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SPECIAL CONNECTIONS IN ALMOST PARACONTACT METRIC GEOMETRY

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Abstract. Two types of properties for linear connections (natural and almost paracontact metric) are discussed in almost paracontact metric geometry with respect to four linear connections: Levi-Civita, canonical (Zamkovoy), Golab and generalized dual. Their relationship is also analyzed with a special view towards their curvature. The particular case of an almost para-cosymplectic manifold gives a major simplification in computations since the paracontact form is closed.

1. **Introduction**

The paracontact geometry appears as a natural counter-part of the almost contact geometry in [9]. Comparing with the huge literature in almost contact geometry, it seems that there are necessary new studies in almost paracontact geometry; a very interesting paper connecting these fields is [4]. The present work is another step in this direction, more precisely from the point of view of linear connections living in the almost paracontact universe; it can be considered as a continuation and generalization of [1].

Since the Levi-Civita connection is a fundamental object in (pseudo-) Riemannian geometry we add to our study a pseudo-Riemannian metric; so, we work in the so-called (*hyperbolical*) *paracontact metric geometry*, see also [6]. In this framework there already exists a *canonical connection* introduced in

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[11] in correspondence with the Tanaka-Webster connection of pseudo-convex CR-geometry; we study the relationship between this linear connection and our connections. For example, in Section 2 we consider the notions of *almost paracontact metric connection* and *natural connection* to which the canonical connection belongs.

An important feature of the canonical connection of Zamkovoy is that it is metrical but not symmetrical. We consider in Section 2 another linear connection which is metrical and not torsion-free. More precisely, a quarter-symmetric connection of Golab type [7] is introduced and its properties are analyzed. The particular case of *almost para-cosymplectic manifolds* is a special situation when the computation is more simple and we obtain a case when the Golab curvature coincides with the Levi-Civita curvature.

A last notion introduced in this paper is that of *generalized dual connections* as a generalization of Norden duality of linear connections. So, the last Section is devoted to the study of the generalized dual of the Golab connection. An important tensor field of $(1,1)$-type studied for various connections is the projector corresponding to the characteristic vector field (also called *the structural vector field*); a natural connection makes parallel this vector field.

2. Almost paracontact metric geometry and some adapted connections

Let M be a $(2n+1)$-dimensional smooth manifold, φ a tensor field of $(1,1)$-type called *the structural endomorphism*, ξ a vector field called *the characteristic vector field*, η a 1-form called *the paracontact form* and g a pseudo-Riemannian metric on M of signature $(n+1,n)$. We say that (φ,ξ,η,g) defines an *almost paracontact metric structure* on M if [11, p. 38], [3]:
1. $\varphi(\xi)=0$, $\eta\circ\varphi=0$,   2. $\eta(\xi)=1$, $\varphi^2=I-\eta\otimes\xi$,
3. φ induces on the $2n$-dimensional distribution $\mathcal{D}:=\ker\eta$ an almost paracomplex structure P i.e. $P^2=1$ and the eigensubbundles T^+, T^-, corresponding to the eigenvalues 1, -1 of P respectively, have equal dimension n; hence $\mathcal{D}=T^+\oplus T^-$,
4. $g(\varphi\cdot,\varphi\cdot)=-g+\eta\otimes\eta$.

For a list of examples of almost paracontact metric structures see [8, p 84]. From the definition it follows that η is the g-dual of ξ i.e. $\eta(X)=g(X,\xi)$, ξ is an unitary vector field, $g(\xi,\xi)=1$, and φ is a g-skew-symmetric operator, $g(\varphi X,Y)=-g(X,\varphi Y)$. The tensor field:

$$\omega(X,Y):=g(X,\varphi Y) \tag{2.1}$$

is skew-symmetric and:

$$\omega(\varphi X, Y) = -\omega(X, \varphi Y), \quad \omega(\varphi X, \varphi Y) = -\omega(X, Y). \tag{2.2}$$

