

Exercise 1:

Construction of PH curves of odd degree.

Let $c_k : [0, 1/k] \rightarrow \mathbb{R}^2$

$c_k(t) = (k^2t^2 - kt, 3kt + 1)$ be a parabola.

Apply the method:

$$c'(t) = (2k^2t - k, 3k)$$

$$c'(t)^2 = (4k^4t^2 - 4k^3t - 8k^2, 6k^2(2kt - 1))$$

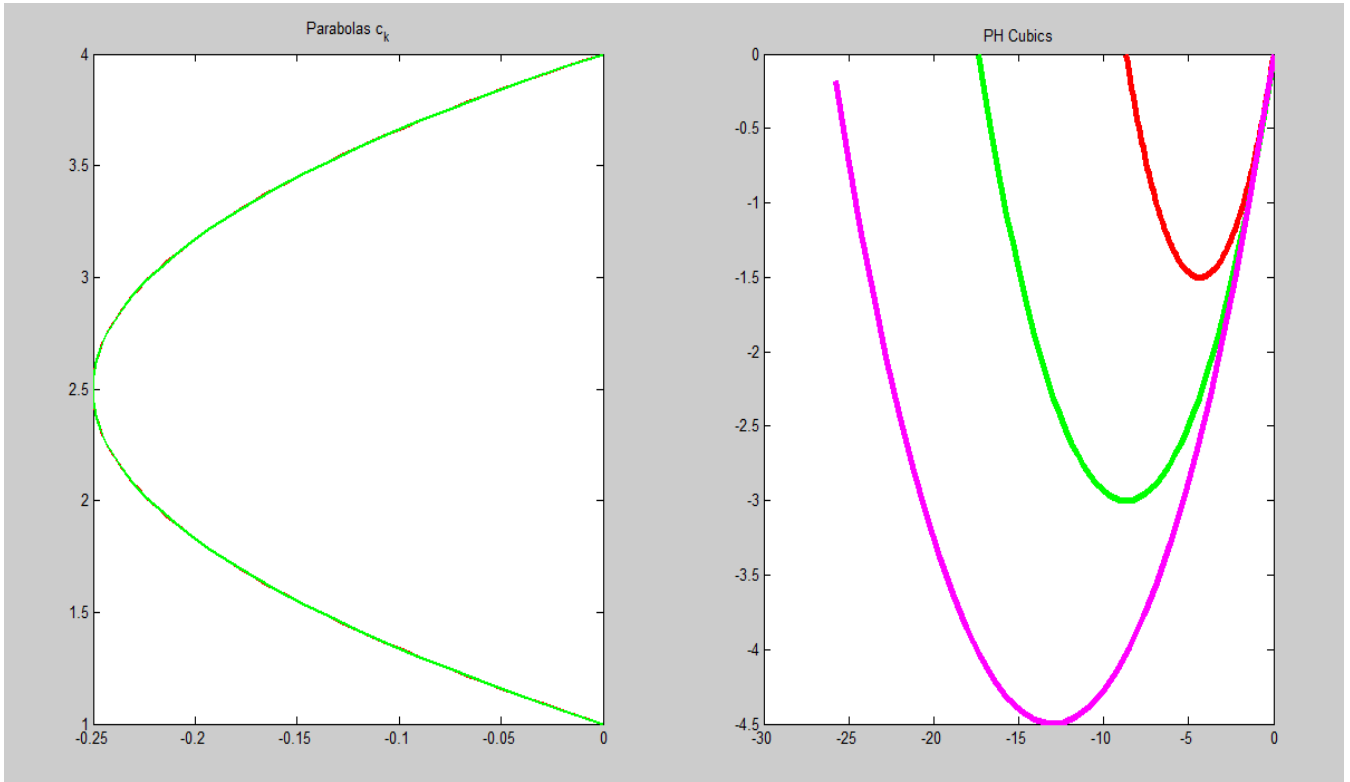
$$r(t) = (4/3t^3k^4 - 2k^3t^2 - 8k^2t, 6k^2(kt^2 - t))$$

```
clear all
k=1;
t=0:0.01:1/k;
x=k^2*t.^2-k*t;
y=3*k*t+1;
subplot(1,2,1)
plot(x,y,'r','LineWidth',2)
hold on
xx=4*k^4/3*t.^3-2*k^3*t.^2-8*k^2*t;
yy=6*k^2*(k*t.^2-t);
subplot(1,2,2)
plot(xx,yy,'r','LineWidth',4)
hold on
```

```
k=2;
t=0:0.01:1/k;
x=k^2*t.^2-k*t;
y=3*k*t+1;
subplot(1,2,1)
plot(x,y,'g','LineWidth',2)
hold on
xx=4*k^4/3*t.^3-2*k^3*t.^2-8*k^2*t;
yy=6*k^2*(k*t.^2-t);
subplot(1,2,2)
title('PH Cubics')
plot(xx,yy,'g','LineWidth',4)
hold on
```

```
k=3;
t=0:0.01:1/k;
x=k^2*t.^2-k*t;
y=3*k*t+1;
subplot(1,2,1)
title('Parabolas c_k')
%plot(x,y,'m')
hold on
xx=4*k^4/3*t.^3-2*k^3*t.^2-8*k^2*t;
yy=6*k^2*(k*t.^2-t);
```

