Scientific Report on the implementation of the project
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Qualitative properties of the solution set of differential inclusions

In the frame of the present project the following activities took place: documentation and research, preparation and writing scientific articles, published in ISI journals, participation at national/international conferences. The proposed objectives were complitely realized. We mention that in 2014 there have been published **13 papers (among them 11 in ISI journals)**. In following we give a presentation of the main results obtained.

The following research objectives have been realized:

Task 1: To get new invariance results.

1.1. *Invariance results for the nonlinear case in Banach spaces, under weaker hypotheses than the Lipschitz condition on the multifunction.*

In a Banach space X with uniformly convex dual, we studied the evolution inclusion $x'(t) \in Ax(t) + F(x(t))$, where A is an operator m-dissipative and F is an upper hemicontinuous multifunction with nonempty convex and weakly compact values.

In the case when F is one-sided Perron (a weaker hypothesis than the Lipschitz condition) with sublinear growth, we established sufficient conditions for the invariance of a set $K \subseteq \overline{D(A)}$. The classical concept of invariance of a set means that all the solutions of the differential inclusion considered, that start in the given set, remain in that set.

Recently, in Cârjă, Postolache (2011), the invariance problem has been studied with respect to the semilinear differential inclusion. The authors established sufficient conditions for invariance, conditiond expressed in terms of a new tangency concept, that involves integrable functions instead of vectors.

Our objective was to prove that the same tangency condition as in Cârjă, Postolache (2011) is sufficient also for the invariance of a closed set with respect to the nonlinear differential inclusion. In the following we present the invariance result obtained.

The function $f \in L^1_{loc}(R_+; X)$ is called *A-tangent to the set K in $\xi \in K$*, if

$$\liminf_{h \downarrow 0} \frac{1}{h} \operatorname{dist}(x(h, 0, \xi, f); K) = 0,$$

where we denoted by $x(h, 0, \xi, f)$ the value of the solution of the problem $x'(t) \in Ax(t) + f(t), x(0) = \xi$, calculated in h. We shall denote by $\mathfrak{F}^A_K(\xi)$ the class of all functions A-tangent to K in $\xi \in K$.

Theorem. If $K \subseteq \overline{D(A)}$ *is a nonempty closed set such that for any* $\xi \in K$ *and for any function* $f \in L^1_{loc}(R_+; X)$ *with the property that* $f(t) \in F(\xi)$, *for a.a.* t, $f \in F^A_K(\xi)$, *then* K *is invariant with respect to* $A + F$.

1.2. *Development of the invariance theory for impulsive differential inclusions in Banach spaces.*

We started the study of the impulsive differential inclusions for the finite dimensional case. We obtained approximation results for the solutions of the studied problems using Euler method and Runge–Kutta method of higher order.

Moreover, we studied fuzzy differential equations with delay, with the right hand side continuous. We established existence and uniqueness rezults and results on the continuous dependence of the initial states. Also, we considered the existence of global solutions and their stability.

Task 2: The study of the continuity properties for the solution map associated to differential inclusions.

2.1. *The continuity of the solution map under weaker hypotheses on the multifunction F.*

i) Let X be a Banach space, $A : D(A) \subseteq X \to X$ the generator of a compact C_0-semigroup, $\{S(t) : X \to X; t \geq 0\}$, which verifies $\|S(t)\| \leq e^{\omega t}$, for $\omega \in \mathbb{R}$ and any $t \geq 0$, and $F : X \rightsquigarrow X$ a nonempty convex, weakly compact valued multifunction. We considered the semilinear inclusion

$$y'(t) \in Ay(t) + F(y(t)). \tag{1}$$

There is an extensive literature regarding the solution map $\mathbb{S}$ for differential inclusions, the starting point being the results of Filippov (1964, 1967) and Plis (1965), for the case $A = 0$. There exists different hypotheses on the multifunction F: Lipschitz, one-sided Lipschitz, one-sided Perron, of continuous type.

Within this project, the basic hypothesis on F is of continuous type with a prescribed continuity modulus, as in Plis (1965). More precisely,

$$F(x_1) \subseteq F(x_2) + G(\|x_1 - x_2\|)\, B, \tag{2}$$

with $G + \omega I$ a Perron function, i.e. ac ontinuous function with $G(0) = 0$ such that the differential equation

$$z'(t) = G(z(t)) + \omega z(t) \tag{3}$$

has the null function as the unique solution with $z(0) = 0$.

