Sisteme de ecuaţii diferenţiale liniare de ordinul 1

Metoda matriceală de rezolvare a sistemelor diferenţiale

1. Pentru următoarele sisteme găsiţi o matrice fundamentală, matricea e^{tA}, soluţia generală şi studiaţi stabilitatea:

$a)\ x' = \begin{pmatrix} 6 & -3 \\ 2 & 1 \end{pmatrix} x, \quad b)\ x' = \begin{pmatrix} -2 & 1 \\ -4 & 3 \end{pmatrix} x, \quad c)\ x' = \begin{pmatrix} -3 & 2 \\ -1 & -1 \end{pmatrix} x,$

$d)\ x' = \begin{pmatrix} 3 & 2 & 4 \\ 2 & 0 & 2 \\ 4 & 2 & 3 \end{pmatrix} x, \quad e)\ x' = \begin{pmatrix} 7 & -1 & 6 \\ -10 & 4 & -12 \\ -2 & 1 & -1 \end{pmatrix} x, \quad f)\ x' = \begin{pmatrix} 1 & -5 & 0 \\ -1 & -3 & 0 \\ 0 & 0 & 1 \end{pmatrix} x, \quad g)\ x' = \begin{pmatrix} 1 & 0 & 0 \\ 3 & 1 & -2 \\ 2 & 2 & 1 \end{pmatrix} x$

$h)\ x' = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{pmatrix} x, \quad i)\ x' = \begin{pmatrix} 1 & 1 & 1 \\ 2 & 1 & -1 \\ -3 & 2 & 4 \end{pmatrix} x, \quad j)\ x' = \begin{pmatrix} 0 & -1 & 1 \\ 2 & -3 & 1 \\ 1 & -1 & -1 \end{pmatrix} x, \quad k)\ x' = \begin{pmatrix} -1 & -1 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -2 \end{pmatrix} x,$

$$l)\begin{cases} x' = x - y + 4z \\ y' = 3x + 2y - z \\ z' = 2x + y - z, \end{cases} \quad m)\begin{cases} x' = -7x + 6z \\ y' = 5y \\ z' = 6x + 2z, \end{cases} \quad n)\begin{cases} x' = x + z \\ y' = y - z \\ z' = -2x - z, \end{cases}$$

2. Pentru următoarele sisteme găsiţi matricea e^{tA} şi rezolvaţi problemele Cauchy:

$$a)\begin{cases} x' = \begin{pmatrix} 1 & 12 \\ 3 & 1 \end{pmatrix} x, \\ x(0) = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \end{cases} \quad b)\begin{cases} x' = \begin{pmatrix} 1 & 1 \\ 4 & 1 \end{pmatrix} x, \\ x(0) = \begin{pmatrix} 2 \\ 3 \end{pmatrix} \end{cases} \quad c)\begin{cases} x' = \begin{pmatrix} 1 & -3 \\ -2 & 2 \end{pmatrix} x, \\ x(0) = \begin{pmatrix} 0 \\ 5 \end{pmatrix} \end{cases}$$

$$d)\begin{cases} x' = \begin{pmatrix} 1 & -1 \\ 5 & -3 \end{pmatrix} x, \\ x(0) = \begin{pmatrix} 1 \\ 2 \end{pmatrix} \end{cases} \quad e)\begin{cases} x' = \begin{pmatrix} 3 & -2 \\ 4 & -1 \end{pmatrix} x, \\ x(0) = \begin{pmatrix} 1 \\ 5 \end{pmatrix} \end{cases}$$

