Geometrie Computationala - Laborator 4

10.03.2016

1) Cu ajutorul cubicelor Bézier, sa se deseneze aproximativ, un cerc.

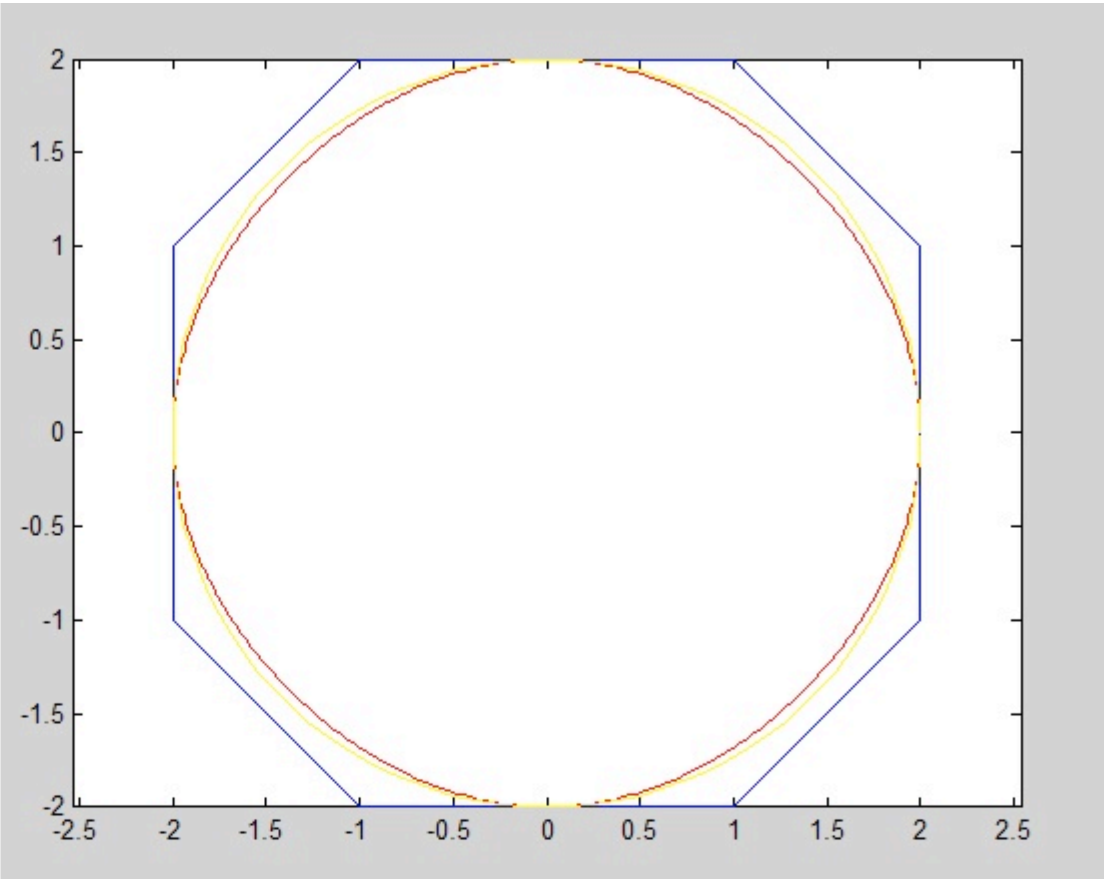
Idee: Sa se gaseasca un poligon de control, care sa determine o cubica Bézier, al carei grafic sa fie aproximativ un sfert de cerc.

Celelalte 3 arce vor fi desenate folosind proprietati de simetrie ale cercului.

Simetrii: fata de Ox: y -> -y
fata de Oy: x -> -x
fata de origine: x -> -x si y -> -y

```
clear all;
a=1;
x0=2*a; % 2a este raza cercului
y0 = 0;
x1=2*a;
y1=a;
x2=a;
y2=2*a;
x3=0;
y3=2*a;

t=0:0.01:1;
b0=(1-t).^3;
b1=3*t.*(1-t).*(1-t);
b2=3*t.*t.*(1-t);
b3=t.^3;

x=x0*b0+x1*b1+x2*b2+x3*b3;
y=y0*b0+y1*b1+y2*b2+y3*b3;
plot(x,y,'r')
hold on
plot([x0 x1 x2 x3],[y0 y1 y2 y3])
hold on

x=x0*b0+x1*b1+x2*b2+x3*b3;
y=-y0*b0-y1*b1-y2*b2-y3*b3;
plot(x,y,'r')
hold on
plot([x0 x1 x2 x3],[-y0 -y1 -y2 -y3])
hold on
x=-x0*b0-x1*b1-x2*b2-x3*b3;
y=y0*b0+y1*b1+y2*b2+y3*b3;
```

```
plot(x,y,'r')
hold on
plot([-x0 -x1 -x2 -x3],[y0 y1 y2 y3])
hold on

x=-x0*b0-x1*b1-x2*b2-x3*b3;
y=-y0*b0-y1*b1-y2*b2-y3*b3;
plot(x,y,'r')
hold on
plot([-x0 -x1 -x2 -x3],[-y0 -y1 -y2 -y3])
hold on

%pentru a compara cu un cerc adevarat:
xx=2*a*cos(2*pi*t);
yy=2*a*sin(2*pi*t);
plot(xx,yy,'y')
hold on
axis equal
```