Then ω is called *the fundamental form*. Remark that the canonical distribution $\mathcal{D}$ is φ-invariant since $\mathcal{D} = Im\varphi$: if $X \in \mathcal{D}$ has the decomposition $X = X^+ + X^-$ with $X^* \in T^*$ then $\varphi X = X^+ - X^-$. Moreover, ξ is orthogonal to $\mathcal{D}$ and therefore the tangent bundle splits orthogonally:

$$TM = T\mathcal{F} \oplus \langle \xi \rangle. \tag{2.3}$$

We are interested now in linear connections compatible with the almost paracontact structure. To this aim we introduce:

Definition 2.1. *A linear connection ∇ is a natural connection on the almost paracontact metric manifold $(M, \varphi, \xi, \eta, g)$ if it satisfies:*

$$\nabla \eta = \nabla g = 0. \tag{2.4}$$

So, a natural connection is a g-metric connection making η a parallel 1-form. A direct consequence of the definition is:

Proposition 2.2. *If ∇ is a natural connection on the almost paracontact metric manifold $(M, \varphi, \xi, \eta, g)$ then ξ is a ∇-parallel vector field: $\nabla \xi = 0$. Hence, the integral curves of ξ are autoparallel curves for ∇.*

Proof. From the conditions (2.4) we obtain:

$$g(\nabla_X Y, \xi) + g(Y, \nabla_X \xi) = X(g(Y, \xi)) = X(\eta(Y)) = \eta(\nabla_X Y) = g(\nabla_X Y, \xi)$$

and whence $\nabla \xi = 0$. □

The next important problem is if φ is ∇-parallel and then with respect to a general linear connection ∇ we introduce a new tensor field of $(0, 3)$-type given by:

$$F_\nabla(X, Y, Z) := g((\nabla_X \varphi)Y, Z). \tag{2.5}$$

F_∇ satisfies:

$$\begin{cases} F_\nabla(X, Y, Z) + F_\nabla(X, Z, Y) = -(\nabla g)(X, \varphi Y, Z) - (\nabla g)(X, Y, \varphi Z) \\ F_\nabla(X, \varphi Y, Z) - F_\nabla(X, Y, \varphi Z) = -\eta(Z)(\nabla_X \eta)Y - \eta(Y) g(\nabla_X \xi, Z) \\ F_\nabla(X, Y, Z) - F_\nabla(X, \varphi Y, \varphi Z) = \eta(Z)\eta((\nabla_X \varphi)Y) + \eta(Y) g(\nabla_X \xi, \varphi Z) \end{cases} \tag{2.6}$$

which yields:

Proposition 2.3. *If ∇ is a natural connection on the almost paracontact metric manifold $(M, \varphi, \xi, \eta, g)$ then its tensor field F_∇ satisfies for any X, Y, $Z \in \mathfrak{X}(M)$:*

$$\begin{cases} F_\nabla(X, Y, Z) = -F_\nabla(X, Z, Y) \\ F_\nabla(X, \varphi Y, Z) = F_\nabla(X, Y, \varphi Z) \\ F_\nabla(X, \varphi Y, \varphi Z) = F_\nabla(X, Y, Z) - \eta(Z)\eta((\nabla_X \varphi)Y). \end{cases} \tag{2.7}$$

The relations (2.7) say that $\Omega^\nabla_X := F_\nabla(X, \cdot, \cdot)$ is a 2-form on M with:

$$\Omega^\nabla_X(\varphi Y, Z) = \Omega^\nabla_X(Y, \varphi Z), \quad \Omega^\nabla_X(\varphi Y, \varphi Z) = \Omega^\nabla_X(Y, Z) - \eta(Z)\eta((\nabla_X \varphi)Y). \tag{2.8}$$

These relations are a counter-part of equations (2.2).

