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Abstract

Various types of statistical structures are introduced in almost paracontact geometry and characterizations for them are given. A large class of examples is provided by an arbitrary 1-form by deforming the Levi-Civita connection by a mixed projective and dual-projective transformation. The particular case of the paracontact form η from the almost paracontact structure leads to simpler conditions in terms of the Levi-Civita connection.

Keywords (separated by '-'): Statistical manifold - Almost paracontact structure - (Dual)projective equivalence

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Statistical Structures in Almost Paracontact Geometry

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Abstract
Various types of statistical structures are introduced in almost paracontact geometry and characterizations for them are given. A large class of examples is provided by an arbitrary 1-form by deforming the Levi-Civita connection by a mixed projective and dual-projective transformation. The particular case of the paracontact form η from the almost paracontact structure leads to simpler conditions in terms of the Levi-Civita connection.

Keywords Statistical manifold · Almost paracontact structure · (Dual)projective equivalence

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1 Introduction

Information geometry has emerged from investigating the geometrical structure of a class of probability distributions depending on various parameters, and has been applied successfully to various areas including statistical inference, control system theory, and multi-terminal information theory, [1]. A main notion of this theory is that of statistical manifold which is a triple (M, g, ∇) with g a Riemannian metric on the manifold M and ∇ a symmetric linear connection for which $C := \nabla g$ is totally symmetric, i.e., the Codazzi equation holds:

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$$(\nabla_X g)(Y, Z) = (\nabla_Y g)(Z, X) \; (= (\nabla_Z g)(X, Y)). \tag{1.1}$$

In this framework, there exists another torsion-free linear connection ∇^* defined by the relation:

$$X(g(Y, Z)) = g(\nabla_X Y, Z) + g(Y, \nabla^*_X Z) \tag{1.2}$$

for any X, Y, and $Z \in \mathfrak{X}(M)$, called the conjugate (or the dual) connection of ∇ with respect to g. From the direct consequence $\nabla^* g = -\nabla g$, it results that (M, ∇^*, g) is a statistical manifold, too. The conjugate of the conjugate connection of ∇ coincides with ∇, i.e., $(\nabla^*)^* = \nabla$. For other applications and generalizations of these concepts, see [16,19], and for the Bochner technique in this setting, see [18].

Until now, the statistical manifolds were studied in a few particular geometries. Therefore, in [20], K. Takano studied the statistical manifolds in almost contact geometry, while statistical structures in almost complex geometry were studied in [21] by the same author and in [12] by Furuhata. In addition, symplectic structures on statistical manifolds are considered in [17]. The present authors deal with statistical structures in golden geometry in [5].

In this paper, we shall study the interplays of a statistical structure on an almost paracontact structure following the studies above as well as [2,3]. A strong motivation for such a study comes from the appearance of paracontact geometry in some physical theories, e.g., para-Sasakian geometry in thermodynamic fluctuation theory conform [6]. More precisely, in the framework of almost paracontact geometry, we introduce various types of statistical structures ∇ according to the behavior of some remarkable tensors associated with the almost paracontact structure, namely the structural endomorphism and the fundamental form, with respect to ∇. Being a study on some special connections in almost paracontact geometry, this work is a natural continuation of [4].

A large class of examples is produced with a differential 1-form λ by deforming the Levi-Civita connection ∇^g of the almost paracontact structure according to a combination of projective and dual-projective transformations. In particular, we search, for examples, using the paracontact form η of the given almost paracontact structure.

2 Paracontact-Statistical Manifolds

An almost paracontact geometry appears as a natural counter-part of the almost contact geometry in [15] and some important studies are [8,9,22]. Let M be a $(2n+1)$-dimensional smooth manifold, φ a tensor field of $(1,1)$-type called the structural endomorphism, ξ a vector field called the characteristic vector field, η a 1-form called the paracontact form, and g a pseudo-Riemannian metric on M of signature $(n+1, n)$. We say that (φ, ξ, η, g) defines an almost paracontact metric structure on M if [22, p. 38], [8]:

1. $\varphi(\xi) = 0$, $\eta \circ \varphi = 0$;
2. $\eta(\xi) = 1$, $\varphi^2 = I - \eta \otimes \xi$;
3. φ induces on the $2n$-dimensional distribution $\mathcal{D} := \ker \eta$ an almost paracomplex structure P, i.e., $P^2 = 1$, and the eigensubbundles T^+, T^-, corresponding to

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the eigenvalues 1, -1 of P, respectively, have equal dimension n; hence, $\mathcal{D} = T^+ \oplus T^-$;

4. $g(\varphi\cdot, \varphi\cdot) = -g + \eta \otimes \eta$.

