Seminar 7

1. Verificati care din urmatoarele aplicatii sunt morfisme de spatii liniare (operatori liniari) $T : \mathbb{R}^n \to \mathbb{R}^m$. Determinati matricele operatorilor liniari in raport cu bazele canonice ale celor doua spatii liniare. Aflati nucleul si imaginea operatorilor liniari, dimensiunile acestora si cate o baza in fiecare dintre aceste subspatii liniare. Precizati care operatori liniari sunt monomorfisme, epimorfisme si respectiv izomorfisme de spatii liniare.

   (a) $T\left(x^1, x^2, x^3\right) = \left(2x^1 + x^2, 4x^3\right)$;

   (b) $T\left(x^1, x^2, x^3\right) = \left(\left(x^1\right)^2 + x^2, 4x^3\right)$;

   (c) $T(x^1, x^2) = (2x^1 - x^2, 4x^1 + x^2)$;

   (d) $T(x^1, x^2) = (e^{x^1} + x^2, x^2)$;

   (e) $T(x^1, x^2) = (2x^1 + x^2, 3x^2, x^2 - 4x^1)$;

   (f) $T(x^1, x^2, x^3) = (x^1 - x^2 + x^3, x^1 + x^3)$;

   (g) $T(x^1, x^2) = (x^1, x^1 + 3x^2, 2x^1 - x^2, x^2)$;

   (h) $T(x^1, x^2, x^3) = (2x^1 + 5x^2, 2x^1 + 3x^3, 4x^2 - 3x^3)$.

2. Pentru urmatorii operatori liniari determinati matricele in raport cu bazele precizate:

   (a) $T\left(x^1, x^2, x^3\right) = \left(2x^1 + x^2, 4x^3\right)$, $B_1 = \{f_1 = (1,0,0), f_2 = (1,1,0), f_3 = (1,1,1)\}$, $B_2 = \{g_1 = (1,2), g_2 = (1,-1)\}$;

   (b) $T(x^1, x^2) = (2x^1 + x^2, 3x^2, x^2 - 4x^1)$, $B_1 = \{f_1 = (2,1), f_2 = (-2,3)\}$, $B_2 = \{g_1 = (1,0,1), g_2 = (1,2,3), g_3 = (0,1,-1)\}$;

   (c) $T(x^1, x^2) = (x^1, x^1 + 3x^2, 2x^1 - x^2, x^2)$, $B_1 = \{f_1 = (1,0), f_2 = (1,1)\}$, $B_2 = \{g_1 = (1,0,1,0), g_2 = (1,2,3,0), g_3 = (0,1,-1,1), g_4 = (1,1,1,1)\}$.

3. Se dau matricele unor operatori liniari in raport cu bazele canonice. Aflati nucleul si imaginea operatorilor, apoi decideti daca operatorii sunt injectivi, surjectivi sau bijectivi:

   (a) $A = \begin{pmatrix} 2 & 7 \\ 4 & 14 \end{pmatrix}$; (b) $A = \begin{pmatrix} -2 & 5 & 3 \\ -2 & 5 & 3 \\ 2 & -5 & -3 \end{pmatrix}$; (c) $A = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix}$; (d) $A = \begin{pmatrix} 1 & 1 & -1 & 1 \\ 1 & -1 & 1 & -1 \\ -3 & -1 & 1 & -1 \\ -3 & 1 & -1 & 1 \end{pmatrix}$.

4. Fie V un spatiu liniar peste $\mathbb{K}$. Determinati spectrul operatorului liniar $T : V \to V$ si subspatiile proprii corespunzatoare, daca matricea lui T in raport cu baza canonica este:

   (a) $A = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$, $\mathbb{K} = \mathbb{R}$; $A = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$, $\mathbb{K} = \mathbb{R}$; $A = \begin{pmatrix} 1 & 1 \\ 0 & i \end{pmatrix}$, $\mathbb{K} = \mathbb{C}$;

   (b) $A = \begin{pmatrix} 3 & 1 & 1 \\ 2 & 2 & 2 \\ 0 & -1 & 2 \end{pmatrix}$, $\mathbb{K} = \mathbb{R}$; $A = \begin{pmatrix} 3 & -1 & 1 \\ -1 & 5 & -1 \\ 1 & 1 & 3 \end{pmatrix}$, $\mathbb{K} = \mathbb{R}$; $A = \begin{pmatrix} 0 & -1 & -1 \\ -1 & 0 & -1 \\ -1 & -1 & 0 \end{pmatrix}$, $\mathbb{K} = \mathbb{R}$;

(c) $A = \begin{pmatrix} 1 & -2 & 0 & 0 \\ 1 & 4 & 0 & 0 \\ 3 & 6 & 2 & 2 \\ 2 & 4 & 1 & 3 \end{pmatrix}$, $\mathbb{K} = \mathbb{R}$.

5. Verificati care dintre urmatoarele matrice sunt diagonalizabile. In caz afirmativ determinati matricea diagonala asemenea cu cea data.

   (a) $A = \begin{pmatrix} 2 & 0 & -4 \\ -2 & 1 & 8 \\ 2 & 1 & 3 \end{pmatrix}$ $A = \begin{pmatrix} 5 & 4 & -4 \\ 8 & 1 & -4 \\ 16 & 8 & -11 \end{pmatrix}$ $A = \begin{pmatrix} 6 & -2 & 1 \\ -2 & 9 & -2 \\ 1 & -2 & -6 \end{pmatrix}$

   (b) $A = \begin{pmatrix} 1 & 2 & -1 \\ 2 & 0 & 2 \\ 1 & -2 & 3 \end{pmatrix}$ $A = \begin{pmatrix} 0 & -4 & -2 \\ 1 & 4 & 1 \\ 0 & 0 & 2 \end{pmatrix}$

6. Verificati ca urmatorul operator liniar este diagonalizabil. Determinati o baza in raport cu care matricea operatorului liniar sa fie diagonala

$$T : \mathbb{R}^3 \to \mathbb{R}^3,\ T(x^1, x^2, x^3) = (4x^1 + 6x^2, -3x^1 - 5x^2, -3x^1 - 6x^2 + x^3).$$

7. Fie operatorul liniar $T : \mathbb{R}^4 \to \mathbb{R}^4$,

$$T(x^1, x^2, x^3, x^4) = (3x^1 + x^2, -4x^1 - x^2, 7x^1 + x^2 + 2x^3 + x^4, -17x^1 - 6x^2 - x^3)$$

   Demonstrati ca T nu este diagonalizabil si aflati subspatiul maximal al lui $\mathbb{R}^4$ invariant prin T.