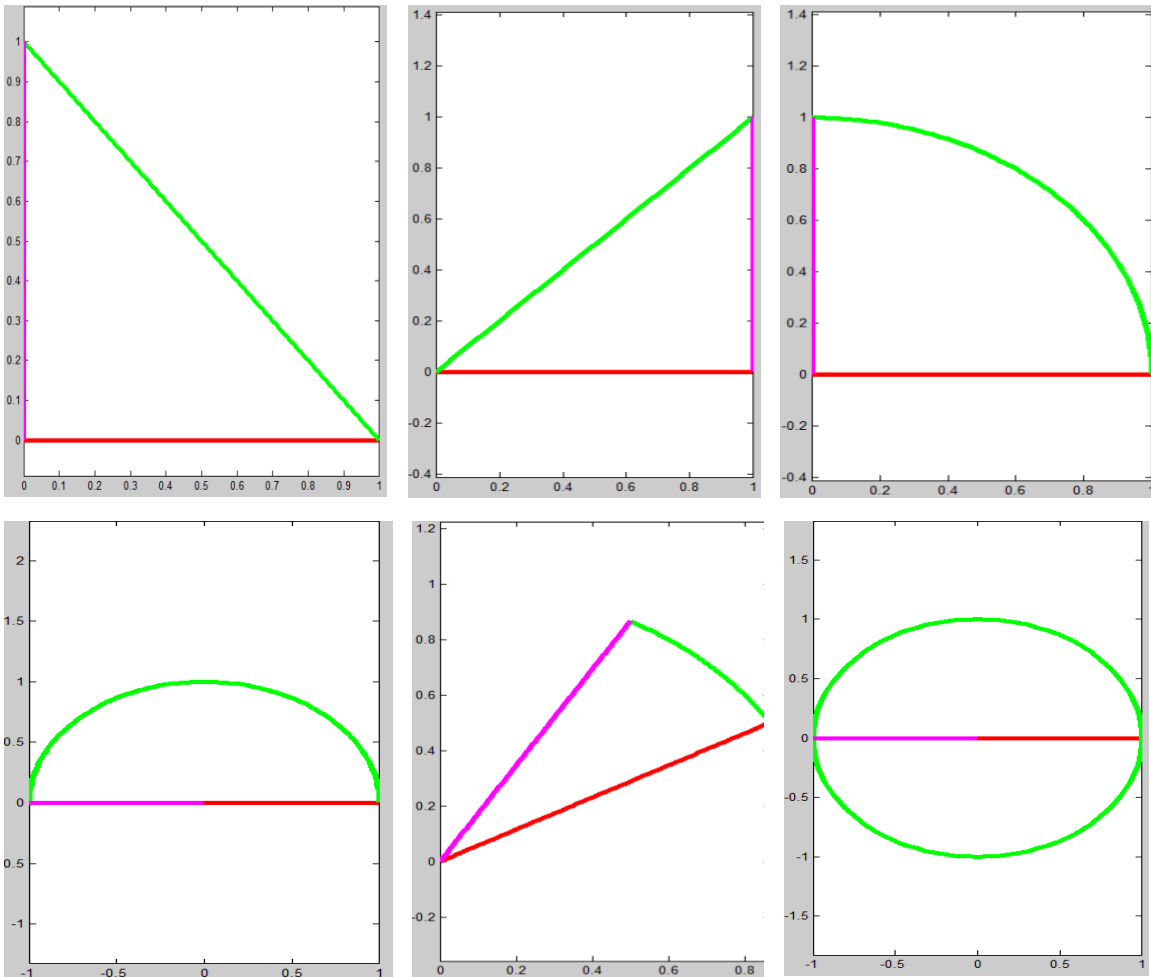
```
subplot(1,2,2)
plot(xx,yy,'m','LineWidth',4)
hold on
```



Remark: In the left picture one can note the change of the parameter $s \rightarrow ks$, which is not preserved under the transformation provided by the algorithm.