2) Sa desenam acum o optime de cerc, dupa care sa folosim simetria fata de (prima) bisectoare pentru a desena sfertul de cerc; apoi se aplica problema precedenta.

y-y3 = -(x-x3) ecuatia tangentei in P3 (panta este -1)

P3 este punctul de intersectie al primei bisectoare cu sfertul de cerc din primul cadran:

M este punctul de intersectie al tangentei de mai sus cu dreapta x=2a (=raza) care este tangent in P0(2a,0) la cerc:

xm=2a

ym=y3 - (2a - x3)

x=y

$x^2 + y^2 = 4a^2 \Rightarrow 2x^2 = 4a^2 \Rightarrow x = a\sqrt{2} = y$

(x3 va fi x-ul de mai sus, iar y3 va fi y)

x0=2*a

y0=0

x1=(x0 + xm)/2

y1=(y0 + ym)/2

x2=(x3 + xm)/2

y2=(y3 + ym)/2

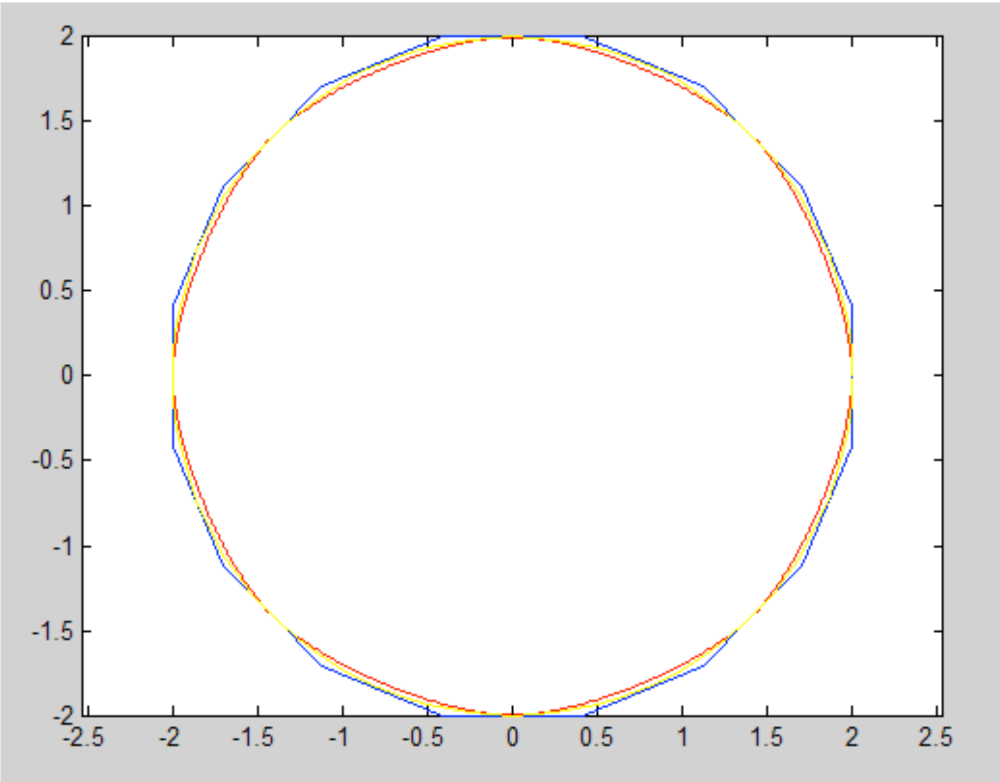
x3=a*sqrt(2)

