SPECIAL CONNECTIONS IN ALMOST PARACONTACT
METRIC GEOMETRY

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Abstract. Two types of properties for linear connections (natural and almost paracontact metric) are discussed in almost paracontact metric geometry with respect to four linear connections: Levi-Civita, canonical (Zamkovoy), Golab and generalized dual. Their relationship is also analyzed with a special view towards their curvature. The particular case of an almost para-cosymplectic manifold gives a major simplification in computations since the paracontact form is closed.

1. Introduction

The paracontact geometry appears as a natural counter-part of the almost contact geometry in [9]. Comparing with the huge literature in almost contact geometry, it seems that there are necessary new studies in almost paracontact geometry; a very interesting paper connecting these fields is [4]. The present work is another step in this direction, more precisely from the point of view of linear connections living in the almost paracontact universe; it can be considered as a continuation and generalization of [1].

Since the Levi-Civita connection is a fundamental object in (pseudo-) Riemannian geometry we add to our study a pseudo-Riemannian metric; so, we work in the so-called (hyperbolical) paracontact metric geometry, see also [6]. In this framework there already exists a canonical connection introduced in

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in correspondence with the Tanaka-Webster connection of pseudo-convex CR-geometry; we study the relationship between this linear connection and our connections. For example, in Section 2 we consider the notions of *almost paracontact metric connection* and *natural connection* to which the canonical connection belongs.

An important feature of the canonical connection of Zamkovoy is that it is metrical but not symmetrical. We consider in Section 2 another linear connection which is metrical and not torsion-free. More precisely, a quarter-symmetric connection of Golab type \cite{7} is introduced and its properties are analyzed. The particular case of *almost para-cosymplectic manifolds* is a special situation when the computation is more simple and we obtain a case when the Golab curvature coincides with the Levi-Civita curvature.

A last notion introduced in this paper is that of *generalized dual connections* as a generalization of Norden duality of linear connections. So, the last Section is devoted to the study of the generalized dual of the Golab connection. An important tensor field of (1,1)-type studied for various connections is the projector corresponding to the characteristic vector field (also called the *structural vector field*); a natural connection makes parallel this vector field.

### 2. Almost paracontact metric geometry and some adapted connections

Let $M$ be a $(2n + 1)$-dimensional smooth manifold, $\varphi$ a tensor field of (1,1)-type called the *structural endomorphism*, $\xi$ a vector field called the *characteristic vector field*, $\eta$ a 1-form called the *paracontact form* and $g$ a pseudo-Riemannian metric on $M$ of signature $(n+1, n)$. We say that $(\varphi, \xi, \eta, g)$ defines an *almost paracontact metric structure* on $M$ if \cite{11, p. 38}, \cite{3}:

1. $\varphi(\xi) = 0$, $\eta \circ \varphi = 0$,
2. $\eta(\xi) = 1$, $\varphi^2 = I - \eta \otimes \xi$,
3. $\varphi$ induces on the $2n$-dimensional distribution $\mathcal{D} := \ker \eta$ an almost paracomplex structure $P$ i.e. $P^2 = 1$ and the eigensubbundles $T^+, T^-$, corresponding to the eigenvalues 1, $-1$ of $P$ respectively, have equal dimension $n$; hence $\mathcal{D} = T^+ \oplus T^-$,
4. $g(\varphi \cdot, \varphi \cdot) = -g + \eta \otimes \eta$.

For a list of examples of almost paracontact metric structures see \cite[p 84]{8}. From the definition it follows that $\eta$ is the $g$-dual of $\xi$ i.e. $\eta(X) = g(X, \xi)$, $\xi$ is an unitary vector field, $g(\xi, \xi) = 1$, and $\varphi$ is a $g$-skew-symmetric operator, $g(\varphi X, Y) = -g(X, \varphi Y)$. The tensor field:

$$\omega(X, Y) := g(X, \varphi Y)$$
is skew-symmetric and:
\[
\omega(\varphi X, Y) = -\omega(X, \varphi Y), \quad \omega(\varphi X, \varphi Y) = -\omega(X, Y).
\] (2.2)