For a list of examples of almost paracontact metric structures, see [10, p. 666], [11, p. 569], and [14, p. 84]. From the definition, it follows that η is the g-dual of ξ, i.e., $\eta(X) = g(X, \xi)$ for any $X \in \Gamma(TM) = \mathfrak{X}(M)$; ξ is an unitary vector field:

$$g(\xi, \xi) = 1, \tag{2.1}$$

and φ is a g-skew-symmetric operator:

$$g(\varphi X, Y) = -g(X, \varphi Y). \tag{2.2}$$

The tensor field:

$$\omega(X, Y) := g(X, \varphi Y) \tag{2.3}$$

is skew-symmetric and:

$$\begin{cases} \omega(\varphi X, Y) = -\omega(X, \varphi Y) \\ \omega(\varphi X, \varphi Y) = -\omega(X, Y). \end{cases} \tag{2.4}$$

ω is called the fundamental form. Remark that the canonical distribution $\mathcal{D}$ is φ-invariant, since $\mathcal{D} = Im\varphi$: if $X \in \mathfrak{X}(M)$ has the decomposition $X = X^+ + X^- + \eta(X)\xi$ with $X^* \in T^*$ (with $* \in \{+, -\}$) then $\varphi X = X^+ - X^-$. Moreover, ξ is orthogonal to $\mathcal{D}$, and therefore, the tangent bundle splits orthogonally:

$$TM = \mathcal{D} \oplus \langle \xi \rangle. \tag{2.5}$$

We introduce now the framework of this paper.

Definition 2.1 The data $(M, \nabla, \varphi, \xi, \eta, g)$ are an almost paracontact-statistical manifold if (M, ∇, g) is a statistical manifold and (φ, ξ, η, g) is an almost paracontact metric structure. Moreover, if $\nabla\varphi = 0$, then we drop the adjective almost.

Let us point out that $(\nabla_X \varphi)(\xi) = -\varphi(\nabla_X \xi)$, and then, in the paracontact-statistical case, we have for every $X \in \mathfrak{X}(M)$ that $\nabla_X \xi = \eta(\nabla_X \xi)\xi$. Concerning the conjugate connection of an almost paracontact-statistical manifold, we can state the following result.

Proposition 2.2 *Let $(M, \nabla, \varphi, \xi, \eta, g)$ be an almost paracontact-statistical manifold and ∇^* the conjugate connection of ∇. Then:*

(i) *$(M, \nabla^*, \varphi, \xi, \eta, g)$ is an almost paracontact-statistical manifold too.*
(ii) *$(M, \nabla, \varphi, \xi, \eta, g)$ is a paracontact-statistical manifold if and only if so is $(M, \nabla^*, \varphi, \xi, \eta, g)$.*
(iii) *η is ∇-parallel if and only if ξ is ∇^*-parallel.*
(iv) *$\nabla^*_X \xi + \nabla_X \xi \in \Gamma(\mathcal{D})$ for any $X \in \mathfrak{X}(M)$, and then, $(\nabla_X \eta + \nabla^*_X \eta)\xi = 0$.*

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Proof i) It results directly from Definition 2.1. ii) A straightforward calculus gives:

$$g((\nabla_X\varphi)Y, Z) + g(Y, (\nabla^*_X\varphi)Z) = 0 \tag{2.6}$$

for any X, Y, and $Z \in \mathfrak{X}(M)$ which have the required consequence. iii) We also have:

$$(\nabla_X\eta)Y = g(\nabla^*_X\xi, Y), \tag{2.7}$$

for any $X, Y \in \mathfrak{X}(M)$ which gives the conclusion. iv) The first part is a direct computation, while the second follows from the first part by adding to 2.7 the dual relation:

$$(\nabla^*_X\eta)Y = g(\nabla_X\xi, Y). \tag{2.8}$$

□

Corollary 2.3 *Let $(M, \nabla, \varphi, \xi, \eta, g)$ be an almost paracontact-statistical manifold and ∇^* the conjugate connection of ∇. If ξ is a ∇-geodesic vector field, i.e., $\nabla_\xi\xi = 0$, then:* 1) $\nabla^*_\xi\xi \in \Gamma(\mathcal{D})$, 2) $\nabla^*_\xi\eta = 0$, 3) $(\nabla_\xi\eta)(\xi) = 0$, 4) $(\nabla_X g)(\xi, \xi) = (\nabla^*_X g)(\xi, \xi) = (\nabla_X\varphi)(\xi) = 0$ *for any* $X \in \mathfrak{X}(M)$.

Proof (1) Results from the first part of iv) above. (2) Follows from 2.8. (3) Results from applying (2) in the second part of (iv) above. (4) Are direct consequences of the geodesic hypothesis. □

An analog of the notion of holomorphic statistical manifold defined in [12] can be considered here:

Definition 2.4 The (almost) paracontact-statistical manifold $(M, \nabla, \varphi, \xi, \eta, g)$ is paraholomorphic if the fundamental 2-form ω is ∇-parallel.

A necessary and sufficient condition for the existence of the above notion is given in the following.

Proposition 2.5 *The almost paracontact-statistical manifold $(M, \nabla, \varphi, \xi, \eta, g)$ is paraholomorphic if and only if for any $X \in \mathfrak{X}(M)$, we have:*

$$(\nabla_X g) \circ (I \times \varphi) = -g \circ (I \times \nabla_X\varphi). \tag{2.9}$$

In particular, the paracontact-statistical manifold M is paraholomorphic if and only if:

$$(\nabla_X g) \circ (I \times \varphi) = 0 \tag{2.10}$$

holds for any $X \in \mathfrak{X}(M)$.

Proof From $(\nabla_X\omega)(Y, Z) = (\nabla_X g)(Y, \varphi Z) + g(Y, (\nabla_X\varphi)Z)$ for any $X, Y, Z \in \mathfrak{X}(M)$ and from the condition $\nabla\omega = 0$, we obtain the required relation. □

Another relationship between ∇ and ∇^* is given by:

$$(\nabla_X\omega + \nabla^*_X\omega)(Y, Z) = g(Y, (\nabla_X\varphi + \nabla^*_X\varphi)Z), \tag{2.11}$$

and then, on a paraholomorphic paracontact-statistical manifold, we have also $\nabla^*\omega = 0$ due to ii) of Proposition 2.2.

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3 Projective Equivalences in Statistical Paracontact Geometry

Geometrically, two torsion-free affine connections are projectively equivalent if they have the same geodesics as unparameterized curves. Thus, they determine a class of equivalence on a given manifold called projective structure.

Fix a 1-form η on M. Two linear connections ∇ and ∇^* on M are called:

i) [7] η-projectively equivalent if:

$$\nabla^* - \nabla = \eta \otimes I + I \otimes \eta; \tag{3.1}$$

ii) [13] η-dual-projectively equivalent if:

$$\nabla^* - \nabla = -g \circ \eta^{\sharp_g}, \tag{3.2}$$

where $\eta^{\sharp_g}$ is the vector field g-dual to η.

Note that if two connections are projectively equivalent or dual-projectively equivalent, then their conjugate connections with respect to a Riemannian metric associated with a statistical structure may not be projectively or dual-projectively equivalent: respectively.

Proposition 3.1 *Let (M, ∇, g) be a statistical manifold of dimension $n \geq 2$ and ∇^* its conjugate connection. Then, ∇ and ∇^* are neither η-projectively equivalent nor η-dual-projectively equivalent.*

Proof i) Replacing the expression 3.1 of ∇^* in 1.2, we get:

$$(\nabla_X g)(Y, Z) = \eta(X)g(Y, Z) + \eta(Z)g(X, Y), \tag{3.3}$$

and taking into account the symmetry 1.1, we obtain:

$$\eta(Z)g(X, Y) = \eta(Y)g(Z, X) \tag{3.4}$$

which implies that:

$$\eta \otimes I = I \otimes \eta. \tag{3.5}$$

Applying this relation on the pair $(X, \xi = \eta^{\sharp_g})$ with $X \perp \xi$, it results $0 = X$ which is impossible.

ii) Replacing 3.2 in 1.2, it results:

$$(\nabla_X g)(Y, Z) = -\eta(Y)g(X, Z), \tag{3.6}$$

and again, the symmetry argument yields 3.5.