In order to prove the lower semicontinuity of the multifunction $\mathbb{S}$, first of all we obtained a Filippov-Plis result, which gives approximation and stability properties of the solutions with respect to perturbations of the initial states. To this aim, we used a viability theorem from Cârjă, Necula, Vrabie (2007, 2009), where it is used a tangency condition that implies functions instead of vectors. In particular, the obtained results prove the power of the mentioned viability theorem, which, at first glance, seems difficult to apply.

In the second step, we applied a result from Hale (1969, p. 24) regarding the upper semicontinuity of the solution set for a differential equation in $\mathbb{R}$.

More precisely, we established the following regularity result.

Theorem. *Let X be a Banach space, $A : D(A) \subseteq X \to X$ the infinitezimal generator of a compact C_0-semigroup, $\{S(t) : X \to X;\ t \geq 0\}$, which satisfies for some $\omega \in \mathbb{R}$, $\|S(t)\| \leq e^{\omega t}$, for any $t \geq 0$, and $F : X \rightsquigarrow X$ is a multifunction with nonempty, weakly compact and convex values. We suppose that there exists $G : [0, \infty) \to [0, \infty)$, a continuous function, such that (2) holds, for any $x_1, x_2 \in X$. Moreover, we suppose that the function $H : [0, \infty) \to [0, \infty)$, $H(x) = G(x) + \omega x$, is a Perron function. Then, for any $\theta > 0$ and any $\varepsilon > 0$, there exists $\delta > 0$ such that, for any $x, \overline{x} \in X$, with $\|\overline{x} - x\| < \delta$, and any solution $y : [0, \sigma) \to X$, $\theta < \sigma$, of the inclusion (1), with $y(0) = x$, there exists a solution $\overline{y} : [0, \tau) \to X$ with $\overline{y}(0) = \overline{x}$ such that $\theta < \tau$ and $\|\overline{y}(t) - y(t)\| < \varepsilon$, for any $t \in [0, \theta]$.*

ii) We continued the study for the nonlinear case in a Banach space X with uniformly convex dual, considering the evolution inclusion $x'(t) \in Ax(t) + F(x(t))$, where A is an m-dissipative operator and F is a upperhemicontinuous multifunction with nonempty, convex and weakly compact values.

In the case when F is one-sided Perron (a weaker hypothesis that the Lipschitz one) with sublinear growth, we peoved the $\varepsilon - \delta$ lower semicontinuity of the solution map associated to the nonlinear differential inclusion considered.

We mention the paper of Tolstonogov, Umanskiĭ (1992), where the authors, under compactness assumptions on the operator, prove the $\varepsilon - \delta$ upper semicontinuity of the solution map.

In comparison with this result, we proved, without assuming the compactness of the operator, the $\varepsilon - \delta$ lower semicontinuity of the solution map.

Theorem. *Let $\xi \in \overline{D(A)}$, $T > 0$ and $\lambda > 0$. Then, there exists $\nu > 0$ such that, for any $\overline{\xi} \in \overline{D(A)}$ with $\|\xi - \overline{\xi}\| \leq \nu$ and any solution $x_\xi(\cdot)$ on $[0, T]$ of the considered problem with the imitial state ξ, there exists a solution $y_{\overline{\xi}}(\cdot)$ on $[0, T]$ of the considered problem with the initial state $\overline{\xi}$ such that, $\|x_\xi(t) - y_{\overline{\xi}}(t)\| \leq \lambda$, for any $t \in [0, T]$.*

2.2. *The continuity of the solution map for problems under state constraints.*

i) First, we considered the differential inclusion

$$y'(t) \in F(y(t)), \tag{4}$$

where $F : K \rightsquigarrow X$ is a nonempty valued multifunction and K is a nonempty subset of a finite dimensional space X. By solution of the differential inclusion (4) we mean an absolutely continuous function $y : [0,T] \to K$ that satisfies (4) for a.a. $t \in [0,T]$. A solution of (4) on the interval $[0,T)$ is defined similarly. For every $x \in K$, we denoted by $\mathbb{S}(x)$ the set of all the solutions of the differential inclusion (4) that start from x and we established conditions for the multifunction $\mathbb{S}$ to be lower semicontinuous.