Exercise 2:

Draw the images of the following curves through the transformation $(x,y) \rightarrow (x^2 - y^2, 2xy)$:



a)

```
clear all
t=0:0.01:1;
x=t;
y=0*t;
xx=x.^2-y.^2;
yy=2*x.*y;
subplot(1,2,1)
plot(x,y,'r','LineWidth',4)
hold on
subplot(1,2,2)
plot(xx,yy,'r','LineWidth',4)
hold on

x=t;
y=1-t;
xx=x.^2-y.^2;
yy=2*x.*y;
```

```

subplot(1,2,1)
plot(x,y,'g','LineWidth',4)
hold on
subplot(1,2,2)
plot(xx,yy,'g','LineWidth',4)
hold on

```

```

x=0*t;
y=t;
xx=x.^2-y.^2;
yy=2*x.*y;
subplot(1,2,1)
plot(x,y,'m','LineWidth',4)
hold on

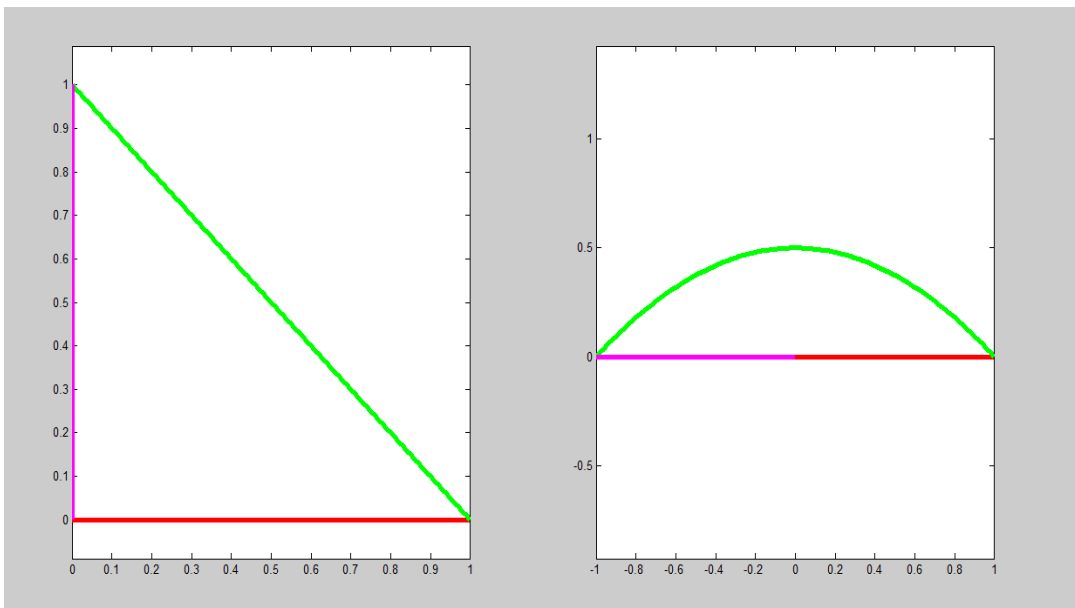
```

```

axis equal
subplot(1,2,2)
plot(xx,yy,'m','LineWidth',4)
hold on

```

```
axis equal
```



b)

```

clear all
t=0:0.01:1;
x=t;
y=0*t;
xx=x.^2-y.^2;
yy=2*x.*y;
subplot(1,2,1)
plot(x,y,'r','LineWidth',4)
hold on
subplot(1,2,2)
plot(xx,yy,'r','LineWidth',4)