Then \(\omega\) is called the fundamental form. Remark that the canonical distribution \(D\) is \(\varphi\)-invariant since \(D = \text{Im}\varphi\); if \(X \in D\) has the decomposition \(X = X^+ + X^-\) with \(X^* \in T^*\) then \(\varphi X = X^+ - X^-\). Moreover, \(\xi\) is orthogonal to \(D\) and therefore the tangent bundle splits orthogonally:
\[
TM = TF \oplus \langle \xi \rangle. \quad (2.3)
\]

We are interested now in linear connections compatible with the almost paracontact structure. To this aim we introduce:

**Definition 2.1.** A linear connection \(\nabla\) is a natural connection on the almost paracontact metric manifold \((M, \varphi, \xi, \eta, g)\) if it satisfies:
\[
\nabla \eta = \nabla g = 0. \quad (2.4)
\]

So, a natural connection is a \(g\)-metric connection making \(\eta\) a parallel 1-form. A direct consequence of the definition is:

**Proposition 2.2.** If \(\nabla\) is a natural connection on the almost paracontact metric manifold \((M, \varphi, \xi, \eta, g)\) then \(\xi\) is a \(\nabla\)-parallel vector field: \(\nabla \xi = 0\). Hence, the integral curves of \(\xi\) are autoparallel curves for \(\nabla\).

**Proof.** From the conditions (2.4) we obtain:
\[
g(\nabla_X Y, \xi) + g(Y, \nabla_X \xi) = X(g(Y, \xi)) = X(\eta(Y)) = \eta(\nabla_X Y) = g(\nabla_X Y, \xi)
\]
and whence \(\nabla \xi = 0\). \(\square\)

The next important problem is if \(\varphi\) is \(\nabla\)-parallel and then with respect to a general linear connection \(\nabla\) we introduce a new tensor field of \((0,3)\)-type given by:
\[
F_{\nabla}(X, Y, Z) := g((\nabla_X \varphi)Y, Z). \quad (2.5)
\]

\(F_{\nabla}\) satisfies:
\[
\begin{cases}
F_{\nabla}(X, Y, Z) + F_{\nabla}(X, Z, Y) = -(\nabla g)(X, \varphi Y, Z) - (\nabla g)(X, Y, \varphi Z) \\
F_{\nabla}(X, \varphi Y, Z) - F_{\nabla}(X, Y, \varphi Z) = -\eta(Z)(\nabla_X \eta)Y - \eta(Y)g(\nabla_X \xi, Z) \\
F_{\nabla}(X, Y, Z) - F_{\nabla}(X, \varphi Y, \varphi Z) = \eta(Z)\eta((\nabla_X \varphi)Y) + \eta(Y)g(\nabla_X \xi, \varphi Z)
\end{cases} \quad (2.6)
\]
which yields:
Proposition 2.3. If $\nabla$ is a natural connection on the almost paracontact metric manifold $(M, \varphi, \xi, \eta, g)$ then its tensor field $F_\nabla$ satisfies for any $X, Y, Z \in \mathfrak{X}(M)$:

$$
\begin{align*}
F_\nabla(X, Y, Z) &= -F_\nabla(X, Z, Y) \\
F_\nabla(X, \varphi Y, Z) &= F_\nabla(X, Y, \varphi Z) \\
F_\nabla(X, \varphi Y, \varphi Z) &= F_\nabla(X, Y, Z) - \eta(Z)\eta((\nabla_X \varphi)Y).
\end{align*}
$$

The relations (2.7) say that $\Omega^\nabla_X := F_\nabla(X, \cdot, \cdot)$ is a 2-form on $M$ with:

$$
\Omega^\nabla_X(\varphi Y, Z) = \Omega^\nabla_X(Y, \varphi Z), \quad \Omega^\nabla_X(\varphi Y, \varphi Z) = \Omega^\nabla_X(Y, Z) - \eta(Z)\eta((\nabla_X \varphi)Y).
$$

These relations are a counter-part of equations (2.2).

