 Approximative quasisubdifferentials

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Abstract. We introduce a kind of approximative quasisubdifferential useful for the characterization of quasiconvex, lower semicontinuous functions. The relationship existing between this notion and some quasisubdifferentials known in the literature is studied.

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1 Introduction.

One of the central problems of quasiconvex analysis is to give a suitable notion of subdifferential which allows to obtain some like results to the convex analysis.

Since the first notion of quasisubdifferential introduced independently by Greenberg and Pierskalla [6] and Zabotin, Koralev and Khabibulin [17] many concepts have been proposed, each of them with specific properties (for more details see [9], [11], [16] and the references therein).

This abundance of “quasi-subdifferentials” may supply a large number of approximative quasi-subdifferentials. We do not propose a complete list here. We find in the literature the ε-quasisubdifferentials introduced using a conjugacy by Balder [1] and extended by Singer [16] by replacing the conjugacy with a general duality. Also, Singer [15] introduced other kinds of ε-quasisubdifferentials, suggested by the θ-subdifferential given in [14].

The aim of the paper is to study some notions of approximative quasisubdifferentials which allows to characterize the lower semicontinuity and quasiconvexity of a function.

The natural way for starting the study is the characterization of the quasiconvex, lower semicontinuous function by its level sets, i.e. a function is quasiconvex and lower semicontinuous if and only if its sublevel sets are closed convex sets.

On the other hand, in a locally convex space, a set is convex and closed if and only if it is an intersection of a nonempty family of closed semispaces (in the extreme case of an empty family we obtain the whole space). Similarly to the convex case, where the level sets are replaced by the epigraph of the function, the continuous linear functionals which describe the closed semi-