Proposition 2.4. *If $\nabla\varphi = 0$ then:*

$$(\nabla_X g)(\xi, Y) = 2(\nabla_X \eta)Y. \tag{2.9}$$

Proof. From hypothesis it follows $F_\nabla = 0$ and then from (2.6)b we get:

$$\eta(Z)(\nabla_X \eta)Y = -\eta(Y)g(\nabla_X \xi, Z)$$

and with $Z = \xi$ it results:

$$(\nabla_X \eta)Y = -\eta(Y)\eta(\nabla_X \xi). \tag{2.10}$$

From (2.6)c we obtain:

$$g(\nabla_X \xi, \varphi Z) = 0$$

which with $Z \to \varphi Y$ yields:

$$g(\nabla_X \xi, Y) = \eta(Y)\eta(\nabla_X \xi). \tag{2.11}$$

Adding (2.10) and (2.11) it results:

$$(\nabla_X \eta)Y = -g(\nabla_X \xi, Y) \tag{2.12}$$

which is equivalent with (2.9). □

The next step is to unify all these conditions in:

Definition 2.5. ∇ *is called almost paracontact metric connection if it satisfies:*

$$\nabla\varphi = \nabla\eta = \nabla g = 0. \tag{2.13}$$

Therefore, ∇ is an almost paracontact metric connection if it is a natural connection with $\nabla\varphi = 0$. The characteristic vector field ξ is parallel with respect to such a linear connection. From Proposition 2.4 a metric linear connection with $\nabla\varphi = 0$ is an almost paracontact metric connection.

S. Zamkovoy [11, p. 49] defined on an almost paracontact metric manifold a connection $\tilde{\nabla}$ using the Levi-Civita connection ∇^g of the structure:

$$\tilde{\nabla}_X Y := \nabla^g_X Y + \eta(X)\varphi Y - \eta(Y)\nabla^g_X \xi + (\nabla^g_X \eta)Y \cdot \xi \tag{2.14}$$

and called it *canonical paracontact connection*. This linear connection is a natural one according to Proposition 4.2 of [11, p. 49] and it is an almost paracontact metric connection if and only if:

$$(\nabla^g_X \varphi)Y = \eta(Y)(X - hX) - g(X - hX, Y)\xi \tag{2.15}$$

where:

$$h = \frac{1}{2}\mathcal{L}_\xi \varphi \tag{2.16}$$

with $\mathcal{L}$ the Lie derivative. The tensor field h vanishes if and only if M is *K-paracontact* i.e. ξ is a Killing vector field with respect to g. If M is K-paracontact then the condition $\nabla^g \varphi = 0$ in (2.15) yields $\eta(Y)X = g(X,Y)\xi$ and the g-product with ξ in this last relation gives $g = \eta \otimes \eta$ an impossible relation since it implies $g|_{\mathcal{D}} = 0$. So, in the K-paracontact case ∇^g and $\tilde{\nabla}$ can not be both almost paracontact metric connections.

We use the conventions of [11]; for example, the exterior differential of η is given by:

$$2d\eta(X,Y) = X(\eta(Y)) - Y(\eta(X)) - \eta([X,Y]) \tag{2.17}$$

respectively [11, p. 39]:

Definition 2.6. *$(M, \varphi, \xi, \eta, g)$ is called paracontact metric manifold if $d\eta = \omega$.*

On a paracontact metric manifold we have [11, p. 41]: $\nabla^g_\xi \varphi = 0$ and ξ is a geodesic vector field i.e. $\nabla^g_\xi \xi = 0$. For the following notion we consider the product manifold $M \times \mathbb{R}$ with the tensor field:

$$J\left(X, f\frac{d}{dt}\right) = \left(\varphi X + f\xi, \eta(X)\frac{d}{dt}\right). \tag{2.18}$$

Definition 2.7. *([11, p. 39], [3]) The paracontact structure (φ, η, ξ) is called normal if J is integrable. Moreover, a normal paracontact metric manifold is called paraSasakian manifold.*

An important feature of a paraSasakian manifold is that it is K-paracontact.