Proposition 2.4. If $\nabla \varphi = 0$ then:

$$
(\nabla g)(\xi, Y) = 2(\nabla \eta)Y.
$$

Proof. From hypothesis it follows $F_\nabla = 0$ and then from (2.6)b we get:

$$
\eta(Z)(\nabla \eta)Y = -\eta(Y)g(\nabla_X \xi, Z)
$$
and with $Z = \xi$ it results:

$$
(\nabla \eta)Y = -\eta(Y)\eta(\nabla_X \xi).
$$

From (2.6)c we obtain:

$$
g(\nabla_X \xi, \varphi Z) = 0
$$
which with $Z \to \varphi Y$ yields:

$$
g(\nabla_X \xi, Y) = \eta(Y)\eta(\nabla_X \xi).
$$

Adding (2.10) and (2.11) it results:

$$
(\nabla \eta)Y = -g(\nabla_X \xi, Y)
$$
which is equivalent with (2.9).

The next step is to unify all these conditions in:

Definition 2.5. $\nabla$ is called almost paracontact metric connection if it satisfies:

$$
\nabla \varphi = \nabla \eta = \nabla g = 0.
$$

Therefore, $\nabla$ is an almost paracontact metric connection if it is a natural connection with $\nabla \varphi = 0$. The characteristic vector field $\xi$ is parallel with respect to such a linear connection. From Proposition 2.4 a metric linear connection with $\nabla \varphi = 0$ is an almost paracontact metric connection.
S. Zamkovoy [11, p. 49] defined on an almost paracontact metric manifold a connection \( \tilde{\nabla} \) using the Levi-Civita connection \( \nabla^g \) of the structure:

\[
(2.14) \quad \tilde{\nabla}_X Y := \nabla^g_X Y + \eta(X) \varphi Y - \eta(Y) \nabla^g_X \xi + (\nabla^g_X \eta) Y \cdot \xi
\]

and called it \textit{canonical paracontact connection}. This linear connection is a natural one according to Proposition 4.2 of [11, p. 49] and it is an almost paracontact metric connection if and only if:

\[
(2.15) \quad (\nabla^g_X \varphi) Y = \eta(Y)(X - hX) - g(X - hX, Y) \xi
\]

where:

\[
(2.16) \quad h = \frac{1}{2} \mathcal{L}_\xi \varphi
\]

with \( \mathcal{L} \) the Lie derivative. The tensor field \( h \) vanishes if and only if \( M \) is \( K \)-paracontact i.e. \( \xi \) is a Killing vector field with respect to \( g \). If \( M \) is \( K \)-paracontact then the condition \( \nabla^g \varphi = 0 \) in (2.15) yields \( \eta(Y)X = g(X, Y) \xi \) and the \( g \)-product with \( \xi \) in this last relation gives \( g = \eta \otimes \eta \) an impossible relation since it implies \( g|_\mathcal{D} = 0 \). So, in the \( K \)-paracontact case \( \nabla^g \) and \( \tilde{\nabla} \) can not be both almost paracontact metric connections.

We use the conventions of [11]; for example, the exterior differential of \( \eta \) is given by:

\[
(2.17) \quad 2 \, d\eta(X, Y) = X(\eta(Y)) - Y(\eta(X)) - \eta([X, Y])
\]

respectively [11, p. 39]:

**Definition 2.6.** \((M, \varphi, \xi, \eta, g)\) is called \textit{paracontact metric manifold} if \( d\eta = \omega \).

On a paracontact metric manifold we have [11, p. 41]: \( \nabla^g_\xi \varphi = 0 \) and \( \xi \) is a geodesic vector field i.e. \( \nabla^g_\xi \xi = 0 \). For the following notion we consider the product manifold \( M \times \mathbb{R} \) with the tensor field:

\[
(2.18) \quad J \left( X, f \frac{d}{dt} \right) = \left( \varphi X + f \xi, \eta(X) \frac{d}{dt} \right)
\]

**Definition 2.7.** ([11, p. 39], [3]) The paracontact structure \((\varphi, \eta, \xi)\) is called \textit{normal} if \( J \) is integrable. Moreover, a normal paracontact metric manifold is called \textit{paraSasakian manifold}.

An important feature of a paraSasakian manifold is that it is \( K \)-paracontact.