The properties of the multifunction $\mathbb{S}$ have been studied by many authors. The first contributions in the field belong to A. Filippov (1964, 1967) and A. Plis (1965). The main hypothesis in Filippov's theorem is the Lipschitz continuity of the right hand side. There are many extensions of these results (H. Frankowska 1990, Q.J. Zhu 1991, W. Kryszewski 2003, T. Donchev, E. Farkhi 2009 etc.) Within this project we relaxed the hypotheses on the multifunction F and we proved the lower semicontinuity of $\mathbb{S}$.

In the following we present the lower semicontinuity result for $\mathbb{S}$.

Theorem. *Let X be a finite dimensional space, K a locally closed subset of X, $F : K \rightsquigarrow X$ a continuous multifunction with convex and compact values. Suppose that there exists a Perron function $G : [0,\infty) \to [0,\infty)$ such that*

$$\sup_{p\in F(x)} \inf_{q\in F(y)} [x-y, p-q]_+ \le G(\|x-y\|), \tag{5}$$

for any $x, y \in K$. Moreover, suppose that, for any $x \in K$,

$$F(x) \subseteq T_K(x). \tag{6}$$

Them for any $x_0 \in K$, for any $y_0 : [0,T] \to K$ solution of (4) with $y_0(0) = x_0$ and for any $\varepsilon > 0$ there exists $\delta > 0$ such that for any $x_1 \in K$ with $\|x_0 - x_1\| < \delta$, there exists $y_1 : [0,T] \to K$ solution of (4) with $y_1(0) = x_1$ such that $\|y_1(s) - y_0(s)\| < \varepsilon$, for any $s \in [0,T]$.

We mention that by $[x,y]_+$ we denoted the right directional derivative of the norm in x in direction y and we denoted by $T_K(\xi)$ the Bouligand tangent cone to K in $\xi \in K$.

If the set K is closed we get the continuity of the solution map with respect to Hausdorff distance.

Theorem. *Let X be a finite dimensional space, K a closed subset of X, $F : K \rightsquigarrow X$ a continuous multifunction, with convex, compact values. Suppose that there exists $G : [0,\infty) \to [0,\infty)$ a Perron function such that (5) holds, for any $x, y \in K$. Moreover, suppose that, for any $x \in K$, (6) holds.* The, for any $\varepsilon > 0$, *there exists $\delta > 0$ such that, for any $x_0, x_1 \in K$, with $\|x_0 - x_1\| < \delta$* and for any *$y_0 : [0,T] \to K$ solution of (4) with $y_0(0) = x_0$ there exists a solution $y_1 : [0,T] \to K$ of (4) with $y_1(0) = x_1$ such that $\|y_1(s) - y_0(s)\| < \varepsilon$, for any $s \in [0,T]$.*

ii) We also considered the nonautonomous version of the problem (4), i.e. the differential inclusion

$$y'(t) \in F(t, y(t)),\ y(t) \in K \subset \mathbb{R}^n,\ t \in [0,T]. \tag{7}$$

We obtained the continuity of the solution map with respect to Hausdorff distance, for this case too, supposing that the multifunction F (with compact values) is one-sided Perron and almost lower semicontinuous, the sufficient condition being expressed by the proximal normal cone of the closed set K.

Also, for the almost upper semicontinuous case, supposing moreover that F has convex values, we proved the continuity of the solution map.

2.3. *Relaxation results for differential inclusions.*

Besides the problem (7), we considered the relaxed version:

$$y'(t) \in \bigcap_{\varepsilon>0} F\left(t, y(t)+\varepsilon\mathbb{B}\right), \; y(t) \in K \subset \mathbb{R}^n, \; t \in [0,T]. \tag{8}$$

We showed that the solution set of the problem (7) is dense in the solution set of the problem (8), supposing that the multifunction F (with compact values) is one-sided Perron and almost lower semicontinuous, the sufficient condition being expressed by the proximal normal cone of the closed set K.

