

Temă pentru acasă. Sume și produse

I. Elaborați câte un program pentru sumarea primilor `nr_term=100` termeni ai seriilor următoare și verificați numeric egalitățile date:

$$\frac{1}{1} + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \frac{1}{6} + \dots = +\infty; \quad (1)$$

$$\frac{1}{1} - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} - \frac{1}{6} + \dots = \ln 2; \quad (2)$$

$$\frac{1}{1} - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9} - \frac{1}{11} + \dots = \frac{\pi}{4}; \quad (3)$$

$$\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \frac{1}{5^2} + \frac{1}{6^2} + \dots = \frac{\pi^2}{6}; \quad (4)$$

$$\frac{1}{1^4} + \frac{1}{2^4} + \frac{1}{3^4} + \frac{1}{4^4} + \frac{1}{5^4} + \frac{1}{6^4} + \dots = \frac{\pi^4}{90}; \quad (5)$$

$$1 + q + q^2 + q^3 + q^4 + q^5 + \dots = \frac{1}{1-q}, \text{ pentru } q \in (-1, 1); \quad (6)$$

$$1 + \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \frac{1}{4!} + \frac{1}{5!} + \dots = e; \quad (7)$$

$$\frac{1}{2 \cdot 3 \cdot 4} - \frac{1}{4 \cdot 5 \cdot 6} + \frac{1}{6 \cdot 7 \cdot 8} - \frac{1}{8 \cdot 9 \cdot 10} + \dots = \frac{\pi - 3}{4}; \quad (8)$$

$$\sum_{i=0}^{\infty} \frac{1}{16^i} \left(\frac{4}{8i+1} - \frac{2}{8i+4} - \frac{1}{8i+5} - \frac{1}{8i+6} \right) = \pi; \quad (9)$$

II. Calculați produsul primilor `nr_fact=100` factori ai următoarelor produse infinite și verificați numeric egalitățile date:

$$\frac{2^2 - 1}{2^2 + 1} \cdot \frac{3^2 - 1}{3^2 + 1} \cdot \frac{4^2 - 1}{4^2 + 1} \cdot \frac{5^2 - 1}{5^2 + 1} \cdot \dots = \frac{\pi}{\sinh \pi}; \quad (10)$$

$$\frac{2^3 - 1}{2^3 + 1} \cdot \frac{3^3 - 1}{3^3 + 1} \cdot \frac{4^3 - 1}{4^3 + 1} \cdot \frac{5^3 - 1}{5^3 + 1} \cdot \dots = \frac{2}{3}; \quad (11)$$

$$\left(1 - \frac{1}{2^2}\right) \cdot \left(1 - \frac{1}{3^2}\right) \cdot \left(1 - \frac{1}{4^2}\right) \cdot \left(1 - \frac{1}{5^2}\right) \cdot \dots = \frac{1}{2}; \quad (12)$$

$$\left(1 + \frac{1}{2^2}\right) \cdot \left(1 + \frac{1}{3^2}\right) \cdot \left(1 + \frac{1}{4^2}\right) \cdot \left(1 + \frac{1}{5^2}\right) \cdot \dots = \frac{\sinh \pi}{2\pi}; \quad (13)$$

$$\left(1 - \frac{1}{2^3}\right) \cdot \left(1 - \frac{1}{3^3}\right) \cdot \left(1 - \frac{1}{4^3}\right) \cdot \left(1 - \frac{1}{5^3}\right) \cdots = \frac{\cosh(\pi\sqrt{3}/2)}{3\pi}; \quad (14)$$

$$\left(1 + \frac{1}{2^3}\right) \cdot \left(1 + \frac{1}{3^3}\right) \cdot \left(1 + \frac{1}{4^3}\right) \cdot \left(1 + \frac{1}{5^3}\right) \cdots = \frac{\cosh(\pi\sqrt{3}/2)}{2\pi}; \quad (15)$$

$$\frac{2}{1} \cdot \frac{2}{3} \cdot \frac{4}{3} \cdot \frac{4}{5} \cdot \frac{6}{5} \cdot \frac{6}{7} \cdot \frac{8}{7} \cdot \frac{8}{9} \cdots = \frac{\pi}{2}; \quad (16)$$

$$\frac{\sqrt{2}}{2} \cdot \frac{\sqrt{2+\sqrt{2}}}{2} \cdot \frac{\sqrt{2+\sqrt{2+\sqrt{2}}}}{2} \cdots = \frac{2}{\pi}; \quad (17)$$