Let us end this section with the following remark for a linear connection \( \nabla \):

- if \( \nabla \) is \( g \)-metric then: \((\mathcal{L}_\xi g)(Y, Z) = (\nabla_X \eta)Y + (\nabla_Y \eta)X\),
- if \( \nabla \) is symmetric then: \( 2d\eta(X, Y) = (\nabla_X \eta)Y - (\nabla_Y \eta)X \).
It results that if $\nabla^g$ is a natural connection then $M$ is a $K$-paracontact manifold and $\eta$ is closed ($d\eta = 0$) which means that $M$ is not a paracontact metric manifold.

3. The Golab connection

In this section we search for a weak version of $\nabla^g$ and $\tilde{\nabla}$. Since the metrical condition is a common property of these two connections we look for a weak condition in terms of torsion.

**Definition 3.1.** The Golab connection $[7]$ associated to the structure $(\varphi, \eta, g)$ is the linear connection $\nabla^G$ satisfying:

\begin{equation}
\nabla^G g = 0, \quad T^G = \varphi \otimes \eta - \eta \otimes \varphi.
\end{equation}

It is known that the unique connection with these properties is given by:

\begin{equation}
\nabla^G = \nabla^g - \eta \otimes \varphi.
\end{equation}

We can express the Golab connection by using the canonical connection (2.14):

\begin{equation}
\nabla^G_X Y = \tilde{\nabla}_X Y - 2\eta(X)\varphi Y + \eta(Y)\nabla^g_X \xi + (\nabla^g_X \eta)Y \cdot \xi
\end{equation}

and then it results that if $\nabla^g$ is a natural connection then:

\begin{equation}
\nabla^G_X Y = \tilde{\nabla}_X Y - 2\eta(X)\varphi Y.
\end{equation}

The Golab connection is different from the Levi-Civita connection; but from (3.3) it coincides with the canonical connection if and only if:

\begin{equation}
2\eta(X)\varphi Y = \eta(Y)\nabla^g_X \xi + (\nabla^g_X \eta)Y \cdot \xi.
\end{equation}

With $Y = \xi$ it results $\nabla^g_{\xi}\xi = -(\nabla^g_{\xi}\eta)\xi \cdot \xi$ and since $\nabla^g_{\xi}\xi$ is $g$-orthogonal on $\xi$ we get $\nabla^g\xi = 0$. Returning to (3.5) it results:

\begin{equation}
2\eta(X)\varphi Y = (\nabla^g_{\xi}\eta)Y \cdot \xi
\end{equation}

and with $X = \xi$ we get:

\begin{equation}
2\varphi Y = (\nabla^g_{\xi}\eta)Y \cdot \xi.
\end{equation}

Then we have $\nabla^g_{\xi}\eta \neq 0$, in particular $\nabla^g$ must not be a natural connection.

Returning to the general case and computing $T^G(\varphi, \varphi) = 0$ we get that $\nabla^G$ is symmetrical on $\text{Im}\varphi = \mathcal{D}$ and therefore it coincides with $\nabla^g$ on $\mathcal{D}$. The main properties of the Golab connection are stated in the next proposition:
Proposition 3.2. The Golab connection of an almost paracontact metric manifold satisfies:

\( \nabla^G \varphi = \nabla^g \varphi, \quad \nabla^G \eta = \nabla^g \eta, \quad \nabla^G \xi = \nabla^g \xi. \)  

(3.8)

Proof. By a direct computation we get 

\[ \nabla^G_X \varphi_Y = \nabla^g_X \varphi_Y - \eta(X) \varphi_2 Y \text{ and:} \]

\[ \varphi(\nabla^G_X Y) = \varphi(\nabla^g_X Y) - \eta(X) \varphi_2 Y \]

respectively:

\[ (\nabla^g_X \eta)Y = \nabla^g_X \eta(Y) - \eta(\nabla^g_X Y) = X(\eta(Y)) - \eta(\nabla^g_X Y) + \eta(X) \eta \circ \varphi Y = (\nabla^g_X \eta)Y. \]

A natural problem is to determine the necessary and sufficient condition for the Golab connection of an almost paracontact metric manifold to be a natural connection. We obtain:

Theorem 3.3. Let \((M, \varphi, \xi, \eta, g)\) be an almost paracontact metric manifold. Then its Golab connection \(\nabla^G\) is a natural connection if and only if the Levi-Civita connection \(\nabla^g\) is a natural connection. This last condition reduces to: \(\nabla^g \eta = 0\). Moreover, \(\nabla^G\) is an almost paracontact metric connection if and only if \(\nabla^g\) is an almost paracontact metric connection.