```

```

hold on

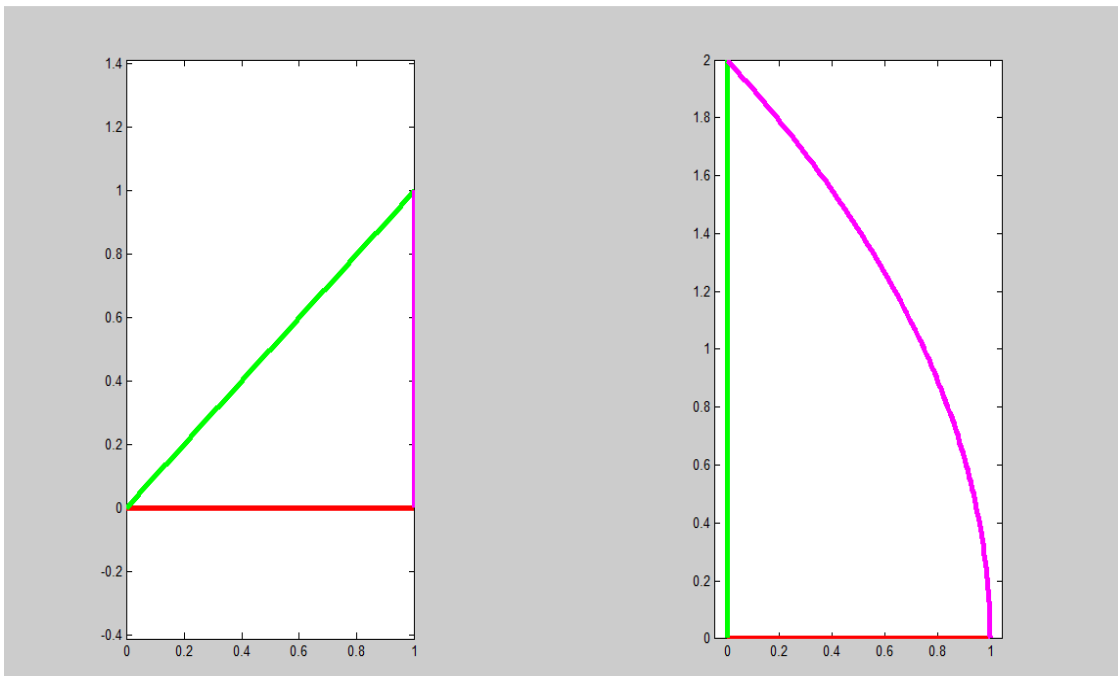
x=t;
y=t;
xx=x.^2-y.^2;
yy=2*x.*y;
subplot(1,2,1)
plot(x,y,'g','LineWidth',4)
hold on
subplot(1,2,2)
plot(xx,yy,'g','LineWidth',4)
hold on

x=1+0*t;
y=t;
xx=x.^2-y.^2;
yy=2*x.*y;
subplot(1,2,1)
plot(x,y,'m','LineWidth',4)
hold on

axis equal
subplot(1,2,2)
plot(xx,yy,'m','LineWidth',4)
hold on

axis equal

```



```

c)
clear all
t=0:0.01:1;
x=t;

```

```

y=0*t;
xx=x.^2-y.^2;
yy=2*x.*y;
subplot(1,2,1)
plot(x,y,'r','LineWidth',4)
hold on
subplot(1,2,2)
plot(xx,yy,'r','LineWidth',4)
hold on

```

```

x=cos(t*pi/2);
y=sin(t*pi/2);
xx=x.^2-y.^2;
yy=2*x.*y;
subplot(1,2,1)
plot(x,y,'g','LineWidth',4)
hold on
subplot(1,2,2)
plot(xx,yy,'g','LineWidth',4)
hold on

```

```

x=0*t;
y=t;
xx=x.^2-y.^2;
yy=2*x.*y;
subplot(1,2,1)
plot(x,y,'m','LineWidth',4)
hold on