Let us end this section with the following remark for a linear connection ∇:
-if ∇ is g-metric then: $(\mathcal{L}_\xi g)(Y,Z) = (\nabla_X \eta)Y + (\nabla_Y \eta)X$,
-if ∇ is symmetric then: $2d\eta(X,Y) = (\nabla_X \eta)Y - (\nabla_Y \eta)X$.

It results that if ∇^g is a natural connection then M is a K-paracontact manifold and η is closed ($d\eta = 0$) which means that M is not a paracontact metric manifold.

3. The Golab connection

In this section we search for a weak version of ∇^g and $\tilde{\nabla}$. Since the metrical condition is a common property of these two connections we look for a weak condition in terms of torsion.

Definition 3.1. *The Golab connection [7] associated to the structure (φ, η, g) is the linear connection ∇^G satisfying:*

$$\nabla^G g = 0, \quad T^G = \varphi \otimes \eta - \eta \otimes \varphi. \tag{3.1}$$

It is known that the unique connection with these properties is given by:

$$\nabla^G = \nabla^g - \eta \otimes \varphi. \tag{3.2}$$

We can express the Golab connection by using the canonical connection (2.14):

$$\nabla^G_X Y = \tilde{\nabla}_X Y - 2\eta(X)\varphi Y + \eta(Y)\nabla^g_X \xi + (\nabla^g_X \eta)Y \cdot \xi \tag{3.3}$$

and then it results that if ∇^g is a natural connection then:

$$\nabla^G_X Y = \tilde{\nabla}_X Y - 2\eta(X)\varphi Y. \tag{3.4}$$

The Golab connection is different from the Levi-Civita connection; but from (3.3) it coincides with the canonical connection if and only if:

$$2\eta(X)\varphi Y = \eta(Y)\nabla^g_X \xi + (\nabla^g_X \eta)Y \cdot \xi. \tag{3.5}$$

With $Y = \xi$ it results $\nabla^g_X \xi = -(\nabla^g_X \eta)\xi \cdot \xi$ and since $\nabla^g_X \xi$ is g-orthogonal on ξ we get $\nabla^g \xi = 0$. Returning to (3.5) it results:

$$2\eta(X)\varphi Y = (\nabla^g_X \eta)Y \cdot \xi \tag{3.6}$$

and with $X = \xi$ we get:

$$2\varphi Y = (\nabla^g_\xi \eta)Y \cdot \xi. \tag{3.7}$$

Then we have $\nabla^g_\xi \eta \neq 0$, in particular ∇^g must not be a natural connection.

Returning to the general case and computing $T^G(\varphi\cdot, \varphi\cdot) = 0$ we get that ∇^G is symmetrical on $Im\varphi = \mathcal{D}$ and therefore it coincides with ∇^g on $\mathcal{D}$. The main properties of the Golab connection are stated in the next proposition:

Proposition 3.2. *The Golab connection of an almost paracontact metric manifold satisfies:*

$$\nabla^G\varphi = \nabla^g\varphi, \quad \nabla^G\eta = \nabla^g\eta, \quad \nabla^G\xi = \nabla^g\xi. \tag{3.8}$$

Proof. By a direct computation we get $\nabla^G_X\varphi Y = \nabla^g_X\varphi Y - \eta(X)\varphi^2 Y$ and:

$$\varphi(\nabla^G_X Y) = \varphi(\nabla^g_X Y) - \eta(X)\varphi^2 Y$$

respectively:

$$(\nabla^G_X\eta)Y = \nabla^G_X\eta(Y) - \eta(\nabla^G_X Y) = X(\eta(Y)) - \eta(\nabla^g_X Y) + \eta(X)\eta\circ\varphi Y = (\nabla^g_X\eta)Y.$$

□

A natural problem is to determine the necessary and sufficient condition for the Golab connection of an almost paracontact metric manifold to be a natural connection. We obtain:

Theorem 3.3. *Let $(M, \varphi, \xi, \eta, g)$ be an almost paracontact metric manifold. Then its Golab connection ∇^G is a natural connection if and only if the Levi-Civita connection ∇^g is a natural connection. This last condition reduces to: $\nabla^g\eta = 0$. Moreover, ∇^G is an almost paracontact metric connection if and only if ∇^g is an almost paracontact metric connection.*

A long but straightforward computation gives also:

Theorem 3.4. *The curvature of the Golab connection is:*

$$R^G_{XYZ} = R^g_{XYZ} - 2d\eta(X, Y)\varphi Z + \eta(X)(\nabla^g_Y\varphi)Z - \eta(Y)(\nabla^g_X\varphi)Z. \tag{3.9}$$

So, if ∇^g is almost paracontact metric connection then $R^G = R^g$.

If the 1-form η and the 2-form ω are closed we say that $(M, \varphi, \xi, \eta, g)$ is an *almost para-cosymplectic manifold* after [5, p. 562].

Proposition 3.5. *Let $(M, \varphi, \xi, \eta, g)$ be an almost para-cosymplectic manifold. Then its curvature satisfies:*

$$R^G_{XYZ} = R^g_{XYZ} + \eta(X)(\nabla^g_Y\varphi)Z - \eta(Y)(\nabla^g_X\varphi)Z. \tag{3.10}$$

Let us point out an application of the formulae (3.8). Let P_0 be the projector corresponding to $\langle\xi\rangle$ in the decomposition (2.3); namely, if $X \in \mathfrak{X}(M)$ has the decomposition:

$$X = X^+ + X^- + \eta(X)\xi \tag{3.11}$$

then $P_0(X) = \eta(X)\xi$. For a general linear connection ∇ we have:

$$(\nabla_X P_0)Y = \nabla_X(\eta(Y)\xi) - \eta(\nabla_X Y)\xi = (\nabla_X\eta)(Y)\cdot\xi + \eta(Y)\nabla_X\xi \tag{3.12}$$

and then (3.8) yields:

$$\nabla^G P_0 = \nabla^g P_0. \tag{3.13}$$

If ∇^g is a natural connection we get that P_0 is covariant constant with respect to both ∇^g and ∇^G. Since the canonical connection $\tilde{\nabla}$ is natural we already have that P_0 is covariant constant with respect to $\tilde{\nabla}$. Another interesting fact is that the P_0-Golab connection i.e. with φ of (3.1) replaced by P_0, it is in fact ∇^g since $P_0 \otimes \eta - \eta \otimes P_0 = 0$.

The projector P_0 ca be used to obtain a more simple formula for the canonical connection $\tilde{\nabla}$. Plugging (3.12) in (2.14) gives:

$$\tilde{\nabla}_X Y = \nabla^g_X Y + \eta(X)\varphi Y - 2\eta(Y)\nabla^g_X \xi + (\nabla^g_X P_0)Y \tag{3.14}$$

and then, for ∇^g a natural connection we get:

$$\tilde{\nabla} = \nabla^g + \eta \otimes \varphi \tag{3.15}$$

yielding a (convex) relationship between all the linear connections studied until now:

$$\tilde{\nabla} + \nabla^G = 2\nabla^g. \tag{3.16}$$

4. Generalized duality for linear connections

Let now ∇ and ∇' be two linear connections on M. We adopt the following notion of generalized conjugation of linear connections from [2, p. 28]:

Definition 4.1. *We say that ∇ and ∇' are generalized dual connections with respect to the pair (g, η) if for any X, Y, $Z \in \mathfrak{X}(M)$:*

$$X(g(Y,Z)) = g(\nabla_X Y, Z) + g(Y, \nabla'_X Z) - \eta(X)g(Y,Z) \tag{4.1}$$

or equivalently:

$$g(\nabla'_X Z - \nabla_X Z - \eta(X)Z, Y) = \nabla g(X,Y,\xi). \tag{4.2}$$

Without the last term, the relation (4.1) reduces to the usual *Norden duality* of linear connections from [10].

