–Laborator 7 –

Interpolare spline cubică. Aplicaţie

Fie punctele: $A_0(0,0,0), A_1(2,1,0), A_2(2,1,2), A_3(0,2,2), A_4(0,0,1)$ astfel încât $P_0 = A_0, P_3 = A_1, P_6 = A_2, P_9 = A_3, P_{12} = A_4$.
Să se construiască o cubică spline (formată din k=4 arce *Bézier*) care să interpoleze punctele date şi care verifică $P_1 = (1,0,0)$ şi $P_{11}(1,0,2)$.

Avem:

$$\begin{array}{cccc} P_0 & P_1 & P_2 & P_3 \\ P_3 & P_4 & P_5 & P_6 \\ P_6 & P_7 & P_8 & P_9 \\ P_9 & P_{10} & P_{11} & P_{12} \end{array}$$

C^1 : coliniaritate, mijloace;
C^2 : existenţa punctelor de Boor, coliniaritate, mijloace;
(k-1 = 3 puncte de Boor, notate d_1, d_2, d_3).

$$\Gamma : u_0 < u_1 < ... < u_k \to \mathbb{R}^3, \Delta_i = u_i - u_{i-1}, i \in \overline{1,k},$$

Γ restricţionat la $[u_{i-1}, u_i]$ va fi cubică *Bézier*.
Considerăm diviziunea echidistantă

$$u_0 = 0, u_1 = 1, \cdots, u_k = k \Rightarrow \Delta_i = 1.$$

$d_0 = P_1, \quad d_k = P_{3k-1}, \quad d_{-1} = A_0, \quad d_{k+1} = A_k \Rightarrow$ în total avem $(k+3) = 7$ puncte de Boor.

Acestea se obţin rezolvând sistemul:

$$\begin{pmatrix} 1 & 0 & 0 & 0 & \cdots & 0 & 0 & 0 \\ \alpha_1 & \beta_1 & \gamma_1 & 0 & \cdots & 0 & 0 & 0 \\ 0 & \alpha_2 & \beta_2 & \gamma 2 & \cdots & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \cdots & \alpha_{k-1} & \beta_{k-1} & \gamma_{k-1} \\ 0 & 0 & 0 & 0 & \cdots & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} d_0 \\ d_1 \\ \vdots \\ \vdots \\ \vdots \\ d_k \end{pmatrix} = \begin{pmatrix} P_1 \\ 2A_1 \\ 2A_2 \\ \vdots \\ 2A_{k-1} \\ P_{3k-1} \end{pmatrix}$$

$r_1 = (\Delta_{i-1} + \Delta_i)A_i;$

$\alpha_1 = \frac{\Delta_1^2}{\Delta_0+\Delta_1} = \frac{1}{2}; \quad \beta_1 = \frac{\Delta_0\Delta_1}{\Delta_0+\Delta_1} + \frac{\Delta_0(\Delta_1+\Delta_2)}{\Delta_0+\Delta_1+\Delta_2} = \frac{7}{6}; \quad \gamma_1 = \frac{\Delta_0^2}{\Delta_0+\Delta_1+\Delta_2} = \frac{1}{3};$

$\alpha_{k-1} = \alpha_3 = \frac{1}{3}; \quad \beta_3 = \frac{7}{6}; \quad \gamma_3 = \frac{1}{2};$

$\alpha_i = \frac{1}{3}; \quad \beta_i = \frac{4}{3}; \quad \gamma_i = \frac{1}{3};$

În cazul nostru:

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ \frac{1}{2} & \frac{7}{6} & \frac{1}{3} & 0 & 0 \\ 0 & \frac{1}{3} & \frac{4}{3} & \frac{1}{3} & 0 \\ 0 & 0 & \frac{1}{3} & \frac{7}{6} & \frac{1}{2} \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} d_0 \\ d_1 \\ d_2 \\ d_3 \\ d_4 \end{pmatrix} = \begin{pmatrix} P_1 \\ 2A_1 \\ 2A_2 \\ 2A_3 \\ P_{11} \end{pmatrix}$$

$d_0 = P_1; \quad d_4 = P_{11}$

$$\begin{cases} \frac{1}{2}d_0 + \frac{7}{6}d_1 + \frac{1}{3}d_2 = 2A_1 / \cdot 3 \\ \frac{1}{3}d_1 + \frac{4}{3}d_2 + \frac{1}{3}d_3 = 2A_2 / \cdot 3 \\ \frac{1}{3}d_2 + \frac{7}{6}d_3 + \frac{1}{2}d_4 = 2A_3 / \cdot 3 \end{cases} \Leftrightarrow \begin{cases} \frac{7}{2}d_1 + d_2 = 6A_1 - \frac{3}{2}P_1 \\ d_1 + 4d_2 + d_3 = 6A_2 \\ d_2 + \frac{7}{2}d_3 = 6A_3 - \frac{3}{2}P_{11} \end{cases}$$

