN \in \Gamma(D^{\perp})$ and $CN \in \Gamma(TM^{\perp})$. Substituting N by FQX in (1.12) and taking into account (1.1), we obtain:

$$BFQX + QX = 0\,; \quad CFQX = 0 \text{ for all } X \in \Gamma(TM). \tag{1.13}$$

Applying F to (1.12) we have

$$C^2N + N + FBN = 0\,; \quad BCN = 0. \tag{1.14}$$

From (1.13) and (1.14), it follows $C^3 + C = 0$ that is, we have

Proposition 1.3. *On the normal bundle of each semi-invariant submanifold of a Sasakian manifold there exist an f-structure* C [5].

Let $\tilde{\nabla}$ and ∇ be the Levi-Civita connections on $\tilde{M}$ and respectively M. Denote by h the second fundamental form of M. Then, the equations of Gauss and Weingarten are given by

$$\tilde{\nabla}_X Y = \nabla_X Y + h(X, Y) \quad \text{and respectively} \tag{1.15}$$

$$\tilde{\nabla}_X N = -A_N X + \nabla^{\perp}_X N \tag{1.16}$$

for all $X, Y \in \Gamma(TM)$, $N \in \Gamma(TM^{\perp})$, where $\nabla^{\perp}$ is the normal connection induced by $\tilde{\nabla}$ on $TM^{\perp}$ and A_N is the fundamental tensor of Weingarten with respect to the normal section N.

It is known that we have

$$g(h(X, Y), N) = g(A_N X, Y). \tag{1.17}$$

Lemma 1.1. *Let M be a semi-invariant submanifold of a Sasakian manifold $\tilde{M}$. Then, we have*

$$P\nabla_X FPY - PA_{FQY}X = FP\nabla_X Y - \eta(Y)PX, \tag{1.18}$$

$$Q\nabla_X FPY - QA_{FQY}X = Bh(X, Y) - \eta(Y)QX, \tag{1.19}$$

$$\eta(\nabla_X FPY - A_{FQY}X) = g(FX, FY), \tag{1.20}$$

$$h(X, FPY) + \nabla^{\perp}_X FQY = Ch(X, Y) + FQ\nabla_X Y, \tag{1.21}$$

for all vector fields X, Y tangent to M.

Proof. Substituing Y by FY in (1.15) and using (1.6) we obtain

$$\nabla_X FPY + h(X, FPY) - A_{FQY}X + \nabla^{\perp}_X FQY = F\tilde{\nabla}_X Y + g(X, Y)\xi - \eta(Y)X. \tag{1.22}$$

By a direct computation using (1.11), (1.12), (1.15), (1.16) and (1.2) from (1.22) follows (1.18)—(1.21).

Lemma 1.2. *Let M be a semi-invariant submanifold of a Sasakian manifold $\tilde{M}$. Then, we have*

$$FPA_N X = PA_{CN}X - P\nabla_X BN, \tag{1.23}$$

$$B\nabla^{\perp}_X N = Q\nabla_X BN - QA_{CN}X, \tag{1.24}$$

$$h(X, BN) + \nabla^{\perp}_X CN + FQA_N X = C\nabla^{\perp}_X N, \tag{1.25}$$

$$\eta(\nabla_X BN - A_{CN}X) = 0, \tag{1.26}$$

for all vector fields X tangent to M and normal section N.

Proof. Differentiating (1.12) with respect to $X \in \Gamma(TM)$ and taking into account (1.6), (1.15) and (1.16), we obtain:

$$F\tilde{\nabla}_X N = \nabla_X BN + h(X, BN) - A_{CN}X + \nabla^{\perp}_X CN. \tag{1.27}$$

Then, (1.23)—(1.26) follows from (1.27) by using (1.11), (1.12) and (1.16).

Lemma 1.3. *Let M be a semi-invariant submanifold of a Sasakian manifold $\tilde{M}$. Then, we have*

$$h(X, \xi) = 0, \tag{1.28}$$

$$\nabla_X \xi = -FX, \text{ for all } X \in \Gamma(D) \text{ and} \tag{1.29}$$

$$h(Y, \xi) = -FY, \tag{1.30}$$

$$\nabla_Y \xi = 0, \text{ for all } Y \in \Gamma(D^{\perp}). \tag{1.31}$$

Proof. By using (1.7) and (1.15) we obtain

$$\nabla_Z \xi + h(Z, \xi) = -FZ. \tag{1.32}$$

for each vector field Z tangent to M. Then, substituiting Z in (1.32) by $X \in \Gamma(D)$ (resp. $Y \in \Gamma(D^{\perp})$), it follows (1.28) and (1.29) (resp. (1.30) and (1.31)).