III. Elaborați câte un program pentru verificarea numerică a următoarelor dezvoltări în serii și produse infinite:

$$e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \frac{x^5}{5!} + \cdots, \quad \forall x \in (-\infty, +\infty); \quad (18)$$

$$\sinh x = \frac{x}{1!} + \frac{x^3}{3!} + \frac{x^5}{5!} + \frac{x^7}{7!} + \frac{x^9}{9!} + \frac{x^{11}}{11!} + \cdots, \quad \forall x \in (-\infty, +\infty); \quad (19)$$

$$\cosh x = 1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \frac{x^6}{6!} + \frac{x^8}{8!} + \frac{x^{10}}{10!} + \cdots, \quad \forall x \in (-\infty, +\infty); \quad (20)$$

$$\sin x = \frac{x}{1!} - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \frac{x^9}{9!} - \frac{x^{11}}{11!} + \cdots, \quad \forall x \in (-\infty, +\infty); \quad (21)$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \frac{x^8}{8!} - \frac{x^{10}}{10!} + \cdots, \quad \forall x \in (-\infty, +\infty); \quad (22)$$

$$\frac{\sin x}{x} = \left(1 - \frac{x^2}{\pi^2}\right) \left(1 - \frac{x^2}{4\pi^2}\right) \left(1 - \frac{x^2}{9\pi^2}\right) \cdots, \quad x \neq 0; \quad (23)$$

$$\frac{\sin x}{x} = \cos \frac{x}{2} \cos \frac{x}{2^2} \cos \frac{x}{2^3} \cdots, \quad x \neq 0; \quad (24)$$

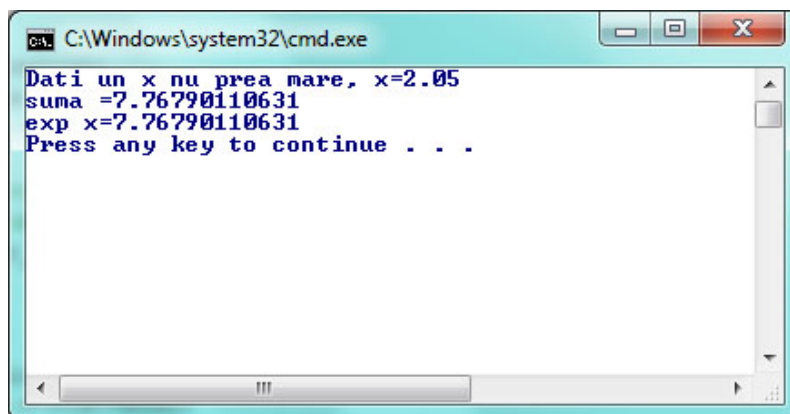
Exemplu de rezolvare:

```
// Exercițiul (18)
// Funcția exponențială

#include<iostream>
#include<math.h>
using namespace std;

int main(){
    int k,nr_term=100;
    double s,t,x;
    cout<<"Dati un x nu prea mare, x=";
    cin>>x;
    t=1.0; //termenul de sumare initial
    s=0; //suma initiala
    for(k=1;k<nr_term;k++){
        s+=t;
        t*=x/k;
    }
    cout.precision(12);
    cout<<"suma ="<<s<<endl;
    cout<<"exp x="<<exp(x)<<endl;
    return 0;
}
```

Exemplu de rulare:



```
C:\Windows\system32\cmd.exe
Dati un x nu prea mare, x=2.05
suma =7.76790110631
exp x=7.76790110631
Press any key to continue . . .
```