$f)$ $\begin{cases} x' = \begin{pmatrix} 3 & 1 & -1 \\ 1 & 3 & -1 \\ 3 & 3 & -1 \end{pmatrix} x, \\ x(0) = \begin{pmatrix} 1 \\ -2 \\ -1 \end{pmatrix} \end{cases}$ $g)$ $\begin{cases} x' = \begin{pmatrix} 1 & -1 & 0 \\ 1 & 2 & 1 \\ 1 & 10 & 2 \end{pmatrix} x, \\ x(0) = \begin{pmatrix} -1 \\ -4 \\ 13 \end{pmatrix} \end{cases}$ $h)$ $\begin{cases} x' = \begin{pmatrix} 1 & -3 & 2 \\ 0 & -1 & 0 \\ 0 & -1 & -2 \end{pmatrix} x, \\ x(0) = \begin{pmatrix} -2 \\ 0 \\ 3 \end{pmatrix} \end{cases}$

$i)$ $\begin{cases} x' = \begin{pmatrix} 3 & 1 & -2 \\ -1 & 2 & 1 \\ 4 & 1 & -3 \end{pmatrix} x, \\ x(0) = \begin{pmatrix} 1 \\ 4 \\ -7 \end{pmatrix} \end{cases}$ $j)$ $\begin{cases} x' = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & 1 & 1 \end{pmatrix} x, \\ x(0) = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \end{cases}$ $k)$ $\begin{cases} x' = \begin{pmatrix} 2 & 1 & 3 \\ 0 & 2 & -1 \\ 0 & 0 & 2 \end{pmatrix} x, \\ x(0) = \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix} \end{cases}$

$l)$ $\begin{cases} x' = \begin{pmatrix} -1 & 1 & 2 \\ -1 & 1 & 1 \\ -2 & 1 & 3 \end{pmatrix} x, \\ x(0) = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} \end{cases}$ $m)$ $\begin{cases} x' = \begin{pmatrix} -4 & -4 & 0 \\ 10 & 9 & 1 \\ -4 & -3 & 1 \end{pmatrix} x, \\ x(0) = \begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix} \end{cases}$ $n)$ $\begin{cases} x' = \begin{pmatrix} 1 & 2 & -3 \\ 1 & 1 & 2 \\ 1 & -1 & 4 \end{pmatrix} x, \\ x(0) = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \end{cases}$

3. Rezolvaţi următoarele probleme Cauchy:

(a) $x' = \begin{pmatrix} 4 & 5 \\ -2 & -2 \end{pmatrix} x + \begin{pmatrix} 4e^t \cos t \\ 0 \end{pmatrix}, \quad x(0) = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

(b) $x' = \begin{pmatrix} 3 & -4 \\ 1 & -1 \end{pmatrix} x + \begin{pmatrix} e^t \\ e^t \end{pmatrix}, \quad x(0) = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

(c) $x' = \begin{pmatrix} 2 & -5 \\ 1 & -2 \end{pmatrix} x + \begin{pmatrix} \sin t \\ \tan t \end{pmatrix}, \quad x(0) = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

(d) $x' = \begin{pmatrix} 1 & 0 & 0 \\ 2 & 1 & -2 \\ 3 & 2 & 1 \end{pmatrix} x + \begin{pmatrix} 0 \\ 0 \\ e^t \cos 2t \end{pmatrix}, \quad x(0) = \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$

(e) $x' = \begin{pmatrix} 2 & 0 & 1 \\ 0 & 2 & 0 \\ 0 & 1 & 3 \end{pmatrix} x + \begin{pmatrix} e^{2t} \\ 0 \\ e^{2t} \end{pmatrix}, \quad x(0) = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$

(f) $x' = \begin{pmatrix} -1 & -1 & -2 \\ 1 & 1 & 1 \\ 2 & 1 & 3 \end{pmatrix} x + \begin{pmatrix} e^t \\ 0 \\ 0 \end{pmatrix}, \quad x(0) = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$

4. Calculaţi matricea $e^{tA} = \sum_{n=0}^{\infty} \frac{t^n}{n!} A^n$ pentru

(a) $A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{pmatrix}$

(b) $A = \begin{pmatrix} 2 & 1 & 0 \\ 0 & 2 & 1 \\ 0 & 0 & 2 \end{pmatrix}$

(c) $A = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$