A long but straightforward computation gives also:

Theorem 3.4. The curvature of the Golab connection is:

\[ R^G_{XYZ} = R^g_{XYZ} - 2d\eta(X,Y) \varphi Z + \eta(X)(\nabla^g_Y \varphi)Z - \eta(Y)(\nabla^g_X \varphi)Z. \]  

(3.9)

So, if \(\nabla^g\) is almost paracontact metric connection then \(R^G = R^g\).

If the 1-form \(\eta\) and the 2-form \(\omega\) are closed we say that \((M, \varphi, \xi, \eta, g)\) is an almost para-cosymplectic manifold after [5, p. 562].

Proposition 3.5. Let \((M, \varphi, \xi, \eta, g)\) be an almost para-cosymplectic manifold. Then its curvature satisfies:

\[ R^G_{XYZ} = R^g_{XYZ} + \eta(X)(\nabla^g_Y \varphi)Z - \eta(Y)(\nabla^g_X \varphi)Z. \]  

(3.10)

Let us point out an application of the formulae (3.8). Let \(P_0\) be the projector corresponding to \(\langle \xi \rangle\) in the decomposition (2.3); namely, if \(X \in \mathfrak{X}(M)\) has the decomposition:

\[ X = X^+ + X^- + \eta(X)\xi \]

then \(P_0(X) = \eta(X)\xi\). For a general linear connection \(\nabla\) we have:

\[ (\nabla_X P_0)Y = \nabla_X (\eta(Y)\xi) - \eta(\nabla_X Y)\xi = (\nabla_X \eta)(Y) \cdot \xi + \eta(Y)\nabla_X \xi \]  

(3.12)
and then (3.8) yields:

\[(3.13) \quad \nabla^G P_0 = \nabla^g P_0.\]

If \(\nabla^g\) is a natural connection we get that \(P_0\) is covariant constant with respect to both \(\nabla^g\) and \(\nabla^G\). Since the canonical connection \(\tilde{\nabla}\) is natural we already have that \(P_0\) is covariant constant with respect to \(\tilde{\nabla}\). Another interesting fact is that the \(P_0\)-Golab connection i.e. with \(\varphi\) of (3.1) replaced by \(P_0\), it is in fact \(\nabla^g\) since \(P_0 \otimes \eta - \eta \otimes P_0 = 0\).

The projector \(P_0\) can be used to obtain a more simple formula for the canonical connection \(\tilde{\nabla}\). Plugging (3.12) in (2.14) gives:

\[(3.14) \quad \tilde{\nabla}_X Y = \nabla^g_X Y + \eta(X) \varphi Y - 2\eta(Y) \nabla^g_X \xi + (\nabla^g_X P_0) Y\]

and then, for \(\nabla^g\) a natural connection we get:

\[(3.15) \quad \tilde{\nabla} = \nabla^g + \eta \otimes \varphi\]

yielding a (convex) relationship between all the linear connections studied until now:

\[(3.16) \quad \tilde{\nabla} + \nabla^G = 2\nabla^g.\]

4. Generalized duality for linear connections

Let now \(\nabla\) and \(\nabla'\) be two linear connections on \(M\). We adopt the following notion of generalized conjugation of linear connections from [2, p. 28]:

**Definition 4.1.** We say that \(\nabla\) and \(\nabla'\) are generalized dual connections with respect to the pair \((g, \eta)\) if for any \(X, Y, Z \in \mathfrak{X}(M)\):

\[(4.1) \quad X(g(Y, Z)) = g(\nabla_X Y, Z) + g(Y, \nabla'_X Z) - \eta(X)g(Y, Z)\]

or equivalently:

\[(4.2) \quad g(\nabla'_X Z - \nabla_X Z - \eta(X)Z, Y) = \nabla g(X, Y, \xi).\]

Without the last term, the relation (4.1) reduces to the usual *Norden duality* of linear connections from [10].