The 2-form Φ of the Sasakian manifold $\tilde{M}$ is defined by

$$\Phi(X, Y) = g(X, FY). \tag{1.33}$$

Then, we have

$$g(X, FY) + g(FX, Y) = 0 \quad X, Y\Gamma \in (T\tilde{M}). \tag{1.34}$$

for all

2. Integrability of distributions on a semi-invariant submanifold. As we have seen, in the definition of a semi-invariant submanifold are involved two distributions D and $D^{\perp}$. The main purpose of this paragraph is to study the integrability of distributions D, $D^{\perp}$, $D \oplus D^{\perp}$, $D \oplus \{\xi\}$, $D^{\perp} \oplus \{\xi\}$. It is well known that the contact distribution on a Sasakian manifold is never involutive. However we shall see in this paragraph that some of the above distributions are always involutive on M.

Lemma 2.1. *Let M be a semi-invariant submanifold of a Sasakian manifold $\tilde{M}$. Then, we have*

$$A_{FX}Y = A_{FY}X \text{ for all } X, Y \in \Gamma(D^{\perp}). \tag{2.1}$$

Proof. By using (1.2), (1.15), (1.17) and (1.34), we obtain

$$g(A_{FX}Y, Z) = -g(\tilde{\nabla}_Z Y, FX) \tag{2.2}$$

for each vector field Z tangent to M. Now, by using (1.6) and (1.16) the second part of (2.2) becomes

$$g(\tilde{\nabla}_Z Y, FX) = -g(F\tilde{\nabla}_Z Y, X) = -g(\tilde{\nabla}_Z FY, X) = g(A_{FY}X, Z). \tag{2.3}$$

Thus, (2.1) follows from (2.2) and (2.3).

Lemma 2.2. *Let M be a semi-invariant submanifold of a Sasakian manifold $\tilde{M}$. Then, we have*

$$[X, Y] \in \Gamma(D \oplus D^{\perp}) \text{ for all } X, Y \in \Gamma(D^{\perp}). \tag{2.4}$$

Proof. Since $\tilde{\nabla}$ is the Levi-Civita connection on $\tilde{M}$, by (1.7) and (1.34) we obtain $g([X, Y], \xi) = g(X, \tilde{\nabla}_Y \xi) - g(Y, \tilde{\nabla}_X \xi) = g(Y, FX) - g(X, FY) = 0$, for all $X, Y \in \Gamma(D^{\perp})$, which proves the assertion of the lemma. Now, we can state

Theorem 2.1. *Let M be a semi-invariant submanifold of a Sasakian manifold $\tilde{M}$. Then, the anti-invariant distribution is involutive.*

Proof. We take $X, Y \in \Gamma(D^{\perp})$ and by using (1.15), (1.16) and (1.6), we obtain

$$F\nabla_Y X + Fh(X, Y) + g(X, Y)\xi = -A_{FX}Y + \nabla^{\perp}_Y FX. \tag{2.5}$$

Substituing Y by X in (2.5) and substracting the obtained relation from (2.5), we have

$$F[Y, X] = A_{FY}X - A_{FX}Y + \nabla^{\perp}_Y FX - \nabla^{\perp}_X FY. \tag{2.6}$$

Then, by (1.1) and lemmas 2.1 and 2.2 we see that (2.6) becomes

$$[X, Y] = F(\nabla^{\perp}_Y FX - \nabla^{\perp}_X FY). \tag{2.7}$$

Now, we denote by $\overline{D}$ the complementary orthogonal subbundle of $F(D^{\perp})$ in $TM^{\perp}$. Then we take an arbitrary normal section $N \in \Gamma(\overline{D})$ and by using (1.6), (1.16) and (1.34) we obtain

$$g(N, \nabla^{\perp}_Y FX) = -g(A_{FN}Y, X). \tag{2.8}$$

Substituting Y by X in (2.8) and substracting the obtained relation from (2.8), we have

$$g(N, \nabla^{\perp}_Y FX - \nabla^{\perp}_X FY) = 0, \tag{2.9}$$

since A_{FN} is a symmetrical operator with respect to g.
Hence $\nabla^{\perp}_Y FX - \nabla^{\perp}_X FY \in \Gamma(FD^{\perp})$, which together with (2.7) implies $[X, Y] \in \Gamma(D^{\perp})$. Thus the proof is complete.

Theorem 2.2. *Let M be a semi-invariant submanifold of a Sasakian manifold $\tilde{M}$. Then, the distribution $D^{\perp} \oplus \{\xi\}$ is involutive.*

Proof. For any $X \in \Gamma(D^{\perp})$ and $Y \in \Gamma(D)$ we have

$$g([X, \xi], Y) = g(\tilde{\nabla}_X \xi - \tilde{\nabla}_\xi X, Y) = g(X, \tilde{\nabla}_\xi Y). \tag{2.10}$$

Now we take $Z \in \Gamma(D)$ such that $FZ = Y$ and by using (1.6), (1.28) and (2.10), we obtain

(2.11) $$g([X, \xi], Y)=g(X, \tilde{\nabla}_\xi FZ)=g(X, F\tilde{\nabla}_\xi Z)=g(X, F\nabla_\xi Z)=0.$$

Consequently, $[X, \xi]\in\Gamma(D^\perp\oplus\{\xi\})$ which together with Theorem 2.1 implies that $D^\perp\oplus\{\xi\}$ is involutive.