y3=a*sqrt(2)

```
clear all;
a=1;
t=0:0.01:1;
x0=2*a;
y0=0;
x3=a*sqrt(2); %x=y ; x^2+y^2 = 4*a^2 => 2x^2=4*a^2=>x=y=a*sqrt(2)
y3=a*sqrt(2);

xm=2*a;
ym=y3-(2*a-x3);

x1=(x0+xm)/2; %mijloc
y1=(y0+ym)/2;
x2=(x3+xm)/2; %mijloc
y2=(y3+ym)/2;

b0=(1-t).^3;
b1=3*t.*(1-t).*(1-t);
```

```
b2=3*t.*t.*(1-t);
b3=t.^3;

x=x0*b0+x1*b1+x2*b2+x3*b3;
y=y0*b0+y1*b1+y2*b2+y3*b3;
plot(x,y,'r')
hold on
plot([x0 x1 x2 x3],[y0 y1 y2 y3])
hold on

x=x0*b0+x1*b1+x2*b2+x3*b3;
y=-y0*b0-y1*b1-y2*b2-y3*b3;
plot(x,y,'r')
hold on
plot([x0 x1 x2 x3],[-y0 -y1 -y2 -y3])
hold on

x=-x0*b0-x1*b1-x2*b2-x3*b3;
y=y0*b0+y1*b1+y2*b2+y3*b3;
plot(x,y,'r')
hold on
plot([-x0 -x1 -x2 -x3],[y0 y1 y2 y3])
hold on

x=-x0*b0-x1*b1-x2*b2-x3*b3;
y=-y0*b0-y1*b1-y2*b2-y3*b3;
plot(x,y,'r')
hold on
plot([-x0 -x1 -x2 -x3],[-y0 -y1 -y2 -y3])
hold on


x=x0*b0+x1*b1+x2*b2+x3*b3;
y=y0*b0+y1*b1+y2*b2+y3*b3;
plot(y,x,'r')
hold on
plot([y0 y1 y2 y3],[x0 x1 x2 x3])
hold on

x=x0*b0+x1*b1+x2*b2+x3*b3;
y=-y0*b0-y1*b1-y2*b2-y3*b3;
plot(y, x,'r')
hold on
plot([-y0 -y1 -y2 -y3],[x0 x1 x2 x3])
```

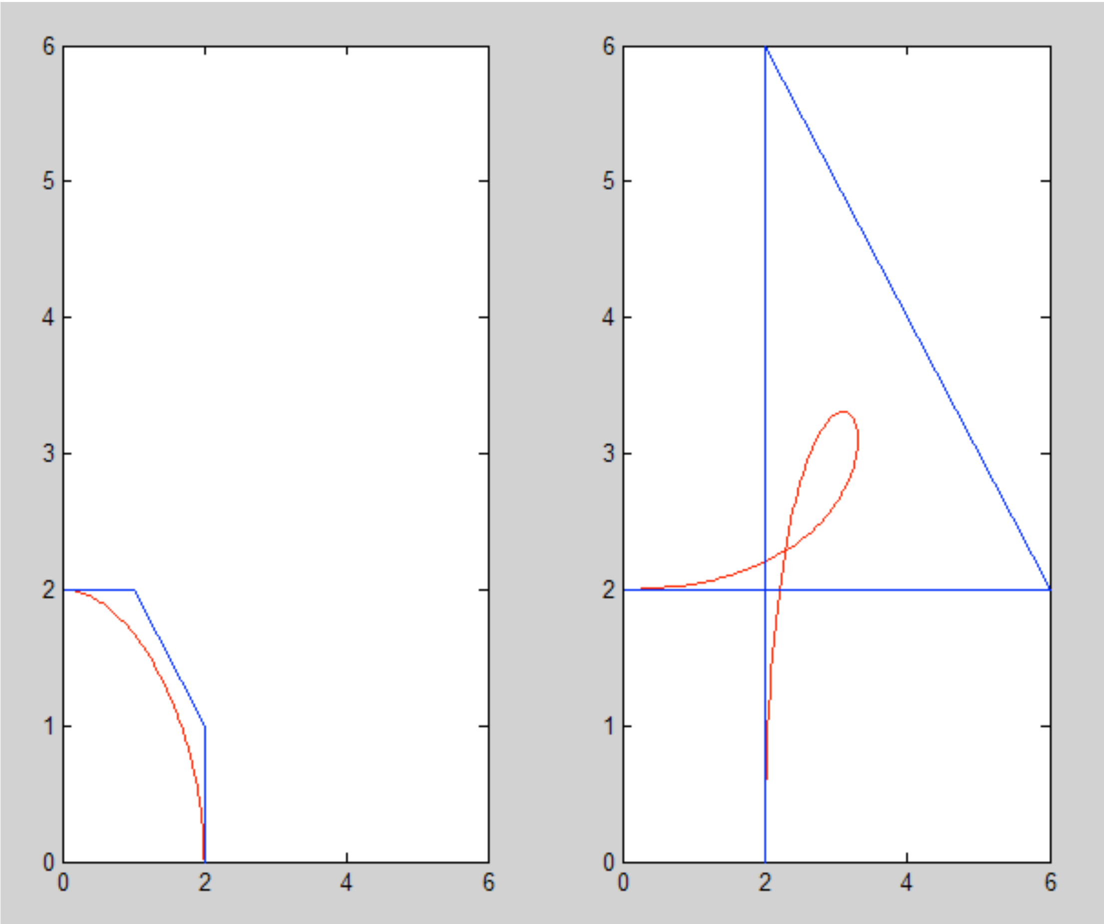
```
hold on

x=-x0*b0-x1*b1-x2*b2-x3*b3;
y=y0*b0+y1*b1+y2*b2+y3*b3;
plot(y, x,'r')
hold on
plot([y0 y1 y2 y3],[-x0 -x1 -x2 -x3])
hold on

x=-x0*b0-x1*b1-x2*b2-x3*b3;
y=-y0*b0-y1*b1-y2*b2-y3*b3;
plot(y, x,'r')
hold on
plot([-y0 -y1 -y2 -y3],[-x0 -x1 -x2 -x3])
hold on

%pentru a compara cu un cerc:
xx=2*a*cos(2*pi*t);
yy=2*a*sin(2*pi*t);
plot(xx,yy,'y')
hold on

axis equal
```