We shall discuss the behavior of the generalized dual connection ∇' of ∇ if we impose certain conditions on ∇. Let us remark the following relations:

$$\begin{cases} \eta(\nabla'_X Y) = \eta(\nabla_X Y) + \eta(X)\eta(Y) + \nabla g(X,Y,\xi) \\ (\nabla'_X \eta)Y = (\nabla_X \eta)Y - \eta(X)\eta(Y) - \nabla g(X,Y,\xi) \\ g((\nabla'_X \varphi)Y, Z) = -g((\nabla_X \varphi)Z, Y). \end{cases} \tag{4.3}$$

Now, if we require the following conditions:
conditions on $\nabla\varphi$:
1) $\nabla\varphi = 0$ implies $\nabla'\varphi = 0$; 2) $\nabla\varphi = \pm\eta\otimes\varphi$ implies $\nabla'\varphi = \pm\eta\otimes\varphi$;
conditions on $\nabla\eta$:
3) $\nabla\eta = 0$ implies $\nabla'\eta = -\eta\otimes\eta - \nabla g(\cdot,\cdot,\xi)$;
4) $\nabla\eta = \eta\otimes\eta$ implies $\nabla'\eta = -\nabla g(\cdot,\cdot,\xi)$;
conditions on ∇g:
5) $\nabla g = 0$ implies $\nabla' = \nabla + \eta\otimes I$; 6) $\nabla g = \eta\otimes g$ implies $\nabla' = \nabla + 2\eta\otimes I$;
7) $\nabla g = -\eta\otimes g$ implies $\nabla' = \nabla$.

Remark 4.2. *If ∇ satisfies 5) and 6) then its generalized dual connection is equal to ∇ on $\mathcal{D}$. Also remark that if ∇ is g-metric then $\nabla_\xi\xi \in \Gamma(\mathcal{D})$ while $g(\nabla'_\xi\xi,\xi) = 1$ and $\nabla'_\xi X - \nabla_\xi X = X$ for any $X \in \mathfrak{X}(M)$.*

The generalized dual connection of the Golab connection has the following properties:

Proposition 4.3. *On the almost paracontact metric manifold (M,φ,ξ,η,g) the generalized dual connection $(\nabla^G)'$ of the Golab connection ∇^G is a quarter-symmetric connection which satisfies:*

$$g(X,(\nabla^G)'_Y\xi) = g((\nabla^G)'_X\xi,Y). \tag{4.4}$$

In the almost para-cosymplectic case $(\nabla^G)'$ has the same curvature as ∇^G.

Proof. Fix X, Y, $Z \in \mathfrak{X}(M)$; the equality (4.4) is a direct consequence of (4.1) and:
-the torsion of $(\nabla^G)'$ is $T^{G'} = \psi\otimes\eta - \eta\otimes\psi$ with $\psi := \varphi - I$.
-the curvature of $(\nabla^G)'$ is $R^{G'}(X,Y,Z) = R^G(X,Y,Z) + d\eta(X,Y)Z$. □

A straightforward computation similar to that of the end of previous Section gives: $\nabla^{G\prime}P_0 = \nabla^G P_0 (= \nabla^g P_0)$ and then a natural ∇^g yields the parallelism of P_0 with respect to all three linear connections ∇^g, ∇^G and $\nabla^{G\prime}$.

Definition 4.4. *The linear connection ∇ is called ξ-metric if: $\nabla g(\cdot,\cdot,\xi) = 0$.*

Of course, a metric linear connection is ξ-metric. Similar to the calculus of Section 3 we get that for a ξ-metric connection ∇ the curvature of the generalized dual connection ∇' is:

$$R'(X,Y,Z) = R(X,Y,Z) + 2d\eta(X,Y)Z. \tag{4.5}$$

So, in the para-cosymplectic case a ξ-metric connection has the same curvature as its generalized dual connection.

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