Definition. *A semi-invariant submanifold M of a Sasakian manifold $\tilde{M}$ is called a proper semi-invariant submanifold, if it is neither an invariant submanifold* (*i.e.* $\dim D_x^\perp=0$ *for each* $x\in M$) *nor an anti-invariant submanifold* (i.e. $\dim D_x=0$ *for each* $x\in M$).

Theorem 2.3. *Let M be a proper semi-invariant submanifold of a Sasakian manifold $\tilde{M}$. Then, the invariant distribution is never involutive.*

Proof. By using (1.7) and (1.33) we have

(2.12) $$g([X, Y], \xi)=g(\tilde{\nabla}_X Y-\tilde{\nabla}_Y X, \xi)=g(X, \tilde{\nabla}_Y\xi)-g(Y, \tilde{\nabla}_X\xi)=$$
$$=g(Y, FX)-g(X, FY)=2\Phi(Y, X), \text{ for all } X, Y\in\Gamma(D).$$

But the 2-form Φ does not vanish on $D\times D$ since for an unit vector field $X\in\Gamma(D)$, we take $Y=FX$ and obtain $\Phi(Y, X)=1$. Hence, by (2.12) the invariant distribution is never involutive. From this theorem we have

Corollary 2.1. *Let M be a proper semi-invariant submanifold of a Sasakian manifold $\tilde{M}$. Then, the distribution $D\oplus D^\perp$ is never involutive.*

Now, we need two lemmas in order to get a necessary and sufficient condition for the integrability of the distribution $D\oplus\{\xi\}$.

Lemma 2.3. *Let M be a semi-invariant submanifold of a Sasakian manifold $\tilde{M}$. Then, we have*

(2.13) $$g(h(X, Y), FZ)=g(\Delta_X Z, FY)$$

for all vector fields $X\in\Gamma(TM), Y\in\Gamma(D)$ *and* $Z\in\Gamma(D^\perp)$.

Proof. By using (1.16) and (1.17), we obtain

(2.14) $$g(h(X, Y), FZ)=g(A_{FZ}X, Y)=g(\nabla_X^\perp FZ-\tilde{\nabla}_X FZ, Y)=-g(\tilde{\nabla}_X FZ, Y).$$

On the other hand, by using (1.15) and (1.34), we have

(2.15) $$g(\nabla_X Z, FY)=g(\tilde{\nabla}_X Z, FY)=-g(F\tilde{\nabla}_X Z, Y)$$

for all $X\in\Gamma(TM)$, $Y\in\Gamma(D)$, $Z\in\Gamma(D^\perp)$. From (2.14) and (2.15) it follows (2.13) by means of (1.6).

Lemma 2.4. *Let M be a semi-invariant submanifold of a Sasakian manifold $\tilde{M}$. Then,* $[X, \xi]\in\Gamma(D\oplus\{\xi\})$ *for each vector field* $X\in\Gamma(D)$.

Proof. By using (1.7) and (1.28) we have

(2.16) $$g([X, \xi], Y)=-g(\tilde{\nabla}_\xi X, Y)=-g(\nabla_\xi X, Y)=g(X, \nabla_\xi Y)$$

for each $Y\in\Gamma(D^\perp)$ and $X\in\Gamma(D)$.
Now, we take $Z\in\Gamma(D)$ such that $X=FZ$ and by using lemma 2.3 and (1.28) obtain

(2.17) $$g(X, \nabla_\xi Y)=g(FZ, \nabla_\xi Y)=g(h(\xi, Z), FY)=0$$

for all $X\in\Gamma(D)$ and $Y\in\Gamma(D^\perp)$. Thus, by (2.16) and (2.17), it follows the assertion of the lemma.

Theorem 2.4. *Let M be a semi-invariant submanifold of a Sasakian manifold $\tilde{M}$. Then, the distribution $D\oplus\{\xi\}$ is involutive if and only if we have*

$$h(X, FY)=h(FX, Y) \text{ for all } X, Y\in\Gamma(D). \tag{2.18}$$

Proof. From (1.21) we obtain $h(X, FY)=Ch(X, Y)+FQ\nabla_X Y$, for all $X, Y\in\Gamma(D)$. Since h is a symmetrical morphism of vector bundles, it follows $h(X, FY)-h(Y, FX)=FQ([X, Y])$. In this way, $[X, Y]\in\Gamma(D\oplus\{\xi\})$ if and only if (2.18) is satisfied. Taking into account lemma 2.4, the proof is complete.

Finally, we establish a result on totally umbilical semi-invariant submanifolds. It is known that M is a totally umbilical submanifold if its second fundamental form satisfies

$$h(X, Y)=g(X, Y)H \tag{2.19}$$

for all $X, Y\in\Gamma(TM)$, where H is the mean curvature vector of M. Then, from (1.30) and (2.19), it follows

Theorem 2.5. *Let M be a tota ly umbilical semi-invariant submanifold of a Sasakian manifold M. Then, $\tilde{M}$ is an invariant submanifold.*

Corollary 2.2. *There does not exist totally umbilical proper semi-invariant submanifolds of a Sasakian manifold.*

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