Tema 13. Integrale prime

Exerciţiul 1. *Să se determine integralele prime ale următoarelor sisteme diferenţiale autonome scrise sub formă simetrică:*

a) $\dfrac{dx}{y+z} = \dfrac{dy}{x+z} = \dfrac{dz}{x+y}$, $\quad x - y = C_1(y-z)$, $\quad (x+y+z)(y-z)^2 = C_2$

b) $\dfrac{dx}{y-x} = \dfrac{dy}{x+y+z} = \dfrac{dz}{x-y}$, $\quad x + z = C_1$, $\quad (x+y+z)(y-3x-z) = C_2$

c) $\dfrac{dx}{y-u} = \dfrac{dy}{z-x} = \dfrac{dz}{u-y} = \dfrac{du}{x-z}$, $\quad x + z = C_1$, $\quad y + u = C_2$, $\quad (x-z)^2 + (y-u)^2 = C_3$

d) $\dfrac{dx}{z} = \dfrac{dy}{xz} = \dfrac{dz}{y}$, $\quad x^2 - 2y = C_1$, $\quad 6xy - 2x^3 - 3z^2 = C_2$

e) $\dfrac{dx}{z^2-y^2} = \dfrac{dy}{z} = \dfrac{dz}{-y}$, $\quad y^2 + z^2 = C_1$, $\quad x - yz = C_2$

f) $\dfrac{dx}{x} = \dfrac{dy}{y} = \dfrac{dz}{xy+z}$, $\quad x = C_1 y$, $\quad xy - z = C_2 x$

g) $\dfrac{dx}{xz} = \dfrac{dy}{yz} = \dfrac{dz}{xy\sqrt{z^2+1}}$, $\quad x = C_1 y$, $\quad xy - 2\sqrt{z^2+1} = C_2$

h) $\dfrac{dx}{x+y^2+z^2} = \dfrac{dy}{y} = \dfrac{dz}{z}$, $\quad y = C_1 z$, $\quad x - y^2 - z^2 = C_2 z$

i) $\dfrac{dx}{x(y^2-z^2)} = \dfrac{dy}{-y(z^2+x^2)} = \dfrac{dz}{z(x^2+y^2)}$, $\quad x^2 + y^2 + z^2 = C_1$, $\quad yz = C_2 x$

j) $\dfrac{dx}{x(z-y)} = \dfrac{dy}{y(y-x))} = \dfrac{dz}{y^2-xz}$, $x > 0$, $\quad x - y + z = C_1$, $\quad y \ln x + z = C_2 y$

Exerciţiul 2. *Să se determine integralele prime ale următoarelor sisteme diferenţiale*

a) $\begin{cases} x' = 2y(2a-x) \\ y' = x^2 + z^2 - y^2 - 4ax \\ z' = -2yz, \quad a \in \mathbb{R}, \end{cases}$ $\quad 2a - x = C_1 z$, $\quad x^2 + y^2 + z^2 = C_2 z$

b) $\begin{cases} x' = x + y \\ y' = x - y \\ z' = \dfrac{y^2 - 2xy - x^2}{z}, \end{cases}$ $\quad x^2 + y^2 + z^2 = C_1$, $\quad y^2 + 2xy - x^2 = C_2$

c) $\begin{cases} x' = x + y - xy^2 \\ y' = x^2 y - x - y \\ z' = y^2 - x^2, \end{cases}$ $\quad x^2 + y^2 + 2z = C_1$, $\quad \ln|1 - xy| + z = C_2$