Task 3: **To get new results on controllability and in optimal time problem.**

3.1. *The regularity of the value function in connection with the optimality principle and the corresponding Hamilton-Jacobi-Bellman equation for evolution systems.*

i) Let $F : K \rightsquigarrow X$ be a nonempty valued multifunction, K a subset of a finite dimensional space X that contains 0 and consider the differential inclusion (4). The optimal time function associated to the differential inclusion (4), $T : K \to [0, +\infty]$, is defined by

$$T(x) = \{\inf T \geq 0; \exists y \text{ solution of } (4) \text{ with } y(t) \in K \; \forall t \in [0,T], \; y(0) = x, \; y(T) = 0\}.$$

If there is no solution from x that gets to the target then$T(x) = +\infty$. We denote by $\mathcal{R}$ the set of points $x \in K$ with the property that $T(x) < +\infty$. Consider the following hypothesis:

(H) For any $\varepsilon > 0$ there exists $\eta(\varepsilon) > 0$ such that any point $x \in X\backslash\{0\}$ with $\|x\| < \eta(\varepsilon)$ can be transferred to the origin by solutions of (4) in time $t \leq \varepsilon$. We prezent the rezult on the propagation of the continuity of the optimal time function.

Theorem. *Let X be a finite dimensional space, K a closed subset of X* with $0 \in K$, $F : X \rightsquigarrow X$ *a continuous multifunction, with convex, compact values. Suppose that there exists $G : [0,\infty) \to [0,\infty)$ a Perronfunction such that* (5) *holds for any $x, y \in K$ and, for any $x \in K$,* (6) holds. *Suppose that hypothesis (H) holds and $0 \in F(0)$. Then the reachable set $\mathcal{R}$ is open in K and the optimal time function T is locally uniformly continuous on $\mathcal{R}$.*

ii) We continued with the study of the Bellman equation for the minimal time problem associated to a semilinear evolution system, from the contingent solutions point of view. To this aim we defined new contingent derivative that involve functions instead of vectors as directions. The study is based on invariance results for appropriate differential inclusions.

Let X be a reflexiv Banach space, $A : D(A) \subseteq X \to X$ the generator of a C_0-semigroup compact, $\{S(t) : X \to X; t \geq 0\}$, with $\|S(t)\| \leq e^{\omega t}$, for some real ω and any $t \geq 0$, and $F : X \rightsquigarrow X$ a multifunction with nonempty, closed, convex and bounded values, such that $0 \in F(0)$. We considered the semilinear differential inclusion

$$y'(t) \in Ay(t) + F(y(t)) \tag{9}$$

and the associated optimal control problem, that consists in reaching the target $\{0\}$, starting from x, in minimal time $T(x)$, with the trajectories of the equation (9). We analysed the minimal time function $T(\cdot)$ with respect to the Bellman optimality principle and the corresponding Hamilton-Jacobi-Bellman equation

$$\inf_{u \in F(x)} \langle DT(x), u\rangle + \langle DT(x), Ax\rangle + 1 = 0, \tag{10}$$

on $\mathcal{R} = \mathrm{dom}(T)$.

We denote by $y_x(\cdot)$ a solution of (9) with $y(0) = x$. For any $x \in X\backslash\{0\}$ and any solution $y_x(\cdot)$ of the problem (9), denote by $\tau(x, y_x(\cdot)) = \min\{t \geq 0 : y_x(t) = 0\} \in [0,\infty]$. Define $\mathcal{R}$ as the set of all points $x \in X \setminus \{0\}$ such that $\tau(x, y_x(\cdot)) < \infty$, for a solution $y_x(\cdot)$. By the minimal time function we understand$T : \mathcal{R} \to [0,\infty)$, $T(x) = \inf\limits_{y_x(\cdot)} \tau(x, y_x(\cdot))$.

Assume the following: (H0) for any $x \in \mathcal{R}$, there exists a solution y of the problem (9) such that $y(0) = x$ and $y(T(x)) = 0$; (H1) the multifunction F is Lipschitz, i.e., there exists $L > 0$ such that $F(x) \subseteq F(y) + L\|x - y\|B$, $\forall x,\ y \in X$.

The obtained result proves that the continuity of $T(\cdot)$ around the target implies the uniform continuity on the whole reachable set.

Theorem. *Suppose* (*H*0), (*H*1) *and* (*H*). *In case* $L+\omega > 0$, *R is open and* $T(\cdot)$ *is locally uniformly continuous on R. In case* $L + \omega \leq 0$, *we have* $R = X\backslash\{0\}$ *and* $T(\cdot)$ is *uniformly continuous on R.*

Moreover, we proved that the Bellman principle together with an appropriate boundary condition uniquely define the minimal time function.