Rezultă că:

$$d_1 = \frac{2}{7}\left(6A_1 - \frac{3}{2}P_1 - d_2\right)$$
$$d_3 = \frac{2}{7}\left(6A_3 - \frac{3}{2}P_{11} - d_2\right)$$

Înlocuind în a doua ecuaţie a sistemului, obţinem:

$4d_2 + \frac{2}{7}\left(6A_1 + 6A_3 - \frac{3}{2}P_1 - \frac{3}{2}P_{11} - 2d_2\right) = 6A_2$

$\Leftrightarrow \frac{24}{7}d_2 = 6\left(A_2 - \frac{2}{7}A_1 - \frac{2}{7}A_3\right) + \frac{3}{7}(P_1 + P_{11})$

$\Leftrightarrow \; d_2 = \frac{7}{4}A_2 - \frac{1}{2}A_1 - \frac{1}{2}A_3 + \frac{1}{8}P_1 + \frac{1}{8}P_{11}$

$P_2 = \frac{1}{2}d_0 + \frac{1}{2}d_1$

$P_{10} = \frac{1}{2}d_3 + \frac{1}{2}d_4$

$P_{3i-1} = \frac{1}{3}d_{i-1} + \frac{2}{3}d_i, i \in \overline{2, k-1} \Leftrightarrow i \in \{2, 3\}$

$P_{3i+1} = \frac{1}{3}d_{i+1} + \frac{2}{3}d_i, i \in \overline{1, k-2} \Leftrightarrow i \in \{1, 2\}$

$P_5 = \frac{1}{3}d_1 + \frac{2}{3}d_2, \qquad P_4 = \frac{1}{3}d_2 + \frac{2}{3}d_1$

$P_8 = \frac{1}{3}d_2 + \frac{2}{3}d_3, \qquad P_7 = \frac{1}{3}d_3 + \frac{2}{3}d_2.$

Matlab:

```
clear all
A0=[0 0 0];
A1=[2 1 0];
A2=[2 1 2];
A3=[0 2 2];
A4=[0 0 1];
P1=[1 0 0];
P11=[1 0 2];
P0=A0;
P3=A1;
d0=P1;
d4=P11;
d2=7/4*A2-1/2*A1-1/2*A3+1/8*P1+1/8*P11
d1=2/7*(6*A1-3/2*P1-d2)
P2=1/2*d0+1/2*d1;
d3=2/7*(6*A3-3/2*P11-d2)
P10=1/2*d3+1/2*d4;
P5=1/3*d1+2/3*d2;
P8=1/3*d2+2/3*d3;
P4=1/3*d2+2/3*d1;
P7=1/3*d3+2/3*d2;
P6=A2;
P9=A3;
P12=A4;

h=0.01;
t=0:h:1;
b0=(1-t).^3;
b1=3*t.*(1-t).*(1-t);
b2=3*t.*t.*(1-t);
b3=t.^3;
h=0.01;
t=0:h:1;
b0=(1-t).^3;
b1=3*t.*(1-t).*(1-t);
b2=3*t.*t.*(1-t);
b3=t.^3;
x1=P0(1)*b0+P1(1)*b1+P2(1)*b2+P3(1)*b3;
y1=P0(2)*b0+P1(2)*b1+P2(2)*b2+P3(2)*b3;
z1=P0(3)*b0+P1(3)*b1+P2(3)*b2+P3(3)*b3;
plot3(x1,y1,z1)
hold on
```

```
x2=P3(1)*b0+P4(1)*b1+P5(1)*b2+P6(1)*b3;
y2=P3(2)*b0+P4(2)*b1+P5(2)*b2+P6(2)*b3;
z2=P3(3)*b0+P4(3)*b1+P5(3)*b2+P6(3)*b3;
plot3(x2,y2,z2)
hold on
x3=P6(1)*b0+P7(1)*b1+P8(1)*b2+P9(1)*b3;
y3=P6(2)*b0+P7(2)*b1+P8(2)*b2+P9(2)*b3;
z3=P6(3)*b0+P7(3)*b1+P8(3)*b2+P9(3)*b3;
plot3(x3,y3,z3)
hold on
x4=P9(1)*b0+P10(1)*b1+P11(1)*b2+P12(1)*b3;
y4=P9(2)*b0+P10(2)*b1+P11(2)*b2+P12(2)*b3;
z4=P9(3)*b0+P10(3)*b1+P11(3)*b2+P12(3)*b3;
plot3(x4,y4,z4)
```