3) Avem P0, P1, P2, P3 ca in problema 1 si P`1 = (2a, 6a), P`2 = (6a, 2a)
Pentru poligoanele de control P0P1P2P3 si P0P`1P`2P3, sa se deseneze in doua subferestre (folosind subplot), curbele Bézier corespunzatoare si sa se compare.

```
clear all;
a=1;
t=0:0.01:1;

x0=2*a;
y0 = 0;
x1=2*a;
y1=a;
x2=a;
y2=2*a;
x3=0;
y3=2*a;
x1p=2*a;
y1p=6*a;
x2p=6*a;
y2p=2*a;

b0=(1-t).^3;
b1=3*t.*(1-t).*(1-t);
b2=3*t.*t.*(1-t);
b3=t.^3;

subplot(1,2,1)
x=x0*b0+x1*b1+x2*b2+x3*b3;
y=y0*b0+y1*b1+y2*b2+y3*b3;
plot(x,y,'r')
hold on
plot([x0 x1 x2 x3],[y0 y1 y2 y3])
hold on
xlim([0,6])
ylim([0,6])


subplot(1,2,2)
x=x0*b0+x1p*b1+x2p*b2+x3*b3;
y=y0*b0+y1p*b1+y2p*b2+y3*b3;
```

```
plot(x,y,'r')
hold on
plot([x0 x1p x2p x3],[y0 y1p y2p y3])
hold on
xlim([0,6])
ylim([0,6])
```



4) Fie punctele

$P_0(0, 0)$			
$P_1(0, 4)$	$P^1_0(0, 1)$		
$P_2(4, -4)$	$P^1_1(1, 2)$	$P^2_0(1/4, 5/4)$	
$P_3(8, 0)$	$P^1_2(5, -3)$	$P^2_1(2, 3/4)$	$P^3_0(11/16, 18/16)$

a) Curba Bézier pentru $\gamma, \gamma_1, \gamma_2$, unde:

γ este format din punctele de pe prima coloana

γ_1 este formata din cele 4 puncte de pe diagonala

γ_2 este formata din punctele pe ultima linie

Mai precis:

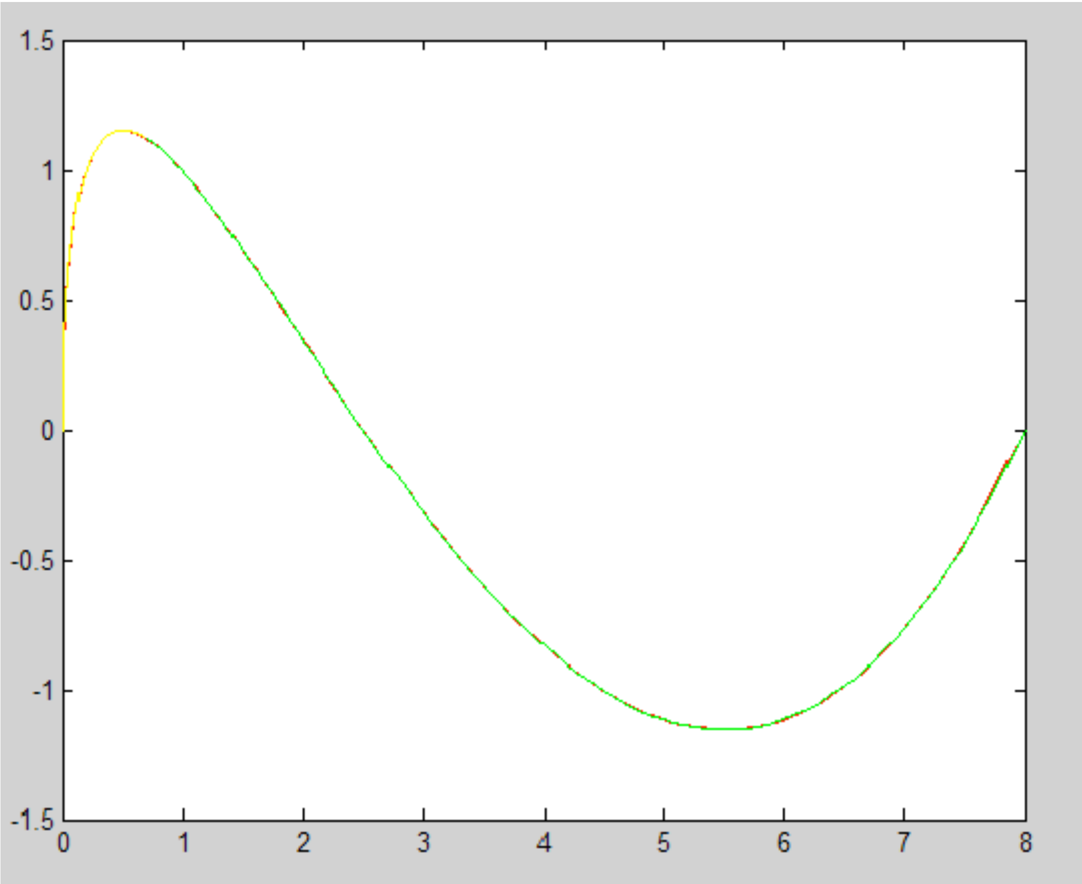
Consideram alte doua poligoane de control: $P_0P^1{}_0P^2{}_0P^3{}_0$ si $P_3P^1{}_2P^2{}_1P^3{}_0$.
Sa se reprezinte grafic cubicele Bézier corespunzatoare, in acelasi sistem de coordonate.

b) Fie $\alpha = \frac{1}{4}$. Sa se calculeze b(α) si sa se compare cu $P^3{}_0$.

a)

```
clear all;

t=0:0.01:1;

x0=0;
x1=0;
x2=4;
x3=8;
y0=0;
y1=4;
y2=-4;
y3=0;

x01=0;
x11=1;
x21=5;
y01=1;
y11=2;
y21=-3;
x02=1/4;
x12=2;
y02=5/4;
y12=3/4;
x03=11/16;
y03=18/16;

b0=(1-t).^3;
b1=3*t.*(1-t).*(1-t);
b2=3*t.*t.*(1-t);
b3=t.^3;

x=x0*b0+x1*b1+x2*b2+x3*b3;
y=y0*b0+y1*b1+y2*b2+y3*b3;
plot(x,y,'r')
hold on

x=x0*b0+x01*b1+x02*b2+x03*b3;
```

```
y=y0*b0+y01*b1+y02*b2+y03*b3;
plot(x,y,'y')
hold on

x=x3*b0+x21*b1+x12*b2+x03*b3;
y=y3*b0+y21*b1+y12*b2+y03*b3;
plot(x,y,'g')
hold on
```




```
% b) In continuarea exercitiului de mai sus, calculam curba

% Bezier in punctul α = 1/4. Adaugam codul:

clear
t=1/4

x0=0;
x1=0;
x2=4;
x3=8;
y0=0;
```

```
y1=4;
y2=-4;
y3=0;

b0=(1-t)^3;
b1=3*t*(1-t)*(1-t);
b2=3*t*t*(1-t);
b3=t^3;
x=x0*b0+x1*b1+x2*b2+x3*b3
y=y0*b0+y1*b1+y2*b2+y3*b3
```

Rezultat:
t =

0.2500

x =

0.6875

y =

1.1250

Unde x si y vor fi egali cu $x03 = \frac{11}{16} = 0.6875$ si $y03 = \frac{18}{16} = 1.1250$