Tema 14. Ecuaţii cu derivate parţiale de ordinul întâi

§1. Ecuaţii liniare

Exerciţiul 1.1 *Aflaţi soluţia generală a următoarelor ecuaţii diferenţiale liniare cu derivate parţiale de ordinul întâi:*

a) $y\frac{\partial u}{\partial x} - x\frac{\partial u}{\partial y} = 0, \qquad u = F(x^2 + y^2)$

b) $x\frac{\partial u}{\partial x} - 2y\frac{\partial u}{\partial y} - z\frac{\partial u}{\partial z} = 0, \qquad u = F(xz, x\sqrt{y})$

c) $xy\frac{\partial u}{\partial x} - y\sqrt{1-y^2}\frac{\partial u}{\partial y} + (z\sqrt{1-y^2} - xy)\frac{\partial u}{\partial z} = 0,$

$$u = F(xe^{-\arcsin y}, 2yz + x(y + \sqrt{1-y^2}))$$

d) $(x-z)\frac{\partial u}{\partial x} + (y-z)\frac{\partial u}{\partial y} + 2z\frac{\partial u}{\partial z} = 0, \qquad u = F\left(\frac{z}{(z+x)^2}, \frac{z}{(z+y)^2}\right)$

e) $(1 + \sqrt{3z-x-y})\frac{\partial u}{\partial x} + 2\frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = 0,$

$$u = F\left(y - 2z, \sqrt{3z-x-y} - \frac{z}{2}\right)$$

f) $(x_2 + x_3)\frac{\partial u}{\partial x_1} + (x_1 + x_3)\frac{\partial u}{\partial x_2} + (x_1 + x_2)\frac{\partial u}{\partial x_3} = 0,$

$$u = F\left(\frac{x_2 - x_1}{x_3 - x_1}, \frac{x_3 - x_1}{x_3 - x_2},\right)$$

g) $x_1\frac{\partial u}{\partial x_1} + x_2\frac{\partial u}{\partial x_2} + x_3\frac{\partial u}{\partial x_3} + \cdots x_n\frac{\partial u}{\partial x_n} = 0,$

$$u = F\left(\frac{x_1}{x_n}, \frac{x_2}{x_n}, \ldots, \frac{x_{n-1}}{x_n}\right)$$

§2. Ecuaţii cvasiliniare

Exerciţiul 2.1 *Aflaţi soluţia generală a următoarelor ecuaţii diferenţiale cvasiliniare cu derivate parţiale de ordinul întâi:*

a) $y\frac{\partial u}{\partial x} + x\frac{\partial u}{\partial y} = x - y, \qquad F(x^2 - y^2, x - y + u) = 0$

b) $e^x\frac{\partial u}{\partial x} + y^2\frac{\partial u}{\partial y} = ye^x, \qquad F\left(e^{-x} - \frac{1}{y}, u + \frac{x - \ln|y|}{e^{-x} - y^{-1}}\right) = 0$

c) $2x\frac{\partial u}{\partial x} + (y-x)\frac{\partial u}{\partial y} = x^2, \qquad F\left(x^2 - 4u, \frac{y-2x}{x^2}\right) = 0$

d) $xy\dfrac{\partial u}{\partial x} - x^2\dfrac{\partial u}{\partial y} = yu,$ $\qquad F\left(x^2+y^2, \dfrac{u}{x}\right) = 0$

e) $(y+z)\dfrac{\partial u}{\partial x} + (z+x)\dfrac{\partial u}{\partial y} + (x+y)\dfrac{\partial u}{\partial z} = u,$

$$F\left((x-y)u, (y-z)u, \frac{x+y+z}{u^2}\right) = 0$$

f) $x\dfrac{\partial u}{\partial x} + y\dfrac{\partial u}{\partial y} + z\dfrac{\partial u}{\partial z} = x^2 + 2u,$ $\qquad F\left(\dfrac{x}{y}, \dfrac{y}{z}, \dfrac{u}{x^2} - \ln x\right) = 0$