We shall discuss the behavior of the generalized dual connection \(\nabla'\) of \(\nabla\) if we impose certain conditions on \(\nabla\). Let us remark the following relations:

\[(4.3) \quad \begin{cases} 
\eta(\nabla'_X Y) = \eta(\nabla_X Y) + \eta(X)\eta(Y) + \nabla g(X, Y, \xi) \\
(\nabla'_X \eta) Y = (\nabla_X \eta) Y - \eta(X)\eta(Y) - \nabla g(X, Y, \xi) \\
g((\nabla'_X \varphi) Y, Z) = -g((\nabla_X \varphi) Z, Y).
\end{cases}\]
Now, if we require the following conditions:

**conditions on \( \nabla \varphi \):**
1) \( \nabla \varphi = 0 \) implies \( \nabla' \varphi = 0 \);
2) \( \nabla \varphi = \pm \eta \otimes \varphi \) implies \( \nabla' \varphi = \pm \eta \otimes \varphi \);

**conditions on \( \nabla \eta \):**
3) \( \nabla \eta = 0 \) implies \( \nabla' \eta = -\eta \otimes \eta - \nabla g(\cdot, \cdot, \xi) \);
4) \( \nabla \eta = \eta \otimes \eta \) implies \( \nabla' \eta = -\nabla g(\cdot, \cdot, \xi) \);

**conditions on \( \nabla g \):**
5) \( \nabla g = 0 \) implies \( \nabla' = \nabla + \eta \otimes I \);
6) \( \nabla g = \eta \otimes g \) implies \( \nabla' = \nabla + 2\eta \otimes I \);
7) \( \nabla g = -\eta \otimes g \) implies \( \nabla' = \nabla \).

**Remark 4.2.** If \( \nabla \) satisfies 5) and 6) then its generalized dual connection is equal to \( \nabla \) on \( \mathcal{D} \). Also remark that if \( \nabla \) is \( g \)-metric then \( \nabla \xi \xi \in \Gamma(\mathcal{D}) \) while \( g(\nabla' \xi, \xi) = 1 \) and \( \nabla' X - \nabla X = X \) for any \( X \in \mathfrak{X}(M) \).

The generalized dual connection of the Golab connection has the following properties:

**Proposition 4.3.** On the almost paracontact metric manifold \((M, \varphi, \xi, \eta, g)\) the generalized dual connection \( (\nabla G)' \) of the Golab connection \( \nabla G \) is a quarter-symmetric connection which satisfies:

\[
g(X, (\nabla G)' Y, \xi) = g((\nabla G)' X, Y, \xi). \tag{4.4}
\]

In the almost para-cosymplectic case \( (\nabla G)' \) has the same curvature as \( \nabla G \).

**Proof.** Fix \( X, Y, Z \in \mathfrak{X}(M) \); the equality (4.4) is a direct consequence of (4.1) and:

- the torsion of \( (\nabla G)' \) is \( T_{G'} = \psi \otimes \eta - \eta \otimes \psi \) with \( \psi := \varphi - I \),
- the curvature of \( (\nabla G)' \) is \( R_{G'}(X, Y, Z) = R_{G}(X, Y, Z) + 2d\eta(X, Y)Z \). \( \square \)

A straightforward computation similar to that of the end of previous Section gives: \( \nabla G P_0 = \nabla G P_0(= \nabla^g P_0) \) and then a natural \( \nabla^g \) yields the parallelism of \( P_0 \) with respect to all three linear connections \( \nabla^g, \nabla^G \) and \( \nabla^{G'} \).

**Definition 4.4.** The linear connection \( \nabla \) is called \( \xi \)-metric if: \( \nabla g(\cdot, \cdot, \xi) = 0 \).

Of course, a metric linear connection is \( \xi \)-metric. Similar to the calculus of Section 3 we get that for a \( \xi \)-metric connection \( \nabla \) the curvature of the generalized dual connection \( \nabla' \) is:

\[
R'(X, Y, Z) = R(X, Y, Z) + 2d\eta(X, Y)Z. \tag{4.5}
\]

So, in the para-cosymplectic case a \( \xi \)-metric connection has the same curvature as its generalized dual connection.
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