Final Activity Report

- **Project Title** Biharmonic and Constant Mean Curvature Submanifolds
- **Contract Number** 1BM / 2019
- **Final Year** 2020
- **The Lenght of The Project** 18 luni
- **Romanian Partner** Prof.dr. Tudorel Toader
- **Principal Investigator** Prof.dr. Cezar Oniciuc
- **Foreign Partner** Prof.dr. Marc Quincampoix

- **General Objectives**
  - The classification of biharmonic and biconservative hypersurfaces in space forms.
  - The construction of non-CMC complete biconservative surfaces in space forms; their uniqueness and the posibility to factorize to a torus.
  - Finding of constructive solutions to the metrizability problem.
  - Finding new characterizations of Finsler spaces of constant curvature.
  - The study of biharmonic and biconservative surfaces in complex space forms.

- **The final indicators of the project**
  - Two or three scientific papers;
  - Two short visits to France;
  - One short and one long visits to Romania.

- **Scientific and technical description of the results and the degree of the achivements**

  During this project one reached the proposed scientific objectives and the following scientific papers were elaborated:

1) Dorel Fetcu, Eric Loubeau, and Cezar Oniciuc, ***Bochner–Simons Formulas and the Rigidity of Biharmonic Submanifolds,*** **The Journal of Geometric Analysis, https://doi.org/10.1007/s12220-019-00323-y.**

2) Simona Nistor and Cezar Oniciuc, ***On the uniqueness of complete biconservative surfaces in 3-dimensional space forms,*** arXiv:1910.04131v2, **Preprint 2019**.

3) Georgeta Crețu, ***New classes of projectively related Finsler metrics of constant flag curvature, Int. J. Geom. Methods Mod. Phys.*** **17 (2020), no. 5, 2050068, 22 pp.**

4) Hiba Bibi, Eric Loubeau, and Cezar Oniciuc, ***Unique continuation property for biharmonic hypersurfaces in spheres,*** arXiv:2007.06527v1, Preprint 2020**.**

5) Hiba Bibi, Bang-Yen Chen, and Dorel Fetcu, ***Biconservative surfaces in complex space forms***, Work in progress**.**

In the first article, one classified all compact biconservative hypersurfaces of constant scalar curvature and non-negative curvature. The cornerstone of the proof was showing that such hypersurfaces have constant mean curvature (CMC) and also parallel shape operator. One obtained then a rigidity result for biharmonic hypersurfaces. In higher codimension, one studied compact biconservative submanifolds with parallel normalized mean curvature vector field (PNMC) in spheres. It was proved that, if the dimension of such a submanifold is less than or equal to 10 and their sectional curvature is non-negative, then they are CMC and, therefore PMC, and the shape operator in the direction of the mean curvature vector field is parallel. Then one applied this result to the case of biharmonic hypersurfaces to obtain a similar result.

In the second paper, one obtained important result related to the global properties of biconservative surfaces. More precisely, one constructed, from an extrinsic and also an intrinsic point of view, non-CMC complete simply connected biconservative surfaces in any 3-dimensional space form; another important problem that was also treated here is that of the uniqueness of these biconservative surfaces. It was proved that the constructed biconservative surfaces are the unique ones with these properties. When the ambient space form has non-positive curvature the non-CMC complete simply connected biconservative surfaces cannot be factorized to a torus, but only to a cylinder.

In the third paper, one defined a Weyl type curvature tensor and then it was used to characterize the Finsler metrics of constant flag curvature. We note that this tensor is projectively invariant only for those projective factors that are not Hamel functions. Taking this fact into account, one constructed new families of projectively equivalent Finsler metrics for which the property of having constant flag curvature is preserved.

In the fourth article, one studied the properties of non-minimal biharmonic in spheres. The main result is a unique continuation theorem for CMC biharmonic hypersurfaces in spheres. Next, one obtained new rigidity theorems supporting the conjecture that states that all biharmonic submanifolds in Euclidean spheres are CMC.

The fifth paper is devoted to the study of PMC surfaces in complex space forms. One finds the necessary and sufficient condition for these surfaces to be biconservative and then, using a formula for the Laplacian of the squared norm of a certain vector field, one obtains properties on the Gaussian curvature of these surfaces, their shape operator, etc. Then, one studies CMC biconservative surfaces in (real) codimension two. Finally, one intends to obtain some characterization results for biconservative submanifolds of dimension greater than 3 in complex space forms.

- **Talks during the bilateral project**

In the period October 31 -- November 3 2019 it took place the International Conference on APPLIED AND PURE MATHEMATICS in Iasi, at the Gheorghe Asachi Technical University. At this conference Dorel Fetcu gave the talk "*Bochner-Simons Formulas And The Rigidity Of Biharmonic Submanifolds*" and Georgeta Crețu, gave the talk "*A new Finslerian version of Schur's Lemma and its applications"*.

Also, Georgeta Crețu attended the conference Sesiunea Națională de Comunicări Științifice Studențești, Ediția a VI-a, July 4--7, 2019, and gave the talk "*Noi metrici Finsler de curbură constantă*".

On April 30, 2020, Simona Nistor (married Barna) gave the talk "*Suprafețe biconservative complete în spațiul hiperbolic*" at the Seminar Informal de Noutăți Geometrice (SING), Faculty of Mathematics, Alexandru Ioan Cuza University of Iasi.

- **Research visits during the project**

Between 24th September and 1st October 2019, Cezar Oniciuc (PI Romania) and Simona Nistor (married Barna) went for a short visit to Universite de Bretagne Occidentale, France.

Between 17th November – 23rd November 2019, Eric Loubeau (PI France) and Michele Benyounes went to Iași, for a short visit.

Between 15th October -- 13rd November 2020, drd. Hiba Bibi went to Iași for a research visit (Hiba Bibi is 3rd PhD Student, advised by Eric Loubeau and Cezar Oniciuc.

The proposed short visits to France of Georgeta Cretu and Dorel Fetcu could not be realized because of the restrictions imposed by the pandemic (the present lockdown in France).

- **Economical posibilities for the results of the project**

The results are theoretical, in the area of Riemannian Geometry, and they do not have economical value.

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[a] Se va scrie titlul proiectului ca în Contract
[b] Se va scrie numele și prenumele Reprezentantului legal al contractorului (conducătorul de proiect)
[c] Se va scrie numele si prenumele Directorului de proiect
[d] Se va scrie numele și prenumele Reprezentantului legal al partenerului străin