□

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Example 3.2 The computations above inspire us to construct a large class of almost paracontact-statistical manifolds. Fix an almost paracontact manifold $(M, \varphi, \xi, \eta, g)$ and an 1-form λ. We define the linear connection:

$$\nabla^{\lambda} = \nabla^{g} + \lambda \otimes I + I \otimes \lambda + g \circ \lambda^{\sharp_g}, \tag{3.7}$$

where ∇^g is the Levi-Civita connection of g. A straightforward computation gives that:

$$(\nabla^{\lambda}_X g)(Y, Z) = -2 \sum_{\text{cyclic}} [\lambda(X) g(Y, Z)] \tag{3.8}$$

and then, ∇^{λ} is a statistical structure on M with the dual connection $(\nabla^{\lambda})^* = \nabla^{-\lambda}$. □

We have:

$$(\nabla^{\lambda}_X \varphi)Y = (\nabla^{g}_X \varphi)Y + \lambda \circ \varphi(Y)X - \lambda(Y)\varphi(X) + \omega(X, Y)\lambda^{\sharp_g} - g(X, Y)\varphi(\lambda^{\sharp_g}), \tag{3.9}$$

and whence:

Proposition 3.3 *The data $(M, \nabla^{\eta}, \varphi, \xi, \eta, g)$ are a paracontact-statistical manifold if and only if:*

$$(\nabla^{g}_X \varphi)Y = -\omega(X, Y)\xi + \eta(Y)\varphi(X). \tag{3.10}$$

Proof The formula 3.9 applied to $\lambda = \eta$ gives:

$$(\nabla^{\eta}_X \varphi)Y = (\nabla^{g}_X \varphi)Y + \omega(X, Y)\xi - \eta(Y)\varphi(X). \tag{3.11}$$

Let us remark that the necessary conditions for 3.10 to holds are:

(i) from $Y = \xi$: $(\nabla^{g}_X \varphi)\xi = \varphi(X)$ which is equivalent to $\varphi(\nabla_X \xi + X) = 0$ for every vector field X;
(ii) from $X = \xi$: $\nabla^{g}_{\xi} \varphi = 0$.

□

In addition, from:

$$\nabla^{\lambda}_{\xi} \xi = \nabla^{g}_{\xi} \xi + 2\lambda(\xi)\xi + \lambda^{\sharp_g}, \tag{3.12}$$

we get $\nabla^{\eta}_{\xi} \xi = \nabla^{g}_{\xi} \xi + 3\xi$ and then ξ is ∇^{η}-geodesic if and only if $\nabla^{g}_{\xi} \xi = -3\xi$.

The condition 2.9 of paraholomorphism means:

$$\begin{aligned} &2\lambda(X)g(Y, \varphi Z) + 3\lambda(Y)g(Z, \varphi X) + \lambda(Z)g(X, \varphi Y) \\ &\quad = g(Y, (\nabla^{g}_X \varphi)Z) + \lambda \circ \varphi(Y)g(X, Z) + \lambda \circ \varphi(Z)g(Y, X) \end{aligned} \tag{3.13}$$

which yields:

Proposition 3.4 *The almost paracontact-statistical manifold $(M, \nabla^{\eta}, \varphi, \xi, \eta, g)$ is paraholomorphic if and only if:*

$$2\eta(X)g(Y, \varphi Z) + 3\eta(Y)g(Z, \varphi X) + \eta(Z)g(X, \varphi Y) = g(Y, (\nabla^{g}_X \varphi)Z). \tag{3.14}$$

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Let us remark that if $\nabla^g\varphi$ is given by 3.10, the last equation becomes:

$$\eta(X)g(Y,\varphi Z)+\eta(Y)g(Z,\varphi X)+\eta(Z)g(X,\varphi Y)=0,$$

which, for $Z=\xi$, gives the impossible relation $g(X,\varphi Y)=0$ and, in conclusion, a paracontact-statistical manifold $(M,\nabla^\eta,\varphi,\xi,\eta,g)$ is not paraholomorphic.

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