In order to properly define a solution of the equation (10), we introduce the concept of inferior derivative A-contingent:

$$\underline{D}^A V(x)(g) = \liminf_{\substack{h\downarrow 0\\ w\to 0}} \frac{V(S(h)x + \int_0^h S(h-s)g(s)ds + hw) - V(x)}{h}, \tag{11}$$

where the direction g is a function in $L^1_{\text{loc}}(\mathbf{R}_+, X)$. We define $\overline{D}^A V(x)(g) = -\underline{D}^A(-V)(x)(g)$.

Definition. A function $T(\cdot)$ is a contingent solution of the equation (10), if for any $x \in \mathcal{R}$ we have $\inf_{g(\cdot)\in F(x)_{L^1}} \underline{D}^A T(x)(g) + 1 \leq 0$ and $\inf_{g(\cdot)\in F(x)_{L^1}} \overline{D}^A T(x)(g) + 1 \geq 0$.

Theorem. *Suppose* (*H*0), (*H*1) *and* (*H*). *Then* $T(\cdot)$ *is the unique contingent continuous solution of the equation* (10) *that verifies the boundary conditions* $T[0] = 0$; $T[x] = \infty$, $\forall x \in \partial R\backslash\{0\}$.

iii) We established a propagation rezultat for the continuity of the minimal time function in connection with Bellman optimality principle.

For any $x \in X$, denote by $\mathbb{S}(x)$ the solution set for the differential inclusion (9) with $y(0) = x$.

An importantrole in geting the results on the propagation of the continuity properties of the minimal time function from around the target to the whole reachable set is played by the Lipschitz dependence of the solutions on the initial states, provided by the Lipschitz hypothesis on F. In fact, the Lipschitz continuity of the multifunction $\mathbb{S}(\cdot)$ is an important tool in getting the regularity results for the value function in optimal control problems. Within this project, we proved that the continuity around the target implies the local uniform continuity of the minimal time function on the whole reachable set, supposing weaker hypotheses on F.

The basic property of the minimal time function is Bellman optimality principle: for any $x \in \mathcal{R}$ and any solution $y(\cdot)$ of (9) that starts from x,

$$T(x) \leq t + T(y(t)),\ \forall t \in [0, \tau(x, y(\cdot))),$$

with equality in the case when $y(\cdot)$ is optimal. This property plays an important role in geting the following result on the propagation of the continuity property for $T(\cdot)$. In [1] it was proved a similar rezult supposing (H_0), (H), and the fact that F is Lipschitz. Within this research we proved that this propagation result holds under weaker hypotheses on F.

Theorem. Let X be a Banach space, $A : D(A) \subseteq X \to X$ generator of a C_0-semigroup compact, $\{S(t) : X \to X;\ t \geq 0\}$, with $\|S(t)\| \leq e^{\omega t}$, for any $t \geq 0$, and $F : X \rightsquigarrow X$ a multifunction with nonempty, weakly compact, convex values, with $0 \in F(0)$. Suppose that there exists $G : [0, \infty) \to [0, \infty)$, a continuous function such that (2) holds for any $x_1,\ x_2 \in X$ and the function $H : [0, \infty) \to [0, \infty)$, $H(x) = G(x) + \omega x$, is a Perron function. Moreover, suppose (H_0) and (H). Then $\mathcal{R}$ is an open set and $T(\cdot)$ is locally uniformly continuous on $\mathcal{R}$.

The regularity of $\mathbb{S}$ is the main tool in getting the above result.

iv) Finally, in a Banach space X with uniformly convex dual, we studied the minimal time function associated to $x'(t) \in Ax(t) + F(x(t))$, where A is an m-dissipative operator and F is an upper hemicontinuous and one-sided Perron multifunction, with nonempty, convex and weakly compact values.

Without imposing that th operator generates a compact semigroup, we showed that (H) implies the upper semicontinuity of the minimal time function on the whole domain.

Theorem. *Suppose* (H). *Then* R *is open in* $\overline{D(A)}$ *and* T *is upper semicontinuous on* R.

3.2. *The evolution of the reachable set.*

In a Banach space X with uniformly convex dual, we studied the evolution (when $t \to +\infty$) of the reachable set associated to $x'(t) \in Ax(t) + F(x(t))$, where A is an m-dissipative operator that generates an echicontinuous semigroup and F is an upper hemicontinuous multifunction, one-sided Lipschitz of negative constant, with nonempty, convexe and weakly compact values.