```

```

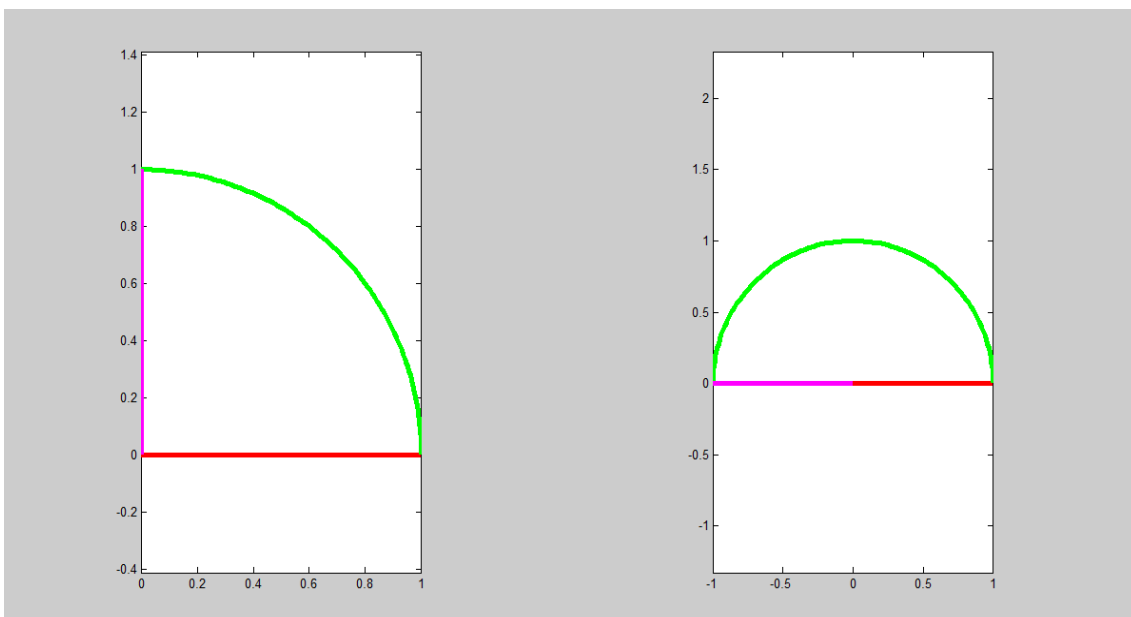
axis equal
subplot(1,2,2)
plot(xx,yy,'m','LineWidth',4)
hold on

```

```

axis equal

```



```

d)
clear all
t=0:0.01:1;

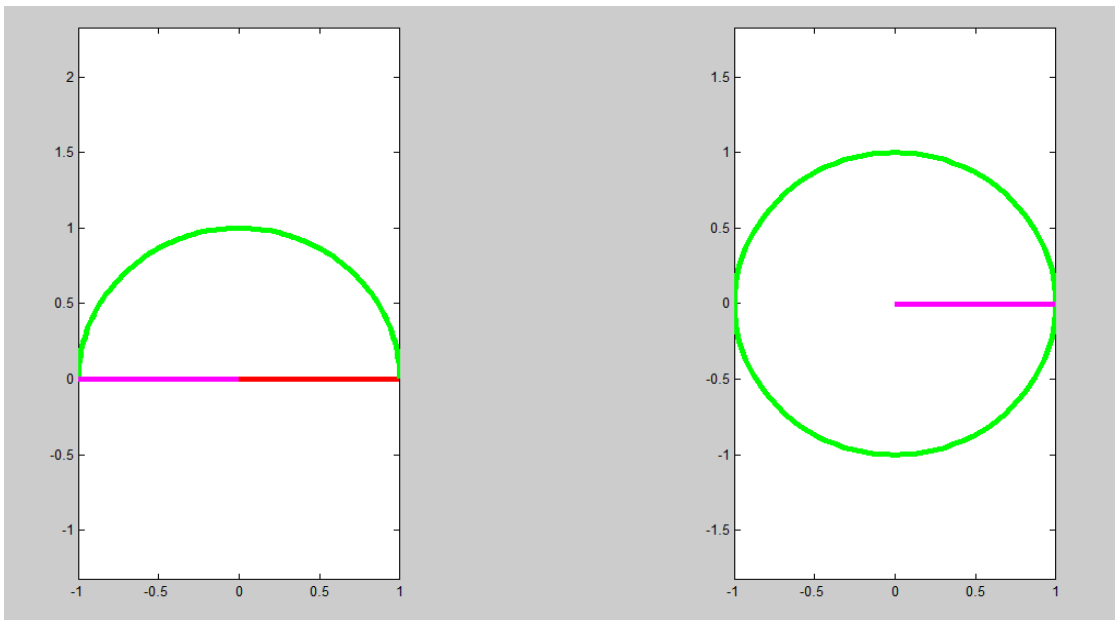
x=t;
y=0*t;
xx=x.^2-y.^2;
yy=2*x.*y;
subplot(1,2,1)
plot(x,y,'r','LineWidth',4)
hold on
subplot(1,2,2)
plot(xx,yy,'r','LineWidth',4)
hold on

x=cos(t*pi);
y=sin(t*pi);
xx=x.^2-y.^2;
yy=2*x.*y;
subplot(1,2,1)
plot(x,y,'g','LineWidth',4)
hold on
subplot(1,2,2)
plot(xx,yy,'g','LineWidth',4)
hold on

x=-t;
y=0*t;
xx=x.^2-y.^2;
yy=2*x.*y;
subplot(1,2,1)
plot(x,y,'m','LineWidth',4)
hold on
axis equal
subplot(1,2,2)
plot(xx,yy,'m','LineWidth',4)
hold on

axis equal

