

Tema 13. Integrale prime

Exercițiul 1. Să se determine integralele prime ale următoarelor sisteme diferențiale autonome scrise sub formă simetrică:

$$a) \frac{dx}{y+z} = \frac{dy}{x+z} = \frac{dz}{x+y}, \quad \begin{aligned} x-y &= C_1(y-z) \\ (x+y+z)(y-z)^2 &= C_2 \end{aligned}$$

$$b) \frac{dx}{y-x} = \frac{dy}{x+y+z} = \frac{dz}{x-y}, \quad \begin{aligned} x+z &= C_1 \\ (x+y+z)(y-3x-z) &= C_2 \end{aligned}$$

$$c) \frac{dx}{y-u} = \frac{dy}{z-x} = \frac{dz}{u-y} = \frac{du}{x-z}, \quad \begin{aligned} x+z &= C_1 \\ y+u &= C_2 \\ (x-z)^2 + (y-u)^2 &= C_3 \end{aligned}$$

$$d) \frac{dx}{z} = \frac{dy}{xz} = \frac{dz}{y}, \quad \begin{aligned} x^2 - 2y &= C_1 \\ 6xy - 2x^3 - 3z^2 &= C_2 \end{aligned}$$

$$e) \frac{dx}{z^2 - y^2} = \frac{dy}{z} = \frac{dz}{-y}, \quad \begin{aligned} y^2 + z^2 &= C_1 \\ x - yz &= C_2 \end{aligned}$$

$$f) \frac{dx}{x} = \frac{dy}{y} = \frac{dz}{xy+z}, \quad \begin{aligned} x &= C_1 y \\ xy - z &= C_2 x \end{aligned}$$

$$g) \frac{dx}{xz} = \frac{dy}{yz} = \frac{dz}{xy\sqrt{z^2+1}}, \quad \begin{aligned} x &= C_1 y \\ xy - 2\sqrt{z^2+1} &= C_2 \end{aligned}$$

$$h) \frac{dx}{x+y^2+z^2} = \frac{dy}{y} = \frac{dz}{z}, \quad \begin{aligned} y &= C_1 z \\ x - y^2 - z^2 &= C_2 z \end{aligned}$$

$$i) \frac{dx}{x(y^2-z^2)} = \frac{dy}{-y(z^2+x^2)} = \frac{dz}{z(x^2+y^2)}, \quad \begin{aligned} x^2 + y^2 + z^2 &= C_1 \\ yz &= C_2 x \end{aligned}$$

$$j) \frac{dx}{x(z-y)} = \frac{dy}{y(y-x)} = \frac{dz}{y^2-xz}, \quad x > 0, \quad \begin{aligned} x - y + z &= C_1 \\ y \ln x + z &= C_2 y \end{aligned}$$

Exercițiul 2. Să se determine integralele prime ale următoarelor sisteme diferențiale

$$a) \begin{cases} x' = 2y(2a-x) \\ y' = x^2 + z^2 - y^2 - 4ax \\ z' = -2yz, \quad a \in \mathbb{R}, \end{cases} \quad \begin{aligned} 2a - x &= C_1 z \\ x^2 + y^2 + z^2 &= C_2 z \end{aligned}$$

$$b) \begin{cases} x' = x + y \\ y' = x - y \\ z' = \frac{y^2 - 2xy - x^2}{z}, \end{cases} \quad \begin{aligned} x^2 + y^2 + z^2 &= C_1 \\ y^2 + 2xy - x^2 &= C_2 \end{aligned}$$

$$c) \begin{cases} x' = x + y - xy^2 \\ y' = x^2 y - x - y \\ z' = y^2 - x^2, \end{cases} \quad \begin{aligned} x^2 + y^2 + 2z &= C_1 \\ \ln |1 - xy| + z &= C_2 \end{aligned}$$