Let $x_0 \in \overline{D(A)}$ and $t \geq 0$. By the *reachable set* we mean:
$Reach_{x_0}(t) = \{v \in X : \exists$ o soluţie $y(\cdot)$ a problemei considerate cu data iniţială x_0 a. î. $y(t) = v\}$.

The obtained result extends the results of Donchev, Farkhi, Reich (2003, 2007), without assuming that A generates a compact semigroup.

Theorem. *The set* $Reach_{x_0}(t)$ *has limit when* $t \to +\infty$ *for any* $x_0 \in \overline{D(A)}$, *that is the unique strong atractor for the considered problem.*

During the period November 2011- December 2013 there have been published **13 articles (among them 11 in ISI journals)**, articles financially supported by the project, confirmed by the corresponding text from the Acknowledgement section:

1. Câ rjă, O., *The minimum time function for semilinear evolutions*, SIAM Journal on Control and Optimization, 2012, 50 (3), pp. 1265–1282.
2. Câ rjă, O., Lazu, A. I., *Approximate weak invariance for differential inclusions in Banach spaces*, Journal of Dynamical and Control Systems, 2012, 18 (2), pp. 215–227.
3. Câ rjă, O., Lazu, A. I., *On the regularity of the solution map for differential inclusions*, Dynamic Systems and Applications, 2012, 21 (2-3), pp. 457–465.
4. Baier, R., Din, Q., Donchev, T., *Higher order Runge-Kutta methods for impulsive differential systems*, Applied Mathematics and Computation, 2012, 218 (24), pp. 11790–11798.
5. Din, Q., Donchev, T., Kolev, D., *Filippov–Pliss lemma and m-dissipative differential inclusions*, J. Glob. Optim., 2013, 56 (4), pp. 1707–1717.
6. Câ rjă, O., Donchev, T., Postolache, V., *Nonlinear Evolution Inclusions with One-sided Perron Right-hand Side*, Journal of Dynamical and Control Systems, 2013, 18 (3), pp. 439-456.
7. Din, Q., Donchev, T., Kolev, D., *Numerical Approximations of Impulsive Delay Differential Equations*, Numerical Functional Analysis and Optimization, 2013, 34 (7), pp. 728-740.
8. Donchev, T., Lazu, A. I., Nosheen, A., *One-sided Perron Differential Inclusions*, Set-Valued Var. Anal., 2013, 21 (2), pp. 283–296.
9. Din, Q., Donchev, T., *Global character of a host-parasite model*, Chaos, Solitons & Fractals, 2013, 54, pp. 1–7.
10. Donchev, T., Nosheen, A., *Fuzzy functional differential equations under dissipative-type conditions*, Ukrainian Mathematical Journal, 2013, 65 (6), pp. 787–795.
11. Câ rjă, O., Lazu, A. I., *Lower semi-continuity of the solution set for semi-linear differential inclusions*, Journal of Mathematical Analysis and Applications, 2012, 385, pp. 865–873.
12. Donchev, T., Nosheen, A., Pecaric, J., *Hardy-Type Inequalities on Time Scale via Convexity in Several Variables*, ISRN Mathematical Analysis, Volume 2013, Article ID 903196, 9 pages, http://dx.doi.org/10.1155/2013/903196.
13. Donchev, T., Kolev, D., Nakagawa, K., *Weakened Condition for the Stability to solutions of Parabolic Equations with "Maxima"*, Journal of Prime Research in Mathematics, 2013, 9, pp. 148-158.

The research results have been presented to the following conferences and workshops: Dhahran, Arabia Saudita (December 2011); Ibn Zohr University Agadir (Aprilie 2012); University of California, Berkeley (August–Septembrie 2012); 12th Viennese Workshop on Optimal Control, Dynamic Games and Nonlinear Dynamics (Mai–Iunie 2012); Research School on Controllability of Deterministic and

Stochastic Systems and its Applications, Iaş i (June 2012); International Conference on Controlled Deterministic and Stochastic Systems, Iaş i (July 2012); Firt RoAIMS Applied and Industrial Mathematics Symposium, Iaş i (May 24–26, 2013); 9th International Conference on "Large-Scale Scientific Computations", Sozopol, (June 3–7, 2013); International Conference "Dynamical Systems: Stability, Control, Optimization"(DSSCO'13), dedicated to the 95th anniversary of Ye.A. Barbashin, Minsk (October 1–5, 2013).

Project leader,
Prof. Dr. Ovidiu CÂRJĂ