```



e)

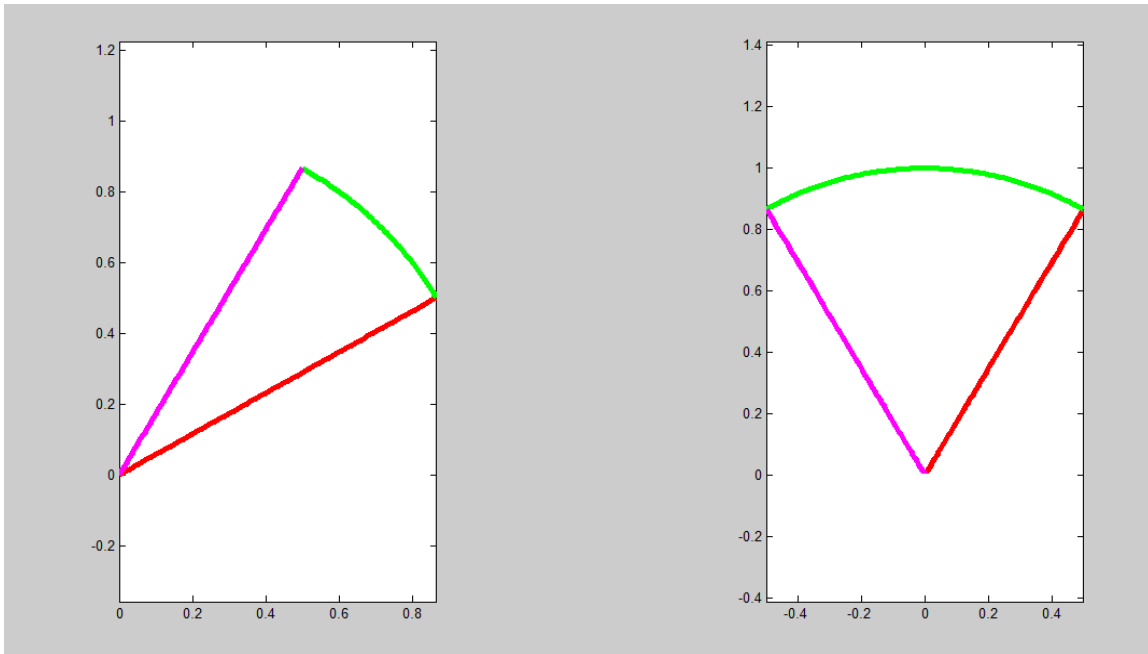
```
clear all
t=0:0.01:1;
```

```
x=sqrt(3)/2*t;
y=1/2*t;
xx=x.^2-y.^2;
yy=2*x.*y;
subplot(1,2,1)
plot(x,y,'r','LineWidth',4)
hold on
subplot(1,2,2)
plot(xx,yy,'r','LineWidth',4)
hold on
```

```
x=cos((t+1)*pi/6);
y=sin((t+1)*pi/6);
xx=x.^2-y.^2;
yy=2*x.*y;
subplot(1,2,1)
plot(x,y,'g','LineWidth',4)
hold on
subplot(1,2,2)
plot(xx,yy,'g','LineWidth',4)
hold on
```

```
x=1/2*t;
y=sqrt(3)/2*t;
xx=x.^2-y.^2;
yy=2*x.*y;
subplot(1,2,1)
plot(x,y,'m','LineWidth',4)
hold on
axis equal
subplot(1,2,2)
plot(xx,yy,'m','LineWidth',4)
hold on
```


axis equal



```
f)
clear all
t=0:0.01:1;

x=t;
y=0*t;
xx=x.^2-y.^2;
yy=2*x.*y;
subplot(1,2,1)
plot(x,y,'r','LineWidth',4)
hold on
subplot(1,2,2)
plot(xx,yy,'r','LineWidth',4)
hold on

x=cos(t*2*pi);
y=sin(t*2*pi);
xx=x.^2-y.^2;
yy=2*x.*y;
subplot(1,2,1)
plot(x,y,'g','LineWidth',4)
hold on
subplot(1,2,2)
plot(xx,yy,'g','LineWidth',4)
hold on

x=-t;
y=0*t;
xx=x.^2-y.^2;
yy=2*x.*y;
```

```
subplot(1,2,1)
plot(x,y,'m','LineWidth',4)
hold on
axis equal
subplot(1,2,2)
plot(xx,yy,'m','LineWidth',4)